Topics in Asset Pricing

Assignment 4: Disagreement and short-sales constraints — To be submitted in pdf format to hombert@hec.fr) before the beginning of the class on March 23rd.

There are two dates $t = 0, 1$. There is a mass one continuum of investors who have CARA utility over date 1 consumption with risk aversion parameter γ. At date 0, investors can take positions in two assets: a risk-free asset in perfectly elastic supply yielding the risk-free rate $r_f = 0$, and a risky asset in supply $S = 1$. The risky asset trades at (endogenous) price P at date 0 and has (exogenous) payoff $V = 1 + \beta Z + \varepsilon$ at date 1, where $\beta > 0$ and $\varepsilon \sim \mathcal{N}(0, \sigma^2)$. Investors disagree on the value of Z: investor i believes that $Z = Z_i$ for sure. Z_i is uniformly distributed over $[-H, H]$ across investors, $H > 0$.

An interpretation of the risky asset's payoff structure is that β is the exposure of the risky asset to an aggregate risk factor such as the stock market return, and investors disagree about future stock market returns.

Question 1 Show that investor i's demand for the risky asset is:

$$X_i = \frac{1 + \beta Z_i - P}{\gamma \sigma^2}. \tag{1}$$

Question 2 What is the equilibrium price of the risky asset if there are no short-sale constraints?

Question 3 What is the equilibrium price if investors cannot short-sell the risky asset, i.e., investors are constrained to choosing $X_i \geq 0$? [Hint: You should find short-sales constraints bind if and only if $\beta H > \gamma \sigma^2$.]

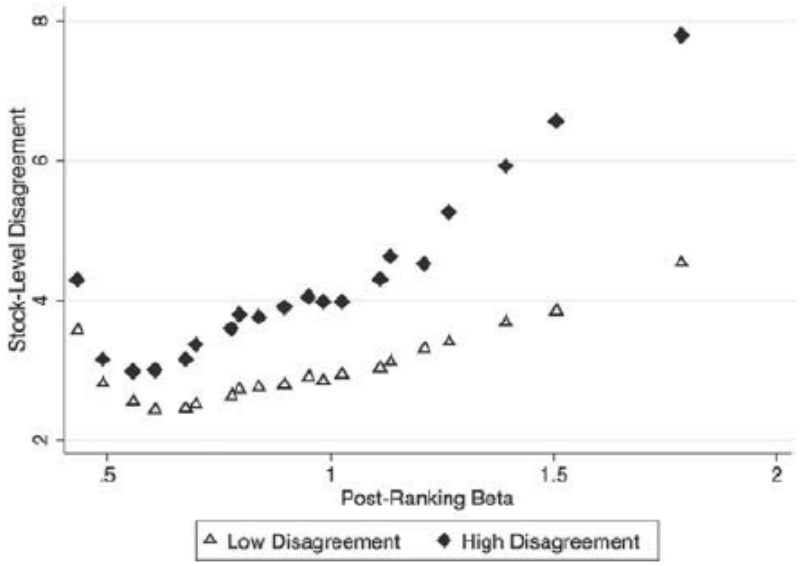
Hong and Sraer (2016) test the predictions of this model. They measure disagreement using data on analyst forecasts of EPS (earnings per share) long-term growth rate. Specifically, for each stock j and at the beginning of each month t, they calculate the dispersion of analyst forecasts $D_{j,t}$. They also calculate the market beta of each stock β_j. Then, for each month, they measure disagreement about future stock market returns $D_{m,t}$ as the beta-weighted average stock-level dispersion:

$$D_{m,t} = \frac{\sum_j \beta_j D_{j,t}}{\sum_j \beta_j}. \tag{2}$$

They classify all months in the sample period into two groups: "low disagreement" month when $D_{m,t}$ is below the sample median and "high disagreement" months when $D_{m,t}$ is above the sample median.

In Figure 1 below, they construct 20 beta-sorted stock portfolios. For each portfolio, they calculate the average stock-level disagreement $D_{j,t}$ separately in low disagreement months and in high disagreement months. The figure shows the relationship between market beta and stock-level disagreement in high vs. low disagreement months.

Figure 1

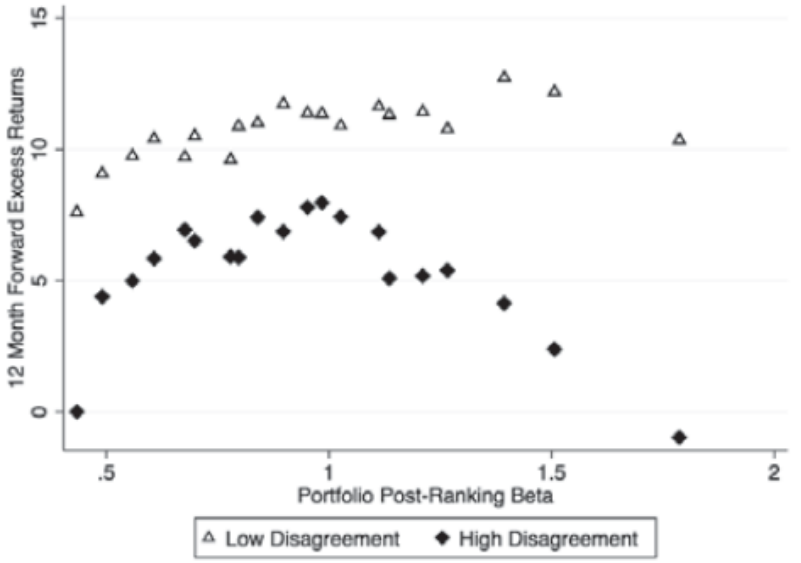


Question 4 Explain why Figure 1 is overall consistent with the model.

Question 5 Are there deviations from the predictions of the model in Figure 1? If yes, how might they be explained?

Figure 2 below shows the relationship between market beta and the average portfolio return over the next 12 months separately for high and low disagreement months.

Figure 2



Question 6 Explain why Figure 2 is overall consistent with the model. Which asset pricing puzzle might this explain?

Question 7 Are there deviations from the predictions of the model in Figure 2? If yes, how might they be explained?