Internet Appendix for "News Trading and Speed"

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This document includes supplementary material to the paper. Section I analyzes the discrete-time versions of the fast and slow speculator models, and shows that the optimal trading strategy of the speculator must be of the form assumed in the baseline continuous-time model in the paper. Section II generalizes some of the main results of the paper when data are sampled at a lower frequency than the trading frequency. Section III generalizes the baseline fast and slow speculator models by (i) relaxing the assumption that trades and news occur at the same rate, and (ii) allowing the delay with which the dealer reacts to news to be longer than one trading round. Section IV generalizes the fast speculator model by allowing the speculator to receive noisy signals about (i) the asset value increment, and (ii) the dealer's news. Section V provides a closed-form solution to the fast speculator model in the particular case when the dealer's news has no noise.

Contents

I. Discrete-Time Models

In this section, we analyze the discrete-time versions of the fast and slow speculator models introduced in Section I of the paper.

A. *Discrete-Time Fast Model*

We consider a discrete version of the fast speculator model presented in Section I of the paper. Denote by $\mathcal{I}_t^q = \{\Delta z_\tau\}_{\tau \le t-1} \cup \{\Delta y_\tau\}_{\tau \le t-1}$ the dealer's information set just before trading at t, and by $\mathcal{I}_t^p = \{\Delta z_\tau\}_{\tau \le t-1} \cup \{\Delta y_\tau\}_{\tau \le t} = \mathcal{I}_t^q \cup \{\Delta y_t\}$ the information set just after trading at t. The zero-profit condition for the competitive dealer translates into the formulas

$$q_t \;=\; \mathsf{E}(v_1 \mid \mathcal{I}_t^q), \qquad p_t \;=\; \mathsf{E}(v_1 \mid \mathcal{I}_t^p). \tag{IA.1}$$

We also define

$$\Omega_t \;=\; \mathsf{Var}(v_t \mid \mathcal{I}_t^p), \qquad \Sigma_t \;=\; \mathsf{Var}(v_t \mid \mathcal{I}_t^q). \tag{IA.2}$$

DEFINITION IA.1: *A pricing rule p_t is called linear if it is of the form $p_t = q_t + \lambda_t \Delta y_t$, for all $t = 1, \ldots, T$.*[1]

The next result shows that if the pricing rule is linear, then the speculator's strategy is also linear, and furthermore it can be decomposed into a forecast-error component, $\beta_t(v_t - q_t)\Delta t$, and a news trading component, $\tilde{\gamma}_t \Delta v_t$, where $\tilde{\gamma}_t \equiv \gamma_t - \beta_t \Delta t = \frac{\alpha_t \Lambda_t \mu_t}{\lambda_t - \alpha_t \Lambda_t^2}$ (see (IA.6)).

THEOREM IA.1: *Any equilibrium with a linear pricing rule must be of the form*

$$\begin{aligned} \Delta x_t \;&=\; \beta_t(v_{t-1} - q_t)\Delta t + \gamma_t \Delta v_t, \\ p_t \;&=\; q_t + \lambda_t \Delta y_t, \\ q_{t+1} \;&=\; p_t + \mu_t(\Delta z_t - \rho_t \Delta y_t), \end{aligned} \tag{IA.3}$$

[1] We could have defined a more general pricing rule to depend on the whole order flow history ($\{\Delta y_\tau\}_{\tau \le t}$), but as shown in Kyle (1985), there is no need for this more general specification, since in equilibrium the speculator's trades are orthogonal to the dealer's information set, and therefore the dealer's pricing rule depends only on the current order flow (Δy_t).

for $t = 1, \ldots, T$, where β_t, γ_t, λ_t, μ_t, ρ_t, Ω_t, and Σ_t are constants that satisfy

$$\begin{aligned}
\lambda_t &= \frac{\beta_t \Sigma_{t-1} + \gamma_t \sigma_v^2}{\beta_t^2 \Sigma_{t-1} \Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2}, \\
\mu_t &= \frac{\left(\sigma_u^2 + \beta_t^2 \Sigma_{t-1} \Delta t - \beta_t \gamma_t \Sigma_{t-1}\right) \sigma_v^2}{\left(\beta_t^2 \Sigma_{t-1} \Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2\right) \sigma_e^2 + \left(\beta_t^2 \Sigma_{t-1} \Delta t + \sigma_u^2\right) \sigma_v^2}, \\
\Lambda_t &= \lambda_t - \rho_t \mu_t \; = \; \frac{\beta_t \Sigma_{t-1}(\sigma_v^2 + \sigma_e^2) + \gamma_t \sigma_v^2 \sigma_e^2}{\left(\beta_t^2 \Sigma_{t-1} \Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2\right) \sigma_e^2 + \left(\beta_t^2 \Sigma_{t-1} \Delta t + \sigma_u^2\right) \sigma_v^2}, \\
\rho_t &= \frac{\gamma_t \sigma_v^2}{\beta_t^2 \Sigma_{t-1} \Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2}, \\
\Omega_t &= \Sigma_{t-1} + \sigma_v^2 \Delta t - \frac{\beta_t^2 \Sigma_{t-1}^2 + 2 \beta_t \gamma_t \Sigma_{t-1} \sigma_v^2 + \gamma_t^2 \sigma_v^4}{\beta_t^2 \Sigma_{t-1} \Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2} \, \Delta t, \\
\Sigma_t &= \Sigma_{t-1} + \sigma_v^2 \Delta t \\
&\quad - \frac{\beta_t^2 \Sigma_{t-1}^2 (\sigma_v^2 + \sigma_e^2) + \beta_t^2 \Sigma_{t-1} \Delta t \sigma_v^4 + \sigma_v^4 \sigma_u^2 + \gamma_t^2 \sigma_v^4 \sigma_e^2 + 2 \beta_t \gamma_t \Sigma_{t-1} \sigma_v^2 \sigma_e^2}{\left(\beta_t^2 \Sigma_{t-1} \Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2\right) \sigma_e^2 + \left(\beta_t^2 \Sigma_{t-1} \Delta t + \sigma_u^2\right) \sigma_v^2} \, \Delta t.
\end{aligned} \tag{IA.4}$$

The value function of the speculator is quadratic for all $t = 1, \ldots, T$:

$$\pi_t \; = \; \alpha_{t-1}(v_{t-1} - q_t)^2 + \alpha'_{t-1}(\Delta v_t)^2 + \alpha''_{t-1}(v_{t-1} - q_t)\Delta v_t + \delta_{t-1}. \tag{IA.5}$$

The coefficients of the optimal trading strategy and the value function satisfy

$$\begin{aligned}
\beta_t \Delta t &= \frac{1 - 2\alpha_t \Lambda_t}{2(\lambda_t - \alpha_t \Lambda_t^2)}, \\
\gamma_t &= \frac{1 - 2\alpha_t \Lambda_t (1 - \mu_t)}{2(\lambda_t - \alpha_t \Lambda_t^2)} \; = \; \beta_t \Delta t + \frac{\alpha_t \Lambda_t \mu_t}{\lambda_t - \alpha_t \Lambda_t^2}, \\
\alpha_{t-1} &= \beta_t \Delta t (1 - \lambda_t \beta_t \Delta t) + \alpha_t (1 - \Lambda_t \beta_t \Delta t)^2, \\
\alpha'_{t-1} &= \alpha_t (1 - \mu_t - \Lambda_t \gamma_t)^2 + \gamma_t (1 - \lambda_t \gamma_t), \\
\alpha''_{t-1} &= \beta_t \Delta t + \gamma_t (1 - 2\lambda_t \beta_t \Delta t) + 2\alpha_t (1 - \Lambda_t \beta_t \Delta t)(1 - \mu_t - \Lambda_t \gamma_t), \\
\delta_{t-1} &= \alpha_t \left(\Lambda_t^2 \sigma_u^2 + \mu_t^2 \sigma_e^2\right) \Delta t + \alpha'_t \sigma_v^2 \Delta t + \delta_t.
\end{aligned} \tag{IA.6}$$

The terminal conditions are

$$\alpha_T \; = \; \alpha'_T \; = \; \alpha''_T \; = \; \delta_T \; = \; 0. \tag{IA.7}$$

The second order condition is

$$\lambda_t - \alpha_t \Lambda_t^2 \; > \; 0. \tag{IA.8}$$

Given Σ_0, conditions (IA.4) to (IA.8) are necessary and sufficient for the existence of a linear equilibrium.

Proof. First, we show that equations (IA.4) are equivalent to the zero-profit condition of the dealer. Second, we show that equations (IA.6) to (IA.8) are equivalent to the speculator's strategy in (IA.3) being optimal. These two steps prove that equations (IA.3) to (IA.8) describe an equilibrium. Conversely, all equilibria with a linear pricing rule must satisfy these equations since the trading strategy in (IA.3) is the best response to the linear pricing rule.

Zero Profit of Dealer: Start with the dealer's update due to the order flow at $t = 1, \ldots, T$. Conditional on $\mathcal{I}_t^q$, the variables $v_{t-1} - q_t$ and Δv_t have a bivariate normal distribution:

$$\begin{bmatrix} v_{t-1} - q_t \\ \Delta v_t \end{bmatrix} \mid \mathcal{I}_{t-1}^q \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{t-1} & 0 \\ 0 & \sigma_v^2 \end{bmatrix} \right). \tag{IA.9}$$

The aggregate order flow at t is of the form

$$\Delta y_t = \beta_t (v_{t-1} - q_t)\Delta t + \gamma_t \Delta v_t + \Delta u_t. \tag{IA.10}$$

Let

$$\Phi_t = \mathsf{Cov}\left(\begin{bmatrix} v_{t-1} - q_t \\ \Delta v_t \end{bmatrix}, \Delta y_t \right) = \begin{bmatrix} \beta_t \Sigma_{t-1} \\ \gamma_t \sigma_v^2 \end{bmatrix} \Delta t. \tag{IA.11}$$

Then, conditional on $\mathcal{I}_t^p = \mathcal{I}_t^q \cup \{\Delta y_t\}$, the joint distribution of $v_{t-1} - q_t$ and Δv_t is bivariate normal,

$$\begin{bmatrix} v_{t-1} - q_t \\ \Delta v_t \end{bmatrix} \mid \mathcal{I}_t^p \sim \mathcal{N}\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \right), \tag{IA.12}$$

where

$$\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \Phi_t \, \mathsf{Var}(\Delta y_t)^{-1} \Delta y_t = \begin{bmatrix} \beta_t \Sigma_{t-1} \\ \gamma_t \sigma_v^2 \end{bmatrix} \frac{1}{\beta_t^2 \Sigma_{t-1} \Delta t + \gamma_t^2 \sigma_v^2 + \sigma_u^2} \Delta y_t, \tag{IA.13}$$

and

$$\begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} = \mathsf{Var}\left(\begin{bmatrix} v_{t-1}-q_t \\ \Delta v_t \end{bmatrix}\right) - \Phi_t \,\mathsf{Var}(\Delta y_t)^{-1}\Phi_t'$$
$$= \begin{bmatrix} \Sigma_{t-1} & 0 \\ 0 & \sigma_v^2\Delta t \end{bmatrix} - \frac{1}{\beta_t^2\Sigma_{t-1}\Delta t + \gamma_t^2\sigma_v^2 + \sigma_u^2}\begin{bmatrix} \beta_t^2\Sigma_{t-1}^2 & \beta_t\gamma_t\Sigma_{t-1}\sigma_v^2 \\ \beta_t\gamma_t\Sigma_{t-1}\sigma_v^2 & \gamma_t^2\sigma_v^4 \end{bmatrix}\Delta t. \tag{IA.14}$$

We compute

$$p_t - q_t = \mathsf{E}(v_t - q_t \mid \mathcal{I}_t^p) = \mu_1 + \mu_2 = \frac{\beta_t\Sigma_{t-1} + \gamma_t\sigma_v^2}{\beta_t^2\Sigma_{t-1}\Delta t + \gamma_t^2\sigma_v^2 + \sigma_u^2}\,\Delta y_t, \tag{IA.15}$$

which proves equation (IA.4) for λ_t. Also,

$$\begin{aligned} \Omega_t &= \mathsf{Var}(v_t - q_t \mid \mathcal{I}_t^p) = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2 \\ &= \Sigma_{t-1} + \sigma_v^2\Delta t - \frac{\beta_t^2\Sigma_{t-1}^2 + 2\beta_t\gamma_t\Sigma_{t-1}\sigma_v^2 + \gamma_t^2\sigma_v^4}{\beta_t^2\Sigma_{t-1}\Delta t + \gamma_t^2\sigma_v^2 + \sigma_u^2}\,\Delta t, \end{aligned} \tag{IA.16}$$

which proves the formula for Ω_t.

Next, to compute $q_{t+1} = \mathsf{E}(v_t \mid \mathcal{I}_{t+1}^q)$, we start from the same prior as in (IA.9), but we consider the impact of both the order flow at t and the dealer's signal at $t+1$:

$$\begin{aligned} \Delta y_t &= \beta_t(v_{t-1} - q_t)\Delta t + \gamma_t\Delta v_t + \Delta u_t, \\ \Delta z_t &= \Delta v_t + \Delta e_t. \end{aligned} \tag{IA.17}$$

Let

$$\begin{aligned} \Psi_t &= \mathsf{Cov}\left(\begin{bmatrix} v_{t-1}-q_t \\ \Delta v_t \end{bmatrix}, \begin{bmatrix} \Delta y_t \\ \Delta z_t \end{bmatrix}\right) = \begin{bmatrix} \beta_t\Sigma_{t-1} & 0 \\ \gamma_t\sigma_v^2 & \sigma_v^2 \end{bmatrix}\Delta t, \\ V_t^{yz} &= \mathsf{Var}\left(\begin{bmatrix} \Delta y_t \\ \Delta z_t \end{bmatrix}\right) = \begin{bmatrix} \beta_t^2\Sigma_{t-1}\Delta t + \gamma_t^2\sigma_v^2 + \sigma_u^2 & \gamma_t\sigma_v^2 \\ \gamma_t\sigma_v^2 & \sigma_v^2 + \sigma_e^2 \end{bmatrix}\Delta t. \end{aligned} \tag{IA.18}$$

Conditional on $\mathcal{I}_{t+1}^q = \mathcal{I}_t^q \cup \{\Delta y_t, \Delta z_t\}$, the joint distribution of $v_{t-1} - q_t$ and Δv_t is bivariate normal,

$$\begin{bmatrix} v_{t-1}-q_t \\ \Delta v_t \end{bmatrix} \mid \mathcal{I}_{t+1}^q \sim \mathcal{N}\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}\right), \tag{IA.19}$$

where

$$\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \Psi_t \, (V_t^{yz})^{-1} \begin{bmatrix} \Delta y_t \\ \Delta z_t \end{bmatrix}$$
$$= \frac{\begin{bmatrix} \beta_t \Sigma_{t-1}(\sigma_v^2 + \sigma_e^2)\Delta y_t \; - \; \beta_t\gamma_t\Sigma_{t-1}\sigma_v^2 \Delta z_t \\ \gamma_t \sigma_v^2 \sigma_e^2 \Delta y_t \; + \; (\beta_t^2 \Sigma_{t-1}\Delta t + \sigma_u^2)\sigma_v^2 \Delta z_t \end{bmatrix}}{(\beta_t^2\Sigma_{t-1}\Delta t + \gamma_t^2\sigma_v^2 + \sigma_u^2)\sigma_e^2 + (\beta_t^2 \Sigma_{t-1}\Delta t + \sigma_u^2)\sigma_v^2}, \tag{IA.20}$$

and

$$\begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} = \mathsf{Var}\left(\begin{bmatrix} v_{t-1} - q_t \\ \Delta v_t \end{bmatrix} \right) - \Psi_t \, (V_t^{yz})^{-1} \, \Psi_t'$$
$$= \begin{bmatrix} \Sigma_{t-1} & 0 \\ 0 & \sigma_v^2 \Delta t \end{bmatrix} - \frac{\begin{bmatrix} \beta_t^2\Sigma_{t-1}^2(\sigma_v^2 + \sigma_e^2) & \beta_t\gamma_t\Sigma_{t-1}\sigma_v^2\sigma_e^2 \\ \beta_t\gamma_t\Sigma_{t-1}\sigma_v^2\sigma_e^2 & (\beta_t^2\Sigma_{t-1}\Delta t + \gamma_t^2\sigma_e^2 + \sigma_u^2)\sigma_v^4 \end{bmatrix}}{(\beta_t^2\Sigma_{t-1}\Delta t + \gamma_t^2\sigma_v^2 + \sigma_u^2)\sigma_e^2 + (\beta_t^2\Sigma_{t-1}\Delta t + \sigma_u^2)\sigma_v^2} \, \Delta t. \tag{IA.21}$$

Therefore,

$$\begin{aligned} q_{t+1} - q_t &= \mu_1 + \mu_2 \\ &= \frac{\big(\beta_t\Sigma_{t-1}(\sigma_v^2 + \sigma_e^2) + \gamma_t\sigma_v^2\sigma_e^2\big)\Delta y_t \; + \; \big(\sigma_u^2 + \beta_t^2\Sigma_{t-1}\Delta t - \beta_t\gamma_t\Sigma_{t-1}\big)\sigma_v^2\Delta z_t}{\big(\beta_t^2\Sigma_{t-1}\Delta t + \gamma_t^2\sigma_v^2 + \sigma_u^2\big)\,\sigma_e^2 + \big(\beta_t^2\Sigma_{t-1}\Delta t + \sigma_u^2\big)\,\sigma_v^2} \\ &= \Lambda_t\Delta y_t \; + \; \mu_t\Delta z_t \; = \; (\lambda_t - \rho_t\mu_t)\Delta y_t \; + \; \mu_t \Delta z_t, \end{aligned} \tag{IA.22}$$

which proves equation (IA.4) for μ_t, Λ_t, and ρ_t. Also,

$$\begin{aligned} \Sigma_t &= \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2 \\ &= \Sigma_{t-1} + \sigma_v^2\Delta t - \frac{\beta_t^2\Sigma_{t-1}^2(\sigma_v^2+\sigma_e^2) + \beta_t^2\Sigma_{t-1}\Delta t\sigma_v^4 + \sigma_v^4\sigma_u^2 + \gamma_t^2\sigma_v^4\sigma_e^2 + 2\beta_t\gamma_t\Sigma_{t-1}\sigma_v^2\sigma_e^2}{\big(\beta_t^2\Sigma_{t-1} + (\beta_t+\gamma_t)^2\sigma_v^2 + \sigma_u^2\big)\,\sigma_e^2 + \big(\beta_t^2\Sigma_{t-1} + \sigma_u^2\big)\,\sigma_v^2} \, \Delta t, \end{aligned} \tag{IA.23}$$

which proves the formula for Σ_t.

Optimal Strategy of Speculator: At each $t = 1, \ldots, T$, the speculator maximizes the expected profit, $\pi_t = \max \sum_{\tau=t}^{T} \mathsf{E}\big((v_T - p_\tau)\Delta x_\tau\big)$. We prove by backward induction that the value function is quadratic and of the form given in (IA.5): $\pi_t = \alpha_{t-1}(v_{t-1} -$

$q_t)^2 + \alpha'_{t-1}(\Delta v_t)^2 + \alpha''_{t-1}(v_{t-1} - q_t)\Delta v_t + \delta_{t-1}$. At the last decision point ($t = T$) the next value function is zero, that is, $\alpha_T = \alpha'_T = \alpha''_T = \delta_T = 0$, which corresponds to the terminal conditions (IA.7). This is the transversality condition, which requires that no money is left on the table. In the induction step, if $t = 1, \ldots, T-1$, we assume that π_{t+1} is of the desired form. The Bellman principle of intertemporal optimization implies

$$\pi_t \;=\; \max_{\Delta x} \; \mathsf{E}\Big((v_t - p_t)\Delta x + \pi_{t+1} \mid \mathcal{I}^q_t, v_t, \Delta v_t \Big). \tag{IA.24}$$

The last two equations in (IA.3) imply that the quote q_t evolves according to $q_{t+1} = q_t + \Lambda_t \Delta y_t + \mu_t \Delta z_t$, where $\Lambda_t = \lambda_t - \rho_t \mu_t$. This implies that the speculator's choice of Δx affects the trading price and the next quote by

$$\begin{aligned} p_t &= q_t + \lambda_t(\Delta x + \Delta u_t), \\ q_{t+1} &= q_t + \Lambda_t(\Delta x + \Delta u_t) + \mu_t \Delta z_t. \end{aligned} \tag{IA.25}$$

Substituting these into the Bellman equation, we get

$$\begin{aligned} \pi_t \;=&\; \max_{\Delta x} \; \mathsf{E}\Big(\Delta x (v_{t-1} + \Delta v_t - q_t - \lambda_t \Delta x - \lambda_t \Delta u_t) \\ &+ \alpha_t (v_{t-1} + \Delta v_t - q_t - \Lambda_t \Delta x - \Lambda_t \Delta u_t - \mu_t \Delta v_t - \mu_t \Delta e_t)^2 \;+\; \alpha'_t \Delta v_{t+1}^2 \\ &+ \alpha''_t (v_{t-1} + \Delta v_t - q_t - \Lambda_t \Delta x - \Lambda_t \Delta u_t - \mu_t \Delta v_t - \mu_t \Delta e_t)\Delta v_{t+1} \;+\; \delta_t \Big) \\ =&\; \max_{\Delta x} \; \Delta x (v_{t-1} - q_t + \Delta v_t - \lambda_t \Delta x) \\ &+ \alpha_t \Big((v_{t-1} - q_t - \Lambda_t \Delta x + (1-\mu_t)\Delta v_t)^2 + (\Lambda_t^2 \sigma_u^2 + \mu_t^2 \sigma_e^2)\Delta t \Big) \;+\; \alpha'_t \sigma_v^2 \Delta t \\ &+\; 0 \;+\; \delta_t. \end{aligned} \tag{IA.26}$$

The first-order condition with respect to Δx is

$$\Delta x \;=\; \frac{1 - 2\alpha_t \Lambda_t}{2(\lambda_t - \alpha_t \Lambda_t^2)}(v_{t-1} - q_t) \;+\; \frac{1 - 2\alpha_t \Lambda_t (1-\mu_t)}{2(\lambda_t - \alpha_t \Lambda_t^2)} \Delta v_t, \tag{IA.27}$$

and the second-order condition for a maximum is $\lambda_t - \alpha_t \Lambda_t^2 > 0$, which is (IA.8). Thus, the optimal Δx is indeed of the form $\Delta x_t = \beta_t (v_{t-1} - q_t)\Delta t + \gamma_t \Delta v_t$, where $\beta_t \Delta t$ and

γ_t are as in (IA.6). We substitute Δx_t in the formula for π_t to obtain

$$\begin{aligned}\pi_t \;=\;& \Big(\beta_t\Delta t(1-\lambda_t\beta_t\Delta t)+\alpha_t(1-\Lambda_t\beta_t\Delta t)^2\Big)(v_{t-1}-q_t)^2\\ &+\Big(\alpha_t(1-\mu_t-\Lambda_t\gamma_t)^2+\gamma_t(1-\lambda_t\gamma_t)\Big)\Delta v_t^2\\ &+\Big(\beta_t\Delta t+\gamma_t(1-2\lambda_t\beta_t\Delta t)+2\alpha_t(1-\Lambda_t\beta_t\Delta t)(1-\mu_t-\Lambda_t\gamma_t)\Big)(v_{t-1}-q_t)\Delta v_t\\ &+\alpha_t\big(\Lambda_t^2\sigma_u^2+\mu_t^2\sigma_e^2\big)\Delta t \;+\; \alpha_t'\sigma_v^2\Delta t \;+\; \delta_t.\end{aligned} \tag{IA.28}$$

This proves that π_t is indeed of the form $\pi_t=\alpha_{t-1}(v_{t-1}-q_t)^2+\alpha_{t-1}'(\Delta v_t)^2+\alpha_{t-1}''(v_{t-1}-q_t)\Delta v_t+\delta_{t-1}$, with α_{t-1}, α_{t-1}', α_{t-1}'' and δ_{t-1} as in (IA.6). $\square$

We now briefly discuss the existence of a solution for the recursive system given in Theorem IA.1. The system of equations (IA.4) to (IA.6) can be numerically solved backwards, starting from the boundary conditions (IA.7). We also start with an arbitrary value of $\Sigma_T>0$.[2] By backward induction, suppose α_t and Σ_t are given. One can verify that equation (IA.4) for Σ_t implies

$$\Sigma_{t-1} \;=\; \frac{\Sigma_t\big(\sigma_v^2\sigma_u^2+\sigma_v^2(\sigma_u^2+\gamma_t^2\sigma_e^2)\big)-\sigma_v^2\sigma_u^2\sigma_e^2\Delta t}{\big(\sigma_u^2\sigma_e^2+\sigma_v^2(\sigma_u^2+\gamma_t^2\sigma_e^2)+\beta_t^2\Delta t^2\sigma_v^2\sigma_e^2-2\gamma_t\beta_t\Delta t\sigma_v^2\sigma_e^2\big)-\Sigma_t\beta_t^2\Delta t\big(\sigma_v^2+\sigma_e^2\big)}. \tag{IA.29}$$

Thus, in equation (IA.4) we can rewrite λ_t,μ_t, and Λ_t as functions of $(\Sigma_t,\beta_t,\gamma_t)$ instead of $(\Sigma_{t-1},\beta_t,\gamma_t)$. Next, we use the formulas for β_t and γ_t to express λ_t,μ_t, and Λ_t as functions of $(\lambda_t,\mu_t,\Lambda_t,\alpha_t,\Sigma_t)$. This gives a system of polynomial equations, whose solution to λ_t,μ_t, and Λ_t depends only on (α_t,Σ_t). Numerical simulations show that the solution is unique under the second-order condition (IA.8), and we conjecture that this result is analytically true for all parameter values. Once the recursive system is computed for all $t=1,\ldots,T$, all that remains is to verify that the value obtained for Σ_0 is the correct one. However, unlike in Kyle (1985), the recursive equation for Σ_t is not linear, and therefore the parameters cannot simply be rescaled. Instead, one must numerically modify the initial choice of Σ_T until the correct value of Σ_0 is reached.

B. Discrete-Time Slow Model

We consider a discrete version of the slow speculator model presented in Section I of the paper. Denote by $\mathcal{I}_t^q=\{\Delta z_\tau\}_{\tau\le t}\cup\{\Delta y_\tau\}_{\tau\le t-1}$ the dealer's information set just

[2] Numerically, it should be of the order of Δt.

before trading at t, and by $\mathcal{I}_t^p = \{\Delta z_\tau\}_{\tau \le t} \cup \{\Delta y_\tau\}_{\tau \le t} = \mathcal{I}_t^q \cup \{\Delta y_t\}$ the information set just after trading at t. The zero profit condition for the competitive dealer translates into the formulas

$$q_t \;=\; \mathsf{E}(v_t \mid \mathcal{I}_t^q), \qquad p_t \;=\; \mathsf{E}(v_t \mid \mathcal{I}_t^p). \tag{IA.30}$$

We also define

$$\Omega_t \;=\; \mathsf{Var}(v_t \mid \mathcal{I}_t^p), \qquad \Sigma_t \;=\; \mathsf{Var}(v_t \mid \mathcal{I}_t^q). \tag{IA.31}$$

The next result shows that if the pricing rule is linear, then the speculator's strategy is also linear, and furthermore it only has a forecast-error component, $\beta_t(v_t - q_t)\Delta t$.

THEOREM IA.2: *Any linear equilibrium must be of the form*

$$\begin{aligned}
\Delta x_t &= \beta_t(v_t - q_t)\Delta t, \\
p_t &= q_t + \lambda_t \Delta y_t, \\
q_t &= p_{t-1} + \mu_{t-1}\Delta z_t,
\end{aligned} \tag{IA.32}$$

for $t = 1, \dots, T$, where by convention $p_0 = 0$, and β_t, γ_t, λ_t, μ_t, Σ_t, and Ω_t are constants that satisfy

$$\begin{aligned}
\lambda_t &= \frac{\beta_t \Omega_t}{\sigma_u^2}, \\
\mu_t &= \frac{\sigma_v^2}{\sigma_v^2 + \sigma_e^2}, \\
\Sigma_t &= \frac{\Omega_t \sigma_u^2}{\sigma_u^2 - \beta_t^2 \Omega_t \Delta t}, \\
\Omega_{t-1} &= \Omega_t + \frac{\beta_t^2 \Omega_t^2}{\sigma_u^2 - \beta_t^2 \Omega_t \Delta t}\,\Delta t - \frac{\sigma_v^2 \sigma_e^2}{\sigma_v^2 + \sigma_e^2}\,\Delta t.
\end{aligned} \tag{IA.33}$$

The value function of the speculator is quadratic for all $t = 1, \dots, T$:

$$\pi_t \;=\; \alpha_{t-1}(v_t - q_t)^2 + \delta_{t-1}. \tag{IA.34}$$

The coefficients of the optimal trading strategy and the value function satisfy

$$\begin{aligned}
\beta_t \Delta t &= \frac{1 - 2\alpha_t \lambda_t}{2\lambda_t(1 - \alpha_t \lambda_t)}, \\
\alpha_{t-1} &= \beta_t \Delta t (1 - \lambda_t \beta_t \Delta t) + \alpha_t (1 - \lambda_t \beta_t \Delta t)^2, \\
\delta_{t-1} &= \alpha_t \big(\lambda_t^2 \sigma_u^2 + \mu_t^2(\sigma_v^2 + \sigma_e^2)\big)\Delta t + \delta_t.
\end{aligned} \tag{IA.35}$$

The terminal conditions are

$$\alpha_T \;=\; \delta_T \;=\; 0. \tag{IA.36}$$

The second-order condition is

$$\lambda_t(1-\alpha_t\lambda_t) \;>\; 0. \tag{IA.37}$$

Given Ω_0*, conditions* (*IA.33*) *to* (*IA.37*) *are necessary and sufficient for the existence of a linear equilibrium.*

Proof. First, we show that equations (IA.33) are equivalent to the zero profit conditions of the dealer. Second, we show that equations (IA.35) to (IA.37) are equivalent to the speculator's strategy being optimal.

Zero Profit of Dealer: Start with the dealer's update due to the order flow at $t = 1, \dots, T$. Conditional on $\mathcal{I}_t^q$, v_t has a normal distribution, $v_t | \mathcal{I}_t^q \;\sim\; \mathcal{N}(q_t, \Sigma_t)$. The aggregate order flow at t is of the form $\Delta y_t = \beta_t(v_t - q_t)\Delta t + \Delta u_t$. Let

$$\Phi_t \;=\; \mathsf{Cov}(v_t - q_t, \Delta y_t) \;=\; \beta_t \Sigma_t \Delta t. \tag{IA.38}$$

Then, conditional on $\mathcal{I}_t^p = \mathcal{I}_t^q \cup \{\Delta y_t\}$, $v_t \sim \mathcal{N}(p_t, \Omega_t)$, with

$$\begin{aligned}
p_t \;&=\; q_t + \lambda_t \Delta y_t, \\
\lambda_t \;&=\; \Phi_t \,\mathsf{Var}(\Delta y_t)^{-1} \;=\; \frac{\beta_t \Sigma_t}{\beta_t^2 \Sigma_t \Delta t + \sigma_u^2}, \\
\Omega_t \;&=\; \mathsf{Var}(v_t - q_t) - \Phi_t \,\mathsf{Var}(\Delta y_t)^{-1} \Phi_t' \;=\; \Sigma_t - \frac{\beta_t^2 \Sigma_t^2}{\beta_t^2 \Sigma_t \Delta t + \sigma_u^2}\,\Delta t \\
\;&=\; \frac{\Sigma_t \sigma_u^2}{\beta_t^2 \Sigma_t \Delta t + \sigma_u^2}.
\end{aligned} \tag{IA.39}$$

To obtain the equation for λ_t, note that the above equations for λ_t and Ω_t imply $\frac{\lambda_t}{\Omega_t} = \frac{\beta_t}{\sigma_u^2}$. The equation for Σ_t is obtained by solving for Ω_t in the last equation of (IA.39).

Next, consider the dealer's update at $t = 1, \dots, T$ due to the signal $\Delta z_t = \Delta v_t + \Delta e_t$. From $v_{t-1} | \mathcal{I}_{t-1}^p \sim \mathcal{N}(p_{t-1}, \Omega_{t-1})$, we have $v_t | \mathcal{I}_{t-1}^p \sim \mathcal{N}(p_{t-1}, \Omega_{t-1} + \sigma_v^2 \Delta t)$. Let

$$\Psi_t \;=\; \mathsf{Cov}(v_t - p_{t-1}, \Delta z_t) \;=\; \sigma_v^2 \Delta t. \tag{IA.40}$$

Then, conditional on $\mathcal{I}^q_t = \mathcal{I}^p_{t-1} \cup \{\Delta z_t\}$, $v_t | \mathcal{I}^q_t \sim \mathcal{N}(q_t, \Sigma_t)$, with

$$\begin{aligned}
q_t &= p_{t-1} + \mu_t \Delta z_t, \\
\mu_t &= \Psi_t \, \mathsf{Var}(\Delta z_t)^{-1} \;=\; \frac{\sigma_v^2}{\sigma_v^2 + \sigma_e^2}, \\
\Sigma_t &= \mathsf{Var}(v_t - p_{t-1}) - \Psi_t \, \mathsf{Var}(\Delta z_t)^{-1} \Psi_t' \;=\; \Omega_{t-1} + \sigma_v^2 \Delta t - \frac{\sigma_v^4}{\sigma_v^2 + \sigma_e^2} \, \Delta t \\
&= \Omega_{t-1} + \frac{\sigma_v^2 \sigma_e^2}{\sigma_v^2 + \sigma_e^2} \, \Delta t.
\end{aligned} \tag{IA.41}$$

Thus, we prove the equation for μ_t. Note that equation (IA.41) gives a formula for Ω_{t-1} as a function of Σ_t, and we have already proved the formula for Σ_t as a function of Ω_t in (IA.33). We therefore get Ω_{t-1} as a function of Ω_t, which is the last equation in (IA.33).

Optimal Strategy of Speculator: At each $t = 1, \ldots, T$, the speculator maximizes the expected profit, $\pi_t = \max \sum_{\tau=t}^{T} \mathsf{E}\big((v_T - p_\tau)\Delta x_\tau\big)$. We prove by backward induction that the value function is quadratic and of the form given in (IA.34): $\pi_t = \alpha_{t-1}(v_t - q_t)^2 + \delta_{t-1}$. At the last decision point ($t = T$) the next value function is zero, that is, $\alpha_T = \delta_T = 0$, which corresponds to the terminal conditions (IA.36). In the induction step, if $t = 1, \ldots, T-1$, we assume that π_{t+1} is of the desired form. The Bellman principle of intertemporal optimization implies

$$\pi_t \;=\; \max_{\Delta x} \; \mathsf{E}\Big((v_t - p_t)\Delta x + \pi_{t+1} \mid \mathcal{I}^q_t, v_t, \Delta v_t \Big). \tag{IA.42}$$

The last two equations in (IA.32) show that the quote q_t evolves according to $q_{t+1} = q_t + \Lambda_t \Delta y_t + \mu_t \Delta z_{t+1}$. This implies that the speculator's choice of Δx affects the trading price and the next quote by

$$\begin{aligned}
p_t &= q_t + \lambda_t(\Delta x + \Delta u_t), \\
q_{t+1} &= q_t + \lambda_t(\Delta x + \Delta u_t) + \mu_t \Delta z_{t+1}.
\end{aligned} \tag{IA.43}$$

Substituting these into the Bellman equation, we get

$$\begin{aligned}\pi_t &= \max_{\Delta x} \ \mathsf{E}\Big(\Delta x(v_t - q_t - \lambda_t \Delta x - \lambda_t \Delta u_t) \\ &\quad + \alpha_t (v_t + \Delta v_{t+1} - q_t - \lambda_t \Delta x - \lambda_t \Delta u_t - \mu_t \Delta z_{t+1})^2 \ + \ \delta_t\Big) \\ &= \max_{\Delta x} \ \Delta x(v_t - q_t - \lambda_t \Delta x) \\ &\quad + \alpha_t \Big((v_t - q_t - \lambda_t \Delta x)^2 + (\lambda_t^2 \sigma_u^2 + \mu_t^2(\sigma_v^2 + \sigma_e^2))\Delta t\Big) \ + \ \delta_t.\end{aligned} \tag{IA.44}$$

The first-order condition with respect to Δx is

$$\Delta x \ = \ \frac{1 - 2\alpha_t \lambda_t}{2\lambda_t(1 - \alpha_t \lambda_t)}(v_t - q_t), \tag{IA.45}$$

and the second-order condition for a maximum is $\lambda_t(1 - \alpha_t \lambda_t) > 0$, which is (IA.37). Thus, the optimal Δx is indeed of the form $\Delta x_t = \beta_t(v_t - q_t)\Delta t$, where $\beta_t \Delta t$ satisfies equation (IA.35). We substitute Δx_t in the formula for π_t to obtain

$$\pi_t \ = \ \Big(\beta_t \Delta t(1 - \lambda_t \beta_t \Delta t) + \alpha_t(1 - \lambda_t \beta_t \Delta t)^2\Big)(v_t - q_t)^2 \ + \ \alpha_t\big(\lambda_t^2 \sigma_u^2 + \mu_t^2(\sigma_v^2 + \sigma_e^2)\big)\Delta t \ + \ \delta_t. \tag{IA.46}$$

This proves that π_t is indeed of the form $\pi_t = \alpha_{t-1}(v_t - q_t)^2 + \delta_{t-1}$, with α_{t-1} and δ_{t-1} as in (IA.35). $\square$

Equations (IA.33) and (IA.35) form a system of equations. As before, this system is solved backwards, starting from the boundary conditions (IA.36), so that $\Omega_t = \Omega_0$ at $t = 0$.

II. Sampling at Lower Frequencies than the Trading Frequency

In this section, we show that Corollaries 4 and 5 in Section III of the paper generalize when data are sampled at a lower frequency than the trading frequency. To do so, suppose that trading takes place in continuous time, but trades are aggregated over $T = 1, 2, \ldots$ discrete intervals of equal length $\frac{1}{T} = \delta$. The data are indexed by $t \in \{1, 2, \ldots, T\}$, which corresponds to calendar time $\tau = t\delta \in [0, 1]$. Denote by

$$\Delta x_t \;=\; x_t - x_{t-1} \;=\; \int_{(t-1)\delta}^{t\delta} \mathrm{d}x_\tau \tag{IA.47}$$

the speculator's aggregate order flow over the t-th time interval.[3]

When data are aggregated every δ-period, the empirical counterpart of the speculator participation rate and the autocorrelation of the speculator's order flow are defined, respectively, as[4]

$$\begin{aligned} \overline{SPR}_t \;&=\; \frac{\mathsf{Var}(\Delta x_t)}{\mathsf{Var}(\Delta x_t) + \mathsf{Var}(\Delta u_t)}, \\ &\overline{\mathsf{Corr}}(\Delta x_t, \Delta x_{t+s}). \end{aligned} \tag{IA.48}$$

The next result provides approximations for these empirical measures when the sampling length δ is small.[5] To simplify formulas, we use the notation from Theorems 1 and 2 of the paper. For instance, let

$$\Lambda^F \;=\; \lambda^F - \mu^F \rho^F. \tag{IA.49}$$

PROPOSITION IA.1: *The empirical speculator participation rate in the slow and fast*

[3] This is related, but not equivalent, to the order flow at the t-th trading round in the discrete model of Section I. In the limit when δ approaches zero, it is reasonable to expect that the two notions are equivalent. This depends on whether the coefficients of the discrete-time model (as described in Internet Appendix Section I) converge to the corresponding coefficients of the continuous-time version. We conjecture this is true, as in Kyle (1985), but we do not formally prove it.

[4] Alternatively, we could define the speculator participation rate as $\frac{\mathsf{Var}(\Delta x_t)}{\mathsf{Var}(\Delta x_t + \Delta u_t)}$. Then, as we show in the proof of Proposition IA.1, we obtain the same formulas as in (IA.50), except that in the second equation the term $\frac{\Lambda^F \beta_t^F}{(g+1)^2}\,\delta$ is replaced by $\frac{\Lambda^F \beta_t^F}{g+1}\,\delta$. We have checked that the numerical results from Table I in the paper are not significantly different under this alternative definition.

[5] In the proof of Proposition IA.1, we also show how to obtain exact formulas or more precise approximations.

models satisfies the following formulas:

$$\begin{aligned}
\overline{SPR}_t^S &= \frac{(\beta_t^S)^2 \Sigma_t}{\sigma_u^2}\, \delta \;+\; O(\delta^2), \\
\overline{SPR}_t^F &= \frac{g}{g+1} \;+\; \frac{\Lambda^F \beta_t^F}{(g+1)^2}\, \delta \;+\; O(\delta^2).
\end{aligned} \tag{IA.50}$$

Therefore, when the sampling length δ is small, the empirical speculator participation rate in the slow model is below its level in the fast model.

The empirical autocorrelation of the speculator's order flow in the slow and fast models satisfies the following formulas ($r > 0$):

$$\begin{aligned}
\overline{\mathsf{Corr}}(\Delta x_t^S, \Delta x_{t+r}^S) &= \left(\frac{1-(t+r)\delta}{1-t\delta}\right)^{\lambda^S \beta_0^S - \frac{1}{2}} + O(\delta), \\
\overline{\mathsf{Corr}}(\Delta x_t^F, \Delta x_{t+r}^F) &= \frac{\Lambda^F \beta_{t+r}^F}{g} \left(\frac{1-(t+r)\delta}{1-t\delta}\right)^{\Lambda^F \beta_0^F} \delta + O(\delta^2).
\end{aligned} \tag{IA.51}$$

Therefore, when the sampling length δ is small, the empirical autocorrelation of the speculator's order flow in the slow model is below its level in the fast model.

Proof. For simplicity, we omit the superscript $k \in \{S, F\}$. Let $r > 0$. Define

$$\begin{aligned}
\Lambda &= \lambda - \mu\rho, \qquad n \;=\; 1 - \mu - \Lambda\gamma, \qquad \alpha \;=\; \Lambda\beta_0, \\
\tau' &= (t-1)\delta, \qquad \tau \;=\; t\delta, \qquad \Delta\tau \;=\; \tau - \tau', \\
\xi' &= (t+r-1)\delta, \qquad \xi \;=\; (t+r)\delta, \qquad \Delta\xi \;=\; \xi - \xi'.
\end{aligned} \tag{IA.52}$$

The aggregate trade over the t-th interval, $[\tau', \tau]$, is

$$\Delta x_t \;=\; \int_{\tau'}^{\tau} \beta_s (v_s - q_s) \mathrm{d}s + \gamma \mathrm{d}v_s. \tag{IA.53}$$

Thus, for $s_1 < s_2 \in (0,1)$, Lemma A1 in the Appendix of the paper implies

$$\begin{aligned}
\mathsf{Cov}(v_{s_1} - q_{s_2}, v_{s_2} - q_{s_2}) &= \Sigma_{s_1} \left(\frac{1-s_2}{1-s_1}\right)^{\Lambda\beta_0}, \\
\mathsf{Cov}(\mathrm{d}v_{s_1}, v_{s_2} - q_{s_2}) &= n\, \sigma_v^2 \left(\frac{1-s_2}{1-s_1}\right)^{\Lambda\beta_0} \mathrm{d}s_1.
\end{aligned} \tag{IA.54}$$

We compute

$$\begin{aligned}
\mathsf{Var}(\Delta x_t) &= 2\int_{\tau'}^{\tau}\int_{s_1}^{\tau}\beta_{s_1}\beta_{s_2}\Sigma_{s_1}\left(\frac{1-s_2}{1-s_1}\right)^{\alpha}\mathrm{d}s_2\mathrm{d}s_1 \\
&\quad + 2\int_{\tau'}^{\tau}\int_{s_1}^{\tau}\gamma\beta_{s_2}\, n\,\sigma_v^2\left(\frac{1-s_2}{1-s_1}\right)^{\alpha}\mathrm{d}s_2\mathrm{d}s_1 + \int_{\tau'}^{\tau}\gamma^2\sigma_v^2\mathrm{d}s_1 \\
&= 2(\beta_0^2\Sigma_0+\gamma\beta_0 n\sigma_v^2)\int_{\tau'}^{\tau}\int_{s_1}^{\tau}(1-s_2)^{\alpha-1}(1-s_1)^{-\alpha}\,\mathrm{d}s_2\mathrm{d}s_1 + \gamma^2\sigma_v^2\int_{\tau'}^{\tau}\mathrm{d}s_1 \\
&= 2(\beta_0^2\Sigma_0+\gamma\beta_0 n\sigma_v^2)\left(\frac{\Delta\tau}{\alpha} - \frac{1-\tau}{\alpha(1-\alpha)}\left(\left(\frac{1-\tau'}{1-\tau}\right)^{1-\alpha}-1\right)\right) + \gamma^2\sigma_v^2\Delta\tau.
\end{aligned} \tag{IA.55}$$

Note that this formula is true for all $0 \le \tau' < \tau \le 1$. In the particular case in which $\tau' = (t-1)\delta$ and $\tau = t\delta$, the Taylor series formula implies

$$\begin{aligned}
\left(\frac{1-\tau'}{1-\tau}\right)^{1-\alpha} &= \left(1+\frac{\delta}{1-\tau}\right)^{1-\alpha} \\
&= 1+\frac{1-\alpha}{1-\tau}\delta + \frac{(1-\alpha)(-\alpha)}{2(1-\tau)^2}\delta^2 + \frac{(1-\alpha)(-\alpha)(-\alpha-1)}{6(1-\tau)^3}\delta^3 + O(\delta^4).
\end{aligned} \tag{IA.56}$$

Hence,

$$\frac{1-\tau}{\alpha(1-\alpha)}\left(\left(\frac{1-\tau'}{1-\tau}\right)^{1-\alpha}-1\right) = \frac{\delta}{\alpha} - \frac{\delta^2}{2(1-\tau)} + \frac{(1+\alpha)\delta^3}{6(1-\tau)^2} + O(\delta^4). \tag{IA.57}$$

Thus, the exact formula for $\mathsf{Var}(\Delta x_t)$ implies the following approximate formula:

$$\begin{aligned}
\mathsf{Var}(\Delta x_t) &= \gamma^2\sigma_v^2\,\delta + \left(\beta_0^2\Sigma_0+\gamma\beta_0 n\sigma_v^2\right)\left(\frac{\delta^2}{1-\tau} - \frac{(1+\alpha)\delta^3}{3(1-\tau)^2} + O(\delta^4)\right) \\
&= \gamma^2\sigma_v^2\,\delta + \left(\beta_t^2\Sigma_t+\gamma\beta_t n\sigma_v^2\right)\delta^2 - \left(\beta_t^2\Sigma_t+\gamma\beta_t n\sigma_v^2\right)\frac{1+\alpha}{3(1-\tau)}\delta^3 + O(\delta^4).
\end{aligned} \tag{IA.58}$$

We also write the formula for the variance of the empirical noise trading order flow:

$$\mathsf{Var}(\Delta u_t) = \sigma_u^2\,\delta. \tag{IA.59}$$

In the slow model, $\gamma = 0$, and thus $\mathsf{Var}(\Delta x_t) = \beta_t^2\Sigma_t\delta^2 - \beta_t^2\Sigma_t\frac{(1+\alpha)}{3(1-\tau)}\delta^3 + O(\delta^4)$. We get

$$\begin{aligned}
\mathsf{Var}(\Delta x_t) &= \delta^2\Big(\beta_t^2\Sigma_t - \beta_t^2\Sigma_0\frac{(1+\alpha)}{3}\delta + O(\delta^2)\Big), \\
\mathsf{Var}(\Delta x_t) + \mathsf{Var}(\Delta u_t) &= \delta\Big(\sigma_u^2 + \beta_t^2\Sigma_t\delta + O(\delta^2)\Big).
\end{aligned} \tag{IA.60}$$

Applying Lemma IA.1, with $f_0 = \beta_t^2\Sigma_t$, $f_1 = -\beta_t^2\Sigma_0\frac{(1+\alpha)}{3}$, $g_0 = \sigma_u^2$, and $g_1 = \beta_t^2\Sigma_t$, we compute $\frac{f_1g_0 - f_0g_1}{g_0^2} = -\frac{\beta_t^2\Sigma_t}{\sigma_u^2}\left(\frac{1+\alpha}{3(1-\tau)} + \frac{\beta_t^2\Sigma_t}{\sigma_u^2}\right)$. Thus, in the slow model we have

$$\overline{SPR}_t^S = \frac{\beta_t^2\Sigma_t}{\sigma_u^2}\,\delta - \frac{\beta_t^2\Sigma_t}{\sigma_u^2}\left(\frac{1+\alpha}{3(1-\tau)} + \frac{\beta_t^2\Sigma_t}{\sigma_u^2}\right)\delta^2 + O(\delta^3), \tag{IA.61}$$

which implies the first equation in (IA.50).

In the fast model, $\gamma > 0$, and thus

$$\begin{aligned} \mathsf{Var}(\Delta x_t) &= \delta\Big(\gamma^2\sigma_v^2 + \left(\beta_t^2\Sigma_t + \gamma\beta_t n\sigma_v^2\right)\delta + O(\delta^2)\Big), \\ \mathsf{Var}(\Delta x_t) + \mathsf{Var}(\Delta u_t) &= \delta\Big(\left(\gamma^2\sigma_v^2 + \sigma_u^2\right) + \left(\beta_t^2\Sigma_t + \gamma\beta_t n\sigma_v^2\right)\delta + O(\delta^2)\Big). \end{aligned} \tag{IA.62}$$

Applying Lemma IA.1, with $f_0 = \gamma^2\sigma_v^2$, $f_1 = \beta_t^2\Sigma_t + \gamma\beta_t n\sigma_v^2$, $g_0 = \gamma^2\sigma_v^2 + \sigma_u^2$, and $g_1 = f_1$, we compute $\frac{f_1g_0 - f_0g_1}{g_0^2} = \frac{(\beta_t^2\Sigma_t + \gamma\beta_t n\sigma_v^2)\sigma_u^2}{(\gamma^2\sigma_v^2 + \sigma_u^2)^2}$. Thus, in the fast model we have

$$\overline{SPR}_t^F = \frac{\gamma^2\sigma_v^2}{\gamma^2\sigma_v^2 + \sigma_u^2} + \frac{(\beta_t^2\Sigma_t + \gamma\beta_t n\sigma_v^2)\sigma_u^2}{(\gamma^2\sigma_v^2 + \sigma_u^2)^2}\,\delta + O(\delta^2). \tag{IA.63}$$

Recall the following formulas from the proof of Theorem 2:

$$\begin{aligned} a &= \frac{\sigma_u^2}{\sigma_v^2}, \qquad b = \frac{\sigma_e^2}{\sigma_v^2}, \qquad c = \frac{\Sigma_0}{\sigma_v^2}, \\ g &= \frac{\gamma^2}{a}, \quad \mu = \frac{1+g}{2+b+bg}, \quad \Lambda = \frac{1}{\gamma}\,\frac{1-g}{2+b+bg}, \\ n &= \frac{b+bg}{2+b+bg}, \quad \psi = \frac{\beta_0\Sigma_0}{\sigma_u^2}\gamma = \frac{1+g}{2+b+bg} - g. \end{aligned} \tag{IA.64}$$

Using these equations, we compute $\frac{\gamma^2\sigma_v^2}{\gamma^2\sigma_v^2 + \sigma_u^2} = \frac{g}{g+1}$, and

$$\frac{1}{\gamma}(\psi + gn) = \frac{1}{\gamma}\frac{1-g}{2+b+bg} = \Lambda. \tag{IA.65}$$

Also, $\beta_t\sigma_u^2\,\frac{\beta_0\Sigma_0 + \gamma n\sigma_v^2}{(\gamma^2\sigma_v^2 + \sigma_u^2)^2} = \frac{\beta_t}{\gamma}\,\frac{\frac{\beta_0\Sigma_0}{\sigma_u^2}\gamma + \gamma^2 n\frac{\sigma_v^2}{\sigma_u^2}}{\left(\gamma^2\frac{\sigma_v^2}{\sigma_u^2} + 1\right)^2} = \frac{\beta_t}{\gamma}\,\frac{\psi + gn}{(g+1)^2} = \frac{\beta_t\Lambda}{(g+1)^2}$. We thus obtain

$$\overline{SPR}_t^F = \frac{g}{g+1} + \frac{\beta_t\Lambda}{(g+1)^2}\,\delta + O(\delta^2), \tag{IA.66}$$

which proves the second equation in (IA.50).

If we use the alternative definition, $\overline{SPR}_t = \frac{\mathsf{Var}(\Delta x_t)}{\mathsf{Var}(\Delta x_t + \Delta u_t)}$, we also need to compute the covariance between Δx_t and Δu_t. Let $s_1 < s_2 \in (0,1)$. Then, using a proof similar to that of Lemma A1 in the Appendix of the paper, we get

$$\mathsf{Cov}(\mathrm{d}u_{s_1}, q_{s_2}) \;=\; \Lambda\sigma_u^2 \left(\frac{1-s_2}{1-s_1}\right)^{\Lambda\beta_0} \mathrm{d}s_1. \tag{IA.67}$$

As in the calculation of $\mathsf{Var}(\Delta x_t)$, we have

$$\begin{aligned}\mathsf{Cov}(\Delta u_t, \Delta x_t) \;&=\; -\int_{\tau'}^{\tau}\int_{s_1}^{\tau} \beta_{s_2}\Lambda\sigma_u^2 \left(\frac{1-s_2}{1-s_1}\right)^{\alpha} \mathrm{d}s_2\mathrm{d}s_1 \\ &=\; -\frac{\beta_t\Lambda\sigma_u^2}{2}\,\delta^2 + \frac{\beta_t\Lambda\sigma_u^2}{2}\,\frac{1+\alpha}{3(1-\tau)}\delta^3 + O(\delta^4).\end{aligned} \tag{IA.68}$$

From (IA.68), it follows that $\mathsf{Cov}(\Delta u_t, \Delta x_t) = O(\delta^2)$. Thus, the computation for $\overline{SPR}_t$ is not affected in the slow model, but it is affected in the fast model. Using the previous formulas, we get

$$\begin{aligned}\mathsf{Var}(\Delta x_t) \;&=\; \delta\Big(\gamma^2\sigma_v^2 + \big(\beta_t^2\Sigma_t + \gamma\beta_t n\sigma_v^2\big)\,\delta + O(\delta^2)\Big), \\ \mathsf{Var}(\Delta x_t + \Delta u_t) \;&=\; \delta\Big(\big(\gamma^2\sigma_v^2 + \sigma_u^2\big) + \big(\beta_t^2\Sigma_t + \gamma\beta_t n\sigma_v^2 - \beta_t\Lambda\sigma_u^2\big)\,\delta + O(\delta^2)\Big).\end{aligned} \tag{IA.69}$$

Applying Lemma IA.1, with $f_0 = \gamma^2\sigma_v^2$, $f_1 = \beta_t^2\Sigma_t + \gamma\beta_t n\sigma_v^2$, $g_0 = \gamma^2\sigma_v^2 + \sigma_u^2$, and $g_1 = f_1 - \beta_t\Lambda\sigma_u^2$, we compute $\frac{f_1 g_0 - f_0 g_1}{g_0^2} = \frac{\left(\beta_t^2\Sigma_t + \gamma\beta_t n\sigma_v^2 + \gamma^2\beta_t\Lambda\sigma_v^2\right)\sigma_u^2}{(\gamma^2\sigma_v^2+\sigma_u^2)^2} = \frac{\left(\beta_t^2\Sigma_t + \gamma\beta_t(1-\mu)\sigma_v^2\right)\sigma_u^2}{(\gamma^2\sigma_v^2+\sigma_u^2)^2}$. Thus, in the fast model we have

$$\frac{\mathsf{Var}(\Delta x_t)}{\mathsf{Var}(\Delta x_t + \Delta u_t)} \;=\; \frac{\gamma^2\sigma_v^2}{\gamma^2\sigma_v^2 + \sigma_u^2} + \frac{\big(\beta_t^2\Sigma_t + \gamma\beta_t(1-\mu)\sigma_v^2\big)\sigma_u^2}{(\gamma^2\sigma_v^2 + \sigma_u^2)^2}\,\delta + O(\delta^2). \tag{IA.70}$$

This shows that under the alternative defition, $\overline{SPR}_t = \frac{\mathsf{Var}(\Delta x_t)}{\mathsf{Var}(\Delta x_t + \Delta u_t)}$, equation (IA.63) still holds, but with n replaced by $1-\mu$. As before, using the formulas for the fast model, we compute

$$\frac{\mathsf{Var}(\Delta x_t)}{\mathsf{Var}(\Delta x_t + \Delta u_t)} \;=\; \frac{g}{g+1} + \frac{\beta_t\Lambda}{g+1}\,\delta + O(\delta^2). \tag{IA.71}$$

We now compute the empirical autocorrelation of the speculator's order flow. The aggregate trade over the $(t+r)$-th interval, $[\xi', \xi]$, is

$$\Delta x_{t+r} \;=\; \int_{\xi'}^{\xi} \beta_s(v_s - q_s)\mathrm{d}s + \gamma\mathrm{d}v_s. \tag{IA.72}$$

With a similar computation as for $\mathsf{Var}(\Delta x_t)$, we get

$$\begin{aligned}
\mathsf{Cov}(\Delta x_t, \Delta x_{t+r}) &= \int_{\tau'}^{\tau}\int_{\xi'}^{\xi} \beta_{s_1}\beta_{s_2}\Sigma_{s_1}\left(\frac{1-s_2}{1-s_1}\right)^{\alpha} \mathrm{d}s_2\mathrm{d}s_1 \\
&\quad + \int_{\tau'}^{\tau}\int_{\xi'}^{\xi} \gamma\beta_{s_2}\, n\, \sigma_v^2 \left(\frac{1-s_2}{1-s_1}\right)^{\alpha} \mathrm{d}s_2\mathrm{d}s_1 \\
&= (\beta_0^2\Sigma_0 + \gamma\beta_0 n\sigma_v^2)\int_{\tau'}^{\tau}(1-s_1)^{-\alpha}\,\mathrm{d}s_1 \int_{\xi'}^{\xi}(1-s_2)^{\alpha-1}\,\mathrm{d}s_2 \qquad \text{(IA.73)} \\
&= (\beta_0^2\Sigma_0 + \gamma\beta_0 n\sigma_v^2) \times \frac{(1-\tau)^{1-\alpha}}{1-\alpha}\left(\left(\frac{1-\tau'}{1-\tau}\right)^{1-\alpha} - 1\right) \\
&\quad \times \frac{(1-\xi)^{\alpha}}{\alpha}\left(\left(\frac{1-\xi'}{1-\xi}\right)^{\alpha} - 1\right).
\end{aligned}$$

Note that this formula is true for all $0 \leq \tau' < \tau \leq \xi' < \xi \leq 1$. In the particular case in which $\tau' = (t-1)\delta$, $\tau = t\delta$, $\xi' = (t+r-1)\delta$, and $\xi = (t+r)\delta$, the Taylor series formula implies

$$\begin{aligned}
\left(\frac{1-\tau'}{1-\tau}\right)^{1-\alpha} &= 1 + \frac{1-\alpha}{1-\tau}\delta + \frac{(1-\alpha)(-\alpha)}{2(1-\tau)^2}\delta^2 + \frac{(1-\alpha)(-\alpha)(-\alpha-1)}{6(1-\tau)^3}\delta^3 + O(\delta^4), \\
\left(\frac{1-\xi'}{1-\xi}\right)^{\alpha} &= \left(1+\frac{\delta}{1-\xi}\right)^{\alpha} = 1 + \frac{\alpha}{1-\xi}\delta + \frac{\alpha(\alpha-1)}{2(1-\xi)^2}\delta^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{6(1-\xi)^3}\delta^3 + O(\delta^4).
\end{aligned} \qquad \text{(IA.74)}$$

Hence,

$$\begin{aligned}
\mathsf{Cov}(\Delta x_t, \Delta x_{t+r}) &= (\beta_0^2\Sigma_0 + \gamma\beta_0 n\sigma_v^2) \times \frac{\delta^2}{(1-\tau)^{\alpha}(1-\xi)^{1-\alpha}} \\
&\quad \times \left(1 - \frac{\alpha}{2(1-\tau)}\delta + \frac{\alpha(\alpha+1)}{6(1-\tau)^2}\delta^2 + O(\delta^3)\right) \\
&\quad \times \left(1 - \frac{(1-\alpha)}{2(1-\xi)}\delta + \frac{(1-\alpha)(2-\alpha)}{6(1-\xi)^2}\delta^2 + O(\delta^3)\right) \\
&= \frac{\beta_0^2\Sigma_0 + \gamma\beta_0 n\sigma_v^2}{(1-\tau)^{\alpha}(1-\xi)^{1-\alpha}}\,\delta^2\left(1 - \left(\frac{\alpha}{1-\tau} + \frac{1-\alpha}{1-\xi}\right)\frac{\delta}{2} + O(\delta^2)\right).
\end{aligned} \qquad \text{(IA.75)}$$

In the slow model, $\gamma = 0$, and thus $\mathsf{Var}(\Delta x_t) = \beta_t^2\Sigma_t\delta^2 - \beta_t^2\Sigma_t\frac{(1+\alpha)}{3(1-\tau)}\delta^3 + O(\delta^4) = \delta^2\Big(\beta_t^2\Sigma_t - \beta_t^2\Sigma_0\frac{(1+\alpha)}{3}\delta + O(\delta^2)\Big)$. Applying Lemma IA.1 with $f_0 = \beta_t^2\Sigma_t$ and $f_1 = -\beta_t^2\Sigma_0\frac{1+\alpha}{3}$, we get $\mathsf{Var}^{1/2}(\Delta x_t) = \delta\left(\beta_t\Sigma_t^{1/2} - \frac{\beta_t^2\Sigma_0\frac{1+\alpha}{3}}{2\beta_t\Sigma_t^{1/2}}\,\delta + O(\delta^2)\right)$, which simplifies to

$\mathsf{Var}^{1/2}(\Delta x_t) = \frac{\delta}{\Sigma_t^{1/2}} \left(\beta_0 \Sigma_0 - \frac{\beta_t \Sigma_0 (1+\alpha)}{6}\,\delta + O(\delta^2)\right)$. We now compute

$$\begin{aligned} \mathsf{Var}^{1/2}(\Delta x_t)\,\mathsf{Var}^{1/2}(\Delta x_{t+r}) &= \frac{\beta_0^2 \Sigma_0^2\,\delta^2}{\Sigma_t^{1/2}\Sigma_{t+r}^{1/2}} \left(1 - \frac{(\beta_t + \beta_{t+r})(1+\alpha)}{6\beta_0}\,\delta + O(\delta^2)\right) \\ &= \frac{\beta_0^2 \Sigma_0\,\delta^2}{(1-\tau)^{1/2}(1-\xi)^{1/2}} \left(1 - \frac{(\beta_t + \beta_{t+r})(1+\alpha)}{6\beta_0}\,\delta + O(\delta^2)\right). \end{aligned} \tag{IA.76}$$

Applying Lemma IA.1, with $f_0 = 1$, $f_1 = -\frac{1}{2}\big(\frac{\alpha}{1-\tau} + \frac{1-\alpha}{1-\xi}\big)$, $g_0 = 1$, and $g_1 = -\frac{1+\alpha}{6}\big(\frac{1}{1-\tau} + \frac{1}{1-\xi}\big)$, we compute $\frac{f_1 g_0 - f_0 g_1}{g_0^2} = f_1 - g_1 = \frac{2\alpha - 1}{6}\big(\frac{2}{1-\xi} - \frac{1}{1-\tau}\big)$. Thus, in the slow model we have

$$\overline{\mathsf{Corr}}_t(\Delta x_t, \Delta x_{t+r}) = \left(\frac{1-\xi}{1-\tau}\right)^{\alpha - 1/2} \left(1 + \frac{\alpha - 1/2}{3}\Big(\frac{2}{1-\xi} - \frac{1}{1-\tau}\Big)\delta + O(\delta^2)\right), \tag{IA.77}$$

which implies the first equation in (IA.51).

In the fast model, $\gamma > 0$. Applying Lemma IA.1 with $f_0 = \gamma^2 \sigma_v^2$ and $f_1 = \beta_t^2 \Sigma_t + \gamma \beta_t n \sigma_v^2$, we get $\mathsf{Var}^{1/2}(\Delta x_t) = \delta^{1/2}\left(\gamma \sigma_v + \frac{\beta_t^2 \Sigma_t + \gamma \beta_t n \sigma_v^2}{2\gamma\sigma_v}\,\delta + O(\delta^2)\right)$, which we rewrite as $\mathsf{Var}^{1/2}(\Delta x_t) = \delta^{1/2}\gamma\sigma_v \left(1 + \beta_t \frac{\beta_0 \Sigma_0 + \gamma n \sigma_v^2}{2\gamma^2\sigma_v^2}\,\delta + O(\delta^2)\right)$. We now compute

$$\begin{aligned} \mathsf{Var}^{1/2}(\Delta x_t)\,\mathsf{Var}^{1/2}(\Delta x_{t+r}) &= \delta \gamma^2 \sigma_v^2 \left(1 + (\beta_t + \beta_{t+r})\frac{\beta_0 \Sigma_0 + \gamma n \sigma_v^2}{2\gamma^2 \sigma_v^2}\,\delta + O(\delta^2)\right) \\ &= \delta \gamma^2 \sigma_v^2 \left(1 + \Big(\frac{1}{1-\tau} + \frac{1}{1-\xi}\Big)\frac{\zeta}{2}\,\delta + O(\delta^2)\right), \end{aligned} \tag{IA.78}$$

where

$$\zeta = \frac{\beta_0^2 \Sigma_0 + \gamma \beta_0 n \sigma_v^2}{\gamma^2 \sigma_v^2} = \frac{\beta_0}{\gamma}\frac{\psi + gn}{g} = \frac{\Lambda \beta_0}{g}. \tag{IA.79}$$

Applying Lemma IA.1, with $f_0 = 1$, $f_1 = -\frac{1}{2}\big(\frac{\alpha}{1-\tau} + \frac{1-\alpha}{1-\xi}\big)$, $g_0 = 1$, and $g_1 = \frac{\zeta}{2}\big(\frac{1}{1-\tau} + \frac{1}{1-\xi}\big)$, we compute $\frac{f_1 g_0 - f_0 g_1}{g_0^2} = f_1 - g_1 = -\frac{1}{2}\big(\frac{\alpha + \zeta}{1-\tau} + \frac{1 - \alpha + \zeta}{1-\xi}\big)$. Thus, in the fast model we have

$$\overline{\mathsf{Corr}}_t(\Delta x_t, \Delta x_{t+r}) = \frac{\zeta}{(1-\tau)^{\alpha}(1-\xi)^{1-\alpha}}\,\delta \left(1 - \frac{1}{2}\Big(\frac{\alpha + \zeta}{1-\tau} + \frac{1-\alpha+\zeta}{1-\xi}\Big)\,\delta + O(\delta^2)\right). \tag{IA.80}$$

This implies

$$
\begin{aligned}
\overline{\mathsf{Corr}}_t(\Delta x_t, \Delta x_{t+r}) &= \frac{\Lambda\beta_0}{g(1-\xi)}\left(\frac{1-\xi}{1-\tau}\right)^{\alpha}\delta + O(\delta^2) \\
&= \frac{\Lambda\beta_{t+r}}{g}\left(\frac{1-\xi}{1-\tau}\right)^{\alpha}\delta + O(\delta^2),
\end{aligned}
\tag{IA.81}
$$

which proves the second equation in (IA.51). □

Finally, we prove some standard Taylor series results that are used in the derivation of the previous formulas.

LEMMA IA.1: *Suppose we have the following identities:*

$$
f = f_0 + f_1\delta + O(\delta^2), \qquad g = g_0 + g_1\delta + O(\delta^2). \tag{IA.82}
$$

Then the following identities are also true:

$$
f^{1/2} = f_0^{1/2} + \frac{f_1}{2f_0^{1/2}}\delta + O(\delta^2), \qquad \frac{f}{g} = \frac{f_0}{g_0} + \frac{f_1 g_0 - f_0 g_1}{g_0^2}\delta + O(\delta^2). \tag{IA.83}
$$

Proof. Write $f = f_0 + f_1\delta + O(\delta^2) = f_0\Big(1 + \frac{f_1}{f_0}\delta + O(\delta^2)\Big)$. In general, $(1+x)^{1/2} = 1 + \frac{x}{2} + O(x^2)$, and thus $\Big(1 + \frac{f_1}{f_0}\delta + O(\delta^2)\Big)^{1/2} = 1 + \frac{f_1}{2f_0}\delta + O(\delta^2)$. We get $f^{1/2} = f_0^{1/2}\Big(1 + \frac{f_1}{2f_0}\delta + O(\delta^2)\Big) = f_0^{1/2} + \frac{f_1}{2f_0^{1/2}}\delta + O(\delta^2)$. Next, write $\frac{f}{g} = \frac{f_0 + f_1\delta + O(\delta^2)}{g_0\left(1 + \frac{g_1}{g_0}\delta + O(\delta^2)\right)} = \Big(\frac{f_0}{g_0} + \frac{f_1}{g_0}\delta + O(\delta^2)\Big)\Big(1 - \frac{g_1}{g_0}\delta + O(\delta^2)\Big) = \frac{f_0}{g_0} + \Big(\frac{f_1}{g_0} - \frac{f_0 g_1}{g_0^2}\Big)\delta + O(\delta^2) = \frac{f_0}{g_0} + \frac{f_1 g_0 - f_0 g_1}{g_0^2}\delta + O(\delta^2)$. □

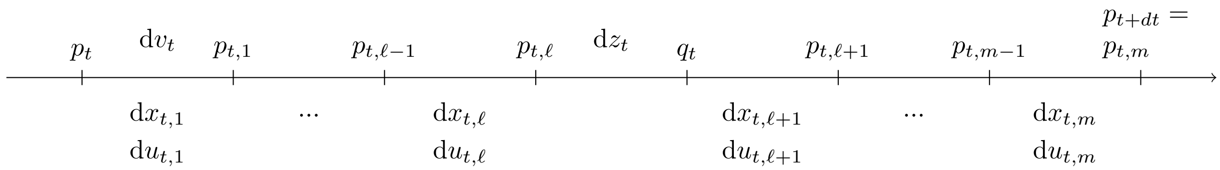

Figure IA.1. Timing of events during $[t, t + \mathrm{d}t]$.

III. Infrequent News Arrivals

In this appendix, we generalize our baseline model in two directions. First, we allow the speculator to receive signals every $m \geq 1$ trading periods. Second, the dealer receives news with a lag $\ell = 0, \ldots, m$. The parameter m is called the *news frequency*, and the parameter ℓ is called the *dealer's latency* or *lag parameter*. The case $m \geq 1$, $\ell = 0$ generalizes our baseline slow model, while the case $m \geq 1$, $\ell \in [1, m)$ generalizes our baseline fast model.

A. Model

As in the baseline model, the speculator is risk neutral and maximizes his expected profit in each trading round. The dealer is risk neutral and competitive, and sets prices equal to expected value v_1 given her information set—which consists of past news and trades. To obtain simpler formulas, we set the model in continuous time. We do so by allowing m trading rounds in the interval $[t, t + \mathrm{d}t]$, rather than just one round as in the baseline model. (See Figure IA.1.) The uncertainty in the model is driven by three diffusion processes, v_t, u_t and e_t, all with zero drift and constant volatility, as well as independent increments. At $t = 0$, before any trading takes place, the speculator learns v_0. Subsequently, at each $t \in [0, 1)$, the speculator learns $\mathrm{d}v_t$. Later in the interval $[t, t + \mathrm{d}t]$, after ℓ more trading rounds, the dealer receives a noisy signal $\mathrm{d}z_t = \mathrm{d}v_t + \mathrm{d}e_t$ (the news). Moreover, each trading round the noise traders submit an i.i.d. order flow, so that the aggregate over the m trading rounds is $\mathrm{d}u_t$.

More precisely, for each $t \in [0, 1)$, we partition the interval $[t, t + \mathrm{d}t]$ into m equal intervals, and index the endpoints by $\{0, 1, \ldots, m\}$. Denote by (t, j) the trading round at the end of the j-th interval in $[t, t + \mathrm{d}t]$, where $j = 1, \ldots, m$. Just before $(t, 1)$, the speculator privately observes $\mathrm{d}v_t$; this has variance $\mathsf{Var}(\mathrm{d}v_t) = \sigma_v^2 \mathrm{d}t$. Then, in trading round (t, j), the speculator submits a market order $\mathrm{d}x_{t,j}$ and the noise traders submit an aggregate market order $\mathrm{d}u_{t,j}$. The noise trader variance over the whole interval $[t, t+\mathrm{d}t]$ is $\sigma_u^2 \mathrm{d}t$, and thus the noise trader variance corresponding to the j-th trading interval in $[t, t+\mathrm{d}t]$ is $\frac{\sigma_u^2}{m}\mathrm{d}t$. To simplify notation, we define

$$\tilde{\sigma}_u^2 \;=\; \frac{\sigma_u^2}{m}. \tag{IA.84}$$

In trading round (t, j), the dealer only observes the aggregate order flow,

$$\mathrm{d}y_{t,j} \;=\; \mathrm{d}x_{t,j} + \mathrm{d}u_{t,j}, \tag{IA.85}$$

and sets the price $p_{t,j}$ at which trading takes place. Furthermore, the dealer publicly receives the news $\mathrm{d}z_t = \mathrm{d}v_t + \mathrm{d}e_t$ immediately after observing the order flow $\mathrm{d}y_{t,\ell}$ at $j = \ell$; the news error has variance $\mathsf{Var}(\mathrm{d}e_t) = \sigma_e^2 \mathrm{d}t$. Figure IA.1 describes the timing of events in the interval $[t, t+\mathrm{d}t]$ in the fast model.

As usual, an equilibrium is defined as a trading strategy of the speculator and a pricing policy of the dealer, such that (i) the speculator's trading strategy maximizes his expected trading profit, given the dealer's pricing policy, and (ii) the dealer's pricing policy is consistent with the equilibrium speculator's trading strategy. A *linear* equilibrium is defined as one in which the speculator's trading strategy is of the form

$$\mathrm{d}x_{t,j} \;=\; \beta_{t,j}(v_t - p_t)\mathrm{d}t + \gamma_{t,j}\big(\mathrm{d}v_t - \mathrm{d}w_{t,j}\big), \quad j = 1, \ldots, m, \tag{IA.86}$$

where $\mathrm{d}w_{t,j}$ is the dealer's expectation of $\mathrm{d}v_t$ given her information until time (t, j).[6]

Throughout the appendix, we use the following notation:

$$a \;=\; \frac{\sigma_u^2}{\sigma_v^2} \;=\; m\frac{\tilde{\sigma}_u^2}{\sigma_v^2} \;=\; m\tilde{a}, \qquad b \;=\; \frac{\sigma_e^2}{\sigma_v^2}, \qquad c \;=\; \frac{\Sigma_0}{\sigma_v^2}, \tag{IA.87}$$

B. Generalization of the Fast Model: $m \geq 1$, $\ell \in [1, m)$

In the generalized fast model, the delay parameter is $\ell \geq 1$. (See Figure IA.1.) Thus, the speculator has at least one trading round to take advantage of his information $\mathrm{d}v_t$ before the dealer learns $\mathrm{d}v_t$.

In Theorem IA.3, we prove two main results. First, we show that the optimal strategy

[6] Using the same type of argument as in Internet Appendix Section I, we rely on the intuition coming from a discrete version of the model to justify this trading strategy. In particular, we can prove that in a discrete-time linear equilibrium, the optimal strategy must be a discrete version of (IA.86).

of the speculator has a news-trading component only during the first ℓ trading rounds—before the news is revealed; after that point, the news trading intensity γ_j equals zero for $j = \ell + 1, \ldots, m$. Second, we prove that a linear equilibrium of the model exists if a certain system of nonlinear equations has a solution.

THEOREM IA.3: *Consider the model in which the speculator observes the value increments every $m \geq 1$ trading periods, and the dealer receives news with a lag $\ell = 1, \ldots, m-1$. Then the speculator's optimal strategy must have $\gamma_j = 0$ for $j = \ell+1, \ldots, m$.*

Furthermore, consider the equations (IA.147) to (IA.152), which are a $(4\ell + 2) \times (4\ell + 2)$-system in the variables $\gamma_j, \rho_j, \lambda_j, \phi_j, \mu, \tilde{\lambda}$, $j = 1, \ldots, \ell$. Then, if this system admits a positive solution, there exists a linear equilibrium of the model.

To describe the resulting equilibrium, for each $t \in [0, 1)$, extend the above solution to $j = \ell + 1, \ldots, m$ as follows: $\gamma_j = 0$, $\rho_j = 0$, $\lambda_j = \tilde{\lambda}$, $\phi_j = \tilde{\lambda}\tilde{a}$. Now, for $j = 1, \ldots, m$ let $B_j = \frac{\phi_j}{c}$ and let β_j be the function of B_i, γ_i, and ρ_i described by equation (IA.104). Then, the speculator's optimal strategy is given by

$$\mathrm{d}x_{t,j} \;=\; \begin{cases} \frac{\beta_j}{1-t}(v_t - p_t)\mathrm{d}t + \gamma_j\big(\mathrm{d}v_t - \mathrm{d}w_{t,j}\big) & \text{if } j = 1, \ldots, \ell, \\ \frac{\beta_j}{1-t}(v_t - p_t)\mathrm{d}t & \text{if } j = \ell + 1, \ldots, m, \end{cases} \tag{IA.88}$$

and the dealer's pricing policy is given by

$$\begin{aligned} p_{t,j} &= p_{t,j-1} + \lambda_j \mathrm{d}y_{t,j}, \quad j = 1, \ldots, \ell, \\ p_{t,\ell+1} &= p_{t,\ell} + \mu\big(\mathrm{d}z_t - \mathrm{d}w^+_{t,\ell}\big) + \lambda_{\ell+1}\mathrm{d}y_{t,\ell+1}, \\ p_{t,j} &= p_{t,j-1} + \lambda_j \mathrm{d}y_{t,j}, \quad j = \ell + 2, \ldots, m, \end{aligned} \tag{IA.89}$$

where the quantities $\mathrm{d}w^+_{t,\ell}$ and $\mathrm{d}w_{t,j}$ $(j = 1, \ldots, m)$ are set according to the rule

$$\begin{aligned} \mathrm{d}w_{t,1} &= 0, \\ \mathrm{d}w_{t,j+1} &= \mathrm{d}w_{t,j} + \rho_j \mathrm{d}y_{t,j}, \quad j = 1, \ldots, \ell - 1, \\ \mathrm{d}w^+_{t,\ell} &= \mathrm{d}w_{t,\ell} + \rho_\ell \mathrm{d}y_{t,\ell}, \\ \mathrm{d}w_{t,\ell+1} &= \mathrm{d}w^+_{t,\ell} + \mu\big(\mathrm{d}z_t - \mathrm{d}w^+_{t,\ell}\big), \\ \mathrm{d}w_{t,j+1} &= \mathrm{d}w_{t,j}, \quad j = \ell + 1, \ldots, m - 1. \end{aligned} \tag{IA.90}$$

Thus, to obtain an equilibrium of the generalized fast model, Theorem IA.3 shows that it is sufficient to solve a $(4\ell+2)$-dimensional system of nonlinear equations. We are not able to analytically prove the existence of a solution to this system. Numerically, however, the Newton method readily produces solutions for all the parameter values we have checked. Figure IA.2 displays the solution corresponding to the parameter values $\sigma_v = 1$, $\sigma_u = 1$, and $\Sigma_0 = 1$, and compares the case $\sigma_e = 0.5$ (news informativeness $\nu = \frac{1}{\sigma_e} = 2$ is high) to the case $\sigma_e = 2$ (news informativeness $\nu = 0.5$ is low).

C. Proof of Theorem IA.3

We divide the proof into two parts. The first part (Section III.C.1) takes the dealer's pricing policy in (IA.89) and (IA.90) as given, and shows that the speculator's strategy in (IA.88) is optimal. The second part (Section III.C.2) takes the speculator's strategy as given, and shows that the dealer's pricing policy in (IA.89) and (IA.90) satisfies the zero-profit condition.

C.1. Speculator's Optimal Strategy (β, γ)

In this section, we assume that the dealer's pricing policy in (IA.89) and (IA.90) is fixed. Since we are interested only in the existence of an equilibrium, we consider pricing functions with constant coefficients: $\lambda_j > 0$ ($j = 1, \ldots, m$), $\rho_j > 0$ ($j = 1, \ldots, \ell$), and $\mu > 0$. For future use, note that equation (IA.90) implies

$$\mathrm{d}w_{t,\ell}^{+} \;=\; \sum_{j=1}^{\ell} \rho_j \mathrm{d}y_{t,j}. \tag{IA.91}$$

Moreover, we substitute (IA.91) into the price equations (IA.89) to obtain

$$\mathrm{d}p_t \;=\; p_{t,m} - p_{t,0} \;=\; \sum_{j=1}^{\ell} (\lambda_j - \mu\rho_j)\mathrm{d}y_{t,j} \;+\; \sum_{j=\ell+1}^{m} \lambda_j \mathrm{d}y_{t,j} \;+\; \mu \mathrm{d}z_t. \tag{IA.92}$$

By the definition of a linear equilibrium, we assume that the speculator chooses for an optimal strategy of the form

$$\mathrm{d}x_{t,j} \;=\; \beta_{t,j}(v_t - p_t)\mathrm{d}t + \gamma_{t,j}\big(\mathrm{d}v_t - \mathrm{d}w_{t,j}\big), \quad j = 1, \ldots, m. \tag{IA.93}$$

We show that $\gamma_{t,j} = 0$ for all t and for $j = \ell + 1, \ldots, m$. To simplify notation, we omit the dependence on time. Let

$$\tau \;=\; (v_t - p_t)\mathrm{d}t. \tag{IA.94}$$

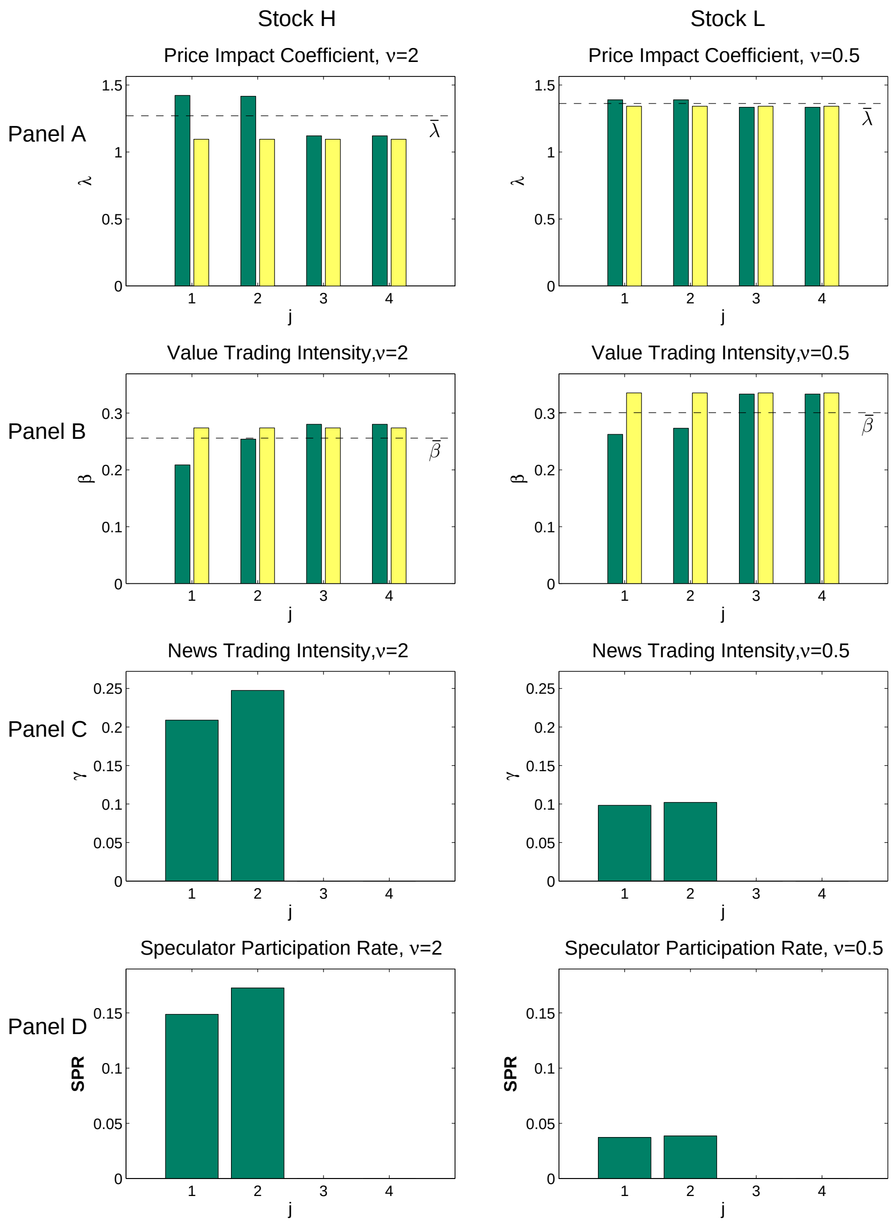

Figure IA.2. Effect of news informativeness in the general case. We plot equilibrium values of various variables of interest in a given interval $[t, t + \mathrm{d}t]$ in each trading round j for two different stocks, H (plots on the left) and L (plots on the right). The news frequency is $m = 4$ for each stock. News informativeness is higher for stock H than for stock L ($\nu = \frac{1}{\sigma_e} = 2$ versus $\nu = \frac{1}{\sigma_e} = 0.5$). For each stock, we show equilibrium values of $\lambda_{t,j}$ (illiquidity), $\beta_{t,j}$ (value trading intensity), $\gamma_{t,j}$ (news trading intensity), and $SPR_{t,j} = \frac{\mathsf{Var}(\mathrm{d}x_{t,j})}{\mathsf{Var}(\mathrm{d}x_{t,j})+Var(\mathrm{d}u_{t,j})}$ in each trading round $j \in \{1, 2, 3, 4\}$ when the dealer's latency is $\ell = 2$ (left dark bars) or $\ell = 0$ (right light bars). The horizontal dotted lines correspond to the average value of the relevant variable over the four trading rounds when the speculator is fast (e.g., $\bar{\lambda} = (\sum_{j=1}^{j=4} \lambda_{t,j})/4$) in the fast model. The other parameter values are $\sigma_u = 1$ (standard deviation of noise traders' order flow), $\sigma_v = 1$ (standard deviation of innovations in the asset value), and $\Sigma_0 = 1$ (variance of asset value conditional on information available at date 0).

We now show that (IA.86) can be decomposed into orthogonal components:

$$\mathrm{d}x_j \;=\; B_j\tau + G_j\mathrm{d}v + \sum_{i=1}^{j-1} A_j^i \mathrm{d}u_i + E_j\mathrm{d}e, \quad j = 1,\ldots,m. \tag{IA.95}$$

To compute B_j, G_j, A_j^i, and E_j, we derive recursive equations for $\mathrm{d}x_j$. For this purpose, we rewrite the recursive equations (IA.90) by eliminating $\mathrm{d}w_\ell^+$:

$$\begin{aligned}
\mathrm{d}w_1 &= 0,\\
\mathrm{d}w_{j+1} &= \mathrm{d}w_j + \rho_j\mathrm{d}y_j, \quad j = 1,\ldots,\ell-1,\\
\mathrm{d}w_{\ell+1} &= (1-\mu)\big(\mathrm{d}w_\ell + \rho_\ell\mathrm{d}y_\ell\big) + \mu\mathrm{d}z,\\
\mathrm{d}w_{j+1} &= \mathrm{d}w_j, \quad j = \ell+1,\ldots,m-1.
\end{aligned} \tag{IA.96}$$

If we define $\tilde{\gamma}_{\ell+1} = \gamma_{\ell+1}(1-\mu)$, then equations (IA.86) and (IA.96) imply the following recursive equations for $\mathrm{d}x_j$:

$$\begin{aligned}
\mathrm{d}x_1 &= \beta_1\tau + \gamma_1\mathrm{d}v,\\
\mathrm{d}x_{j+1} &= \frac{\gamma_{j+1}}{\gamma_j}(1-\rho_j\gamma_j)\mathrm{d}x_j + \Big(\beta_{j+1} - \frac{\gamma_{j+1}}{\gamma_j}\beta_j\Big)\tau - \gamma_{j+1}\rho_j\mathrm{d}u_j, \quad j = 1,\ldots,\ell-1,\\
\mathrm{d}x_{\ell+1} &= \frac{\tilde{\gamma}_{\ell+1}}{\gamma_\ell}(1-\rho_\ell\gamma_\ell)\mathrm{d}x_\ell + \Big(\beta_{\ell+1} - \frac{\tilde{\gamma}_{\ell+1}}{\gamma_\ell}\beta_\ell\Big)\tau - \tilde{\gamma}_{\ell+1}\rho_\ell\mathrm{d}u_\ell - \gamma_{\ell+1}\mu\mathrm{d}e,\\
\mathrm{d}x_{j+1} &= \frac{\gamma_{j+1}}{\gamma_j}\mathrm{d}x_j + \Big(\beta_{j+1} - \frac{\gamma_{j+1}}{\gamma_j}\beta_j\Big)\tau, \quad j = \ell+1,\ldots,m-1.
\end{aligned} \tag{IA.97}$$

Putting together equations (IA.95) and (IA.97), we obtain recursive equations for B_j, G_j, A_j^i, and E_j. The equations for B_j are

$$\begin{aligned}
B_1 &= \beta_1,\\
B_{j+1} &= \frac{\gamma_{j+1}}{\gamma_j}(1-\rho_j\gamma_j)\,B_j + \Big(\beta_{j+1} - \frac{\gamma_{j+1}}{\gamma_j}\beta_j\Big), \quad j = 1,\ldots,\ell-1,\\
B_{\ell+1} &= \frac{\gamma_{\ell+1}(1-\mu)}{\gamma_\ell}(1-\rho_\ell\gamma_\ell)\,B_\ell + \Big(\beta_{\ell+1} - \frac{\gamma_{\ell+1}(1-\mu)}{\gamma_\ell}\beta_\ell\Big),\\
B_{j+1} &= \frac{\gamma_{j+1}}{\gamma_j}\,B_j + \Big(\beta_{j+1} - \frac{\gamma_{j+1}}{\gamma_j}\beta_j\Big), \quad j = \ell+1,\ldots,m-1.
\end{aligned} \tag{IA.98}$$

The equations for G_j are

$$\begin{aligned} G_1 &= \gamma_1, \\ G_{j+1} &= \frac{\gamma_{j+1}}{\gamma_j}(1-\rho_j\gamma_j)\, G_j, \quad j = 1, \ldots, \ell-1, \\ G_{\ell+1} &= \frac{\gamma_{\ell+1}(1-\mu)}{\gamma_\ell}(1-\rho_\ell\gamma_\ell)\, G_\ell, \\ G_{j+1} &= \frac{\gamma_{j+1}}{\gamma_j}\, G_j, \quad j = \ell+1, \ldots, m-1. \end{aligned} \tag{IA.99}$$

The equations for A^i_j are

$$\begin{aligned} A^j_{j+1} &= -\rho_j\gamma_{j+1}, \quad A^i_{j+1} = \frac{\gamma_{j+1}}{\gamma_j}(1-\rho_j\gamma_j)\, A^i_j, \quad i < j = 1, \ldots, \ell-1, \\ A^\ell_{\ell+1} &= -\rho_\ell\gamma_{\ell+1}(1-\mu), \quad A^i_{\ell+1} = \frac{\gamma_{\ell+1}(1-\mu)}{\gamma_\ell}(1-\rho_\ell\gamma_\ell)\, A^i_\ell, \quad i < \ell, \\ A^j_{j+1} &= 0, \quad A^i_{j+1} = \frac{\gamma_{j+1}}{\gamma_j}\, A^i_j, \quad i < j = \ell+1, \ldots, m-1. \end{aligned} \tag{IA.100}$$

The equations for E_j are

$$\begin{aligned} E_1 &= 0, \\ E_{j+1} &= \frac{\gamma_{j+1}}{\gamma_j}(1-\rho_j\gamma_j)\, E_j, \quad j = 1, \ldots, \ell-1, \\ E_{\ell+1} &= \frac{\gamma_{\ell+1}(1-\mu)}{\gamma_\ell}(1-\rho_\ell\gamma_\ell)\, E_\ell - \gamma_{\ell+1}\mu, \\ E_{j+1} &= \frac{\gamma_{j+1}}{\gamma_j}\, E_j, \quad j = \ell+1, \ldots, m-1. \end{aligned} \tag{IA.101}$$

To obtain explicit formulas for G and A, define

$$C^i_j = \begin{cases} \prod_{k=i}^{j}(1-\rho_k\gamma_k) & \text{if } i \leq j = 1, \ldots, m, \\ 1 & \text{if } i > j. \end{cases} \tag{IA.102}$$

Then B, G, A, and E satisfy the following equations:

$$
\begin{aligned}
B_j &= \begin{cases} \beta_j + \sum_{i=1}^{j-1} A_j^i \beta_i & \text{if } j = 1, \dots, \ell, \\ \beta_j + \sum_{i=1}^{\ell} A_j^i \beta_i & \text{if } j = \ell+1, \dots, m. \end{cases} \\
G_j &= \begin{cases} \gamma_j C_{j-1}^1 & \text{if } j = 1, \dots, \ell, \\ (1-\mu)\gamma_j C_\ell^1 & \text{if } j = \ell+1, \dots, m. \end{cases} \\
A_j^i &= \begin{cases} -\rho_i \gamma_j C_{j-1}^{i+1} & \text{if } i < j = 1, \dots, \ell, \\ -(1-\mu)\rho_i \gamma_j C_\ell^{i+1} & \text{if } i < j = \ell+1, \dots, m. \end{cases} \\
E_j &= \begin{cases} 0 & \text{if } j = 1, \dots, \ell, \\ -\mu\gamma_j & \text{if } j = \ell+1, \dots, m. \end{cases}
\end{aligned}
\tag{IA.103}
$$

We can also express β_j as a function of B_j as follows:

$$
\beta_j = \begin{cases} B_j + \gamma_j \sum_{i=1}^{j-1} \rho_i B_i & \text{if } j = 1, \dots, \ell, \\ B_j & \text{if } j = \ell+1, \dots, m. \end{cases}
\tag{IA.104}
$$

Define

$$
\tilde{\lambda}_j = \begin{cases} \lambda_j - \mu\rho_j, & \text{if } \quad j = 1, \dots, \ell, \\ \lambda_j & \text{if } \quad j = \ell+1, \dots, m. \end{cases}
\tag{IA.105}
$$

We now make explicit again the dependence on time. Let

$$
\mathrm{D}p_{t,j} = p_{t,j} - p_{t,0}, \quad j = 1, \dots, m.
\tag{IA.106}
$$

Then equation (IA.89) implies

$$
\begin{aligned}
\mathrm{D}p_{t,j} &= \sum_{i=1}^{j} \lambda_i \mathrm{d}y_{t,i}, \quad j = 1, \dots, \ell, \\
\mathrm{D}p_{t,j} &= \sum_{i=1}^{j} \tilde{\lambda}_i \mathrm{d}y_{t,i} + \mu \mathrm{d}z_t, \quad j = \ell+1, \dots, m.
\end{aligned}
\tag{IA.107}
$$

Using equation (IA.95), we get $\mathrm{d}x_{t,j} = B_{t,j}(v_t - p_t)\mathrm{d}t + G_{t,j}\mathrm{d}v_t + \sum_{i=1}^{j-1} A_{t,j}^i \mathrm{d}u_{t,i} + E_{t,j}\mathrm{d}e_t$. We compute

$$
\mathrm{D}p_{t,k} = X_{t,k}(v_t - p_t)\mathrm{d}t + Y_{t,k}\,\mathrm{d}v_t + \sum_{i=1}^{k} Z_{t,k}^i\,\mathrm{d}u_{t,i} + W_{t,k}\,\mathrm{d}e_t,
\tag{IA.108}
$$

where for $k = 1, \dots, \ell$ and $i = 1, \dots, k$,

$$\begin{aligned} X_{t,k} &= \sum_{j=1}^{k} \lambda_j B_{t,j}, \quad Y_{t,k} = \sum_{j=1}^{k} \lambda_j G_{t,j}, \quad W_{t,k} = \sum_{j=1}^{k} \lambda_j E_{t,j}, \\ Z^i_{t,k} &= \lambda_i + \sum_{j=i+1}^{k} \lambda_j A^i_{t,j}, \end{aligned} \tag{IA.109}$$

and for $k = \ell+1, \dots, m$ and $i = 1, \dots, k$,

$$\begin{aligned} X_{t,k} &= \sum_{j=1}^{k} \tilde{\lambda}_j B_{t,j}, \quad Y_{t,k} = \mu_t + \sum_{j=1}^{k} \tilde{\lambda}_j G_{t,j}, \quad W_{t,k} = \mu_t + \sum_{j=1}^{k} \tilde{\lambda}_j E_{t,j}, \\ Z^i_{t,k} &= \tilde{\lambda}_i + \sum_{j=i+1}^{k} \tilde{\lambda}_j A^i_{t,j}. \end{aligned} \tag{IA.110}$$

In particular, $\mathrm{D}p_{t,m} = p_{t,m} - p_{t,0} = p_{t+\mathrm{d}t} - p_t = \mathrm{d}p_t$. Therefore,

$$\mathrm{d}p_t = X_{t,m}(v_t - p_t)\mathrm{d}t + Y_{t,m}\,\mathrm{d}v_t + \sum_{i=1}^{m} Z^i_{t,m}\,\mathrm{d}u_{t,i} + W_{t,m}\,\mathrm{d}e_t. \tag{IA.111}$$

For $\tau \geq t$, define

$$V_{t,\tau} = \mathsf{E}_t\big((v_\tau - p_\tau)^2\big), \tag{IA.112}$$

where the expectation is conditional on the speculator's information set at t. Then $V_{t,\tau}$ evolves according to

$$\begin{aligned} V_{t,\tau+\mathrm{d}\tau} &= \mathsf{E}_t\Big((v_{\tau+\mathrm{d}\tau} - p_{\tau+\mathrm{d}\tau})^2\Big) \\ &= V_{t,\tau} + 2\,\mathsf{E}_t\Big((v_\tau - p_\tau)(\mathrm{d}v_\tau - \mathrm{d}p_\tau)\Big) + \mathsf{E}_t\Big((\mathrm{d}v_\tau - \mathrm{d}p_\tau)^2\Big) \\ &= V_{t,\tau} - 2X_{\tau,m}V_{t,\tau}\,\mathrm{d}\tau + (1 - Y_{\tau,m})^2\sigma_v^2\,\mathrm{d}\tau + \sum_{j=1}^{m}(Z^j_{\tau,m})^2\tilde{\sigma}_u^2\,\mathrm{d}\tau + W^2_{\tau,m}\sigma_e^2\,\mathrm{d}\tau. \end{aligned} \tag{IA.113}$$

Since $X_{\tau,m} = \sum_{j=1}^{m} \tilde{\lambda}_j B_{\tau,j}$, $V_{t,\tau}$ satisfies the first-order linear ODE:

$$\frac{\mathrm{d}V_{t,\tau}}{\mathrm{d}\tau} = -2\left(\sum_{j=1}^{m} \tilde{\lambda}_j B_{\tau,j}\right) V_{t,\tau} + (1 - Y_{\tau,m})^2\sigma_v^2 + \sum_{j=1}^{m}(Z^j_{\tau,m})^2\tilde{\sigma}_u^2 + W^2_{\tau,m}\sigma_e^2. \tag{IA.114}$$

For $t \in [0,1)$, we compute the speculator's expected profit

$$\pi_t \;=\; \mathsf{E}_t\left(\int_t^1 \sum_{j=1}^m (v_1 - p_{\tau,j})\mathrm{d}x_{\tau,j}\right) \;=\; \mathsf{E}_t\left(\int_t^1 \sum_{j=1}^m \Big(v_\tau + \mathrm{d}v_\tau - p_\tau - \mathrm{D}p_{\tau,j}\Big)\mathrm{d}x_{\tau,j}\right), \tag{IA.115}$$

where the second equality follows from the law of iterated expectations. From (IA.95) and (IA.108), we get

$$\pi_t \;=\; \int_t^1 \sum_{j=1}^m \left[B_{\tau,j}V_{t,\tau} + G_{\tau,j}(1-Y_{\tau,j})\sigma_v^2 - \sum_{i=1}^{j-1} A_{\tau,j}^i Z_{\tau,j}^i \tilde{\sigma}_u^2 - E_{\tau,j}W_{\tau,j}\sigma_e^2\right]\mathrm{d}\tau. \tag{IA.116}$$

Note that the profit integrand at τ depends only on the sum $\sum_{j=1}^m B_{\tau,j}$, while $V_{t,\tau}$ evolves according to the sum $\sum_{j=1}^m \tilde{\lambda}_j B_{\tau,j}$. Then, by the usual argument, the existence of a maximum implies that for all $\tau \in [t,1)$, $\tilde{\lambda}_j$ does not depend on j. To see this, note that the condition $\tilde{\lambda}_j > \tilde{\lambda}_k$ for some $j \neq k$ is incompatible with equilibrium. Indeed, the speculator could then decrease $B_{\tau,j}$ by $\varepsilon > 0$ and increase $B_{\tau,k}$ by $\varepsilon\tilde{\lambda}_j/\tilde{\lambda}_k$. This would not affect the evolution of $V_{t,\tau}$, as the sum $\tilde{\lambda}_j B_{\tau,j} + \tilde{\lambda}_k B_{\tau,k}$ is the same, but it would increase the sum $B_{\tau,j} + B_{\tau,k}$ by $\varepsilon(\tilde{\lambda}_j/\tilde{\lambda}_k - 1) > 0$, and hence increase the expected profit. Therefore, for all $j = 1,\dots,m$, we have

$$\tilde{\lambda}_{\tau,j} \;=\; \text{constant} \;=\; \tilde{\lambda}. \tag{IA.117}$$

Next, consider the variance of the forecast error,

$$\Sigma_t \;=\; \mathsf{E}_0\big((v_t - p_t)^2\big) \;=\; V_{0,\tau}. \tag{IA.118}$$

Since $\tilde{\lambda}$ is constant, equation (IA.114) implies that Σ_t satisfies the ODE

$$\frac{\mathrm{d}\Sigma_t}{\mathrm{d}t} \;=\; -2\tilde{\lambda}\left(\sum_{j=1}^m B_{t,j}\right)\Sigma_t + (1-Y_{t,m})^2\sigma_v^2 + \sum_{j=1}^m (Z_{t,m}^j)^2\tilde{\sigma}_u^2 + W_{t,m}^2\sigma_e^2. \tag{IA.119}$$

Using (IA.116), the speculator's expected profit at $t = 0$ becomes

$$\pi_0 \;=\; \int_0^1 \left[\left(\sum_{j=1}^m B_{t,j}\right)\Sigma_t + \sum_{j=1}^m \left(G_{t,j}(1-Y_{t,j})\sigma_v^2 - \sum_{i=1}^{j-1} A_{t,j}^i Z_{t,j}^i \tilde{\sigma}_u^2 - E_{t,j}W_{t,j}\sigma_e^2\right)\right]\mathrm{d}t. \tag{IA.120}$$

From (IA.119), we have

$$\left(\sum_{j=1}^{m} B_{t,j}\right)\Sigma_t \;=\; \frac{(1-Y_{t,m})^2\sigma_v^2 + \sum_{j=1}^m (Z_{t,m}^j)^2\tilde{\sigma}_u^2 + W_{t,m}^2\sigma_e^2}{2\tilde{\lambda}} - \frac{\Sigma_t'}{2\tilde{\lambda}}. \tag{IA.121}$$

Substituting $\left(\sum_{j=1}^m B_{t,j}\right)\Sigma_t$ into (IA.120) and integrating by parts, we obtain:

$$\begin{aligned}\pi_0 \;=\; & \frac{\Sigma_0}{2\tilde{\lambda}} - \frac{\Sigma_1}{2\tilde{\lambda}} + \frac{1}{2\tilde{\lambda}}\int_0^1 \left((1-Y_{t,m})^2\sigma_v^2 + \sum_{j=1}^m (Z_{t,m}^j)^2\tilde{\sigma}_u^2 + W_{t,m}^2\sigma_e^2\right)\mathrm{d}t \\ & + \sum_{j=1}^m \int_0^1 \left(G_{t,j}(1-Y_{t,j})\sigma_v^2 - \sum_{i=1}^{j-1} A_{t,j}^i Z_{t,j}^i \tilde{\sigma}_u^2 - E_{t,j}W_{t,j}\sigma_e^2\right)\mathrm{d}t.\end{aligned} \tag{IA.122}$$

Since $\Sigma_t = V_{0,t} > 0$ can be chosen arbitrarily by the speculator, we obtain the following condition for a maximum (the transversality condition):

$$\Sigma_1 \;=\; 0. \tag{IA.123}$$

We now use equation (IA.122) to compute the speculator's first-order condition with respect to the other choice variables: $\gamma_{t,j}$ for $j = \ell+1, \ldots, m$. From (IA.103), we see that for $j > \ell$ the integrand of (IA.122) is a quadratic function of $\gamma_{t,j}$. Let $1_{\mathcal{P}}$ be either one if $\mathcal{P}$ is true or zero otherwise. From (IA.103) and (IA.110) we derive the following formulas, for $j, k \geq \ell + 1$:

$$\begin{aligned}&\frac{\partial G_{t,j}}{\partial \gamma_{t,k}} \;=\; 1_{j=k}(1-\mu)C_{t,L}^1, \quad \frac{\partial A_{t,j}^i}{\partial \gamma_{t,k}} \;=\; -1_{j=k}(1-\mu)\rho_i C_{t,L}^{i+1}, \quad \frac{\partial E_{t,j}}{\partial \gamma_{t,k}} \;=\; -1_{j=k}\mu, \\ &\frac{\partial Y_{t,j}}{\partial \gamma_{t,k}} \;=\; 1_{j\geq k}\tilde{\lambda}(1-\mu)C_{t,L}^1, \quad \frac{\partial Z_{t,j}^i}{\partial \gamma_{t,k}} \;=\; -1_{i+1\geq k\geq j}\tilde{\lambda}(1-\mu)\rho_i C_{t,L}^{i+1}, \quad \frac{\partial W_{t,j}}{\partial \gamma_{t,k}} \;=\; -1_{j\geq k}\tilde{\lambda}\mu.\end{aligned} \tag{IA.124}$$

Using (IA.124), we compute

$$\frac{\partial \pi_0}{\partial \gamma_{t,k}} \;=\; -\tilde{\lambda}\left((1-\mu)^2(C_{t,L}^1)^2\sigma_v^2 + (1-\mu)^2\sum_{i=1}^{k-1}\rho_i^2(C_{t,L}^{i+1})^2\tilde{\sigma}_u^2 + \mu^2\sigma_e^2\right)\gamma_{t,k}, \quad k = \ell{+}1, \ldots, m. \tag{IA.125}$$

Since the term in parentheses does not depend on γ_k, the first-order condition implies $\gamma_{t,k} = 0$. Moreover, the second-order condition for a maximum is also satisfied. Therefore, for all t the speculator optimally chooses

$$\gamma_{t,\ell+1} \;=\; \cdots \;=\; \gamma_{t,m} \;=\; 0. \tag{IA.126}$$

We have now finished the proof of the first part of the theorem.

But (IA.126) implies that the formulas in (IA.103) simplify for $j > \ell$:

$$G_{t,j} \;=\; A^i_{t,j} \;=\; E_{t,j} \;=\; 0 \text{ if } j = \ell+1, \ldots, m. \tag{IA.127}$$

Equation (IA.122) becomes (after imposing the transversality condition $\Sigma_1 = 0$)

$$\begin{aligned}\pi_0 \;=\; & \frac{\Sigma_0}{2\tilde{\lambda}} + \frac{1}{2\tilde{\lambda}} \int_0^1 \Big((1 - Y_{t,m})^2 \sigma_v^2 + \sum_{j=1}^m (Z^j_{t,m})^2 \tilde{\sigma}_u^2 + \mu^2 \sigma_e^2 \Big) \mathrm{d}t \\ & + \sum_{j=1}^{\ell} \int_0^1 \Big(G_{t,j} (1 - Y_{t,j}) \sigma_v^2 - \sum_{i=1}^{j-1} A^i_{t,j} Z^i_{t,j} \tilde{\sigma}_u^2 \Big) \mathrm{d}t.\end{aligned} \tag{IA.128}$$

Using equations (IA.109) and (IA.110), we compute the following equality up to a constant C_0:[7]

$$\begin{aligned}\pi_0 \;=\; & \frac{1}{2\tilde{\lambda}} \int_0^1 \Bigg[\Big(1 - \mu - \tilde{\lambda} \sum_{j=1}^{\ell} G_{t,j} \Big)^2 \sigma_v^2 + \sum_{j=1}^{\ell-1} \tilde{\lambda}^2 \Big(1 + \sum_{k=j+1}^{\ell} \big(A^j_{t,k} \big) \Big)^2 \tilde{\sigma}_u^2 \Bigg] \mathrm{d}t \\ & + \sum_{j=1}^{\ell} \int_0^1 \Bigg[G_{t,j} \Big(1 - \sum_{i=1}^{j} \lambda_i G_{t,i} \Big) \sigma_v^2 - \sum_{i=1}^{j-1} A^i_{t,j} \Big(\lambda_i + \sum_{k=i+1}^{j} \lambda_k A^i_{t,k} \Big) \tilde{\sigma}_u^2 \Bigg] \mathrm{d}t \;+\; C_0.\end{aligned} \tag{IA.129}$$

This allows us to compute the first-order conditions with respect to $\gamma_{t,j}$, $j = 1, \ldots, \ell$. Since the formulas are too complicated to be written explicitly, we simply summarize the first-order conditions by writing $\frac{\partial \pi_0}{\partial \gamma_j} = 0$, $j = 1, \ldots, \ell$, which are equations (IA.148).

C.2. Dealer's Pricing Functions (λ, ρ, μ)

In this section, we assume that the speculator's trading strategy is given by (IA.88)

$$\mathrm{d}x_{t,j} \;=\; \begin{cases} \frac{\beta_j}{1-t}(v_t - p_t)\mathrm{d}t + \gamma_j \big(\mathrm{d}v_t - \mathrm{d}w_{t,j} \big) & \text{if } j = 1, \ldots, \ell, \\ \frac{\beta_j}{1-t}(v_t - p_t)\mathrm{d}t & \text{if } j = \ell+1, \ldots, m. \end{cases} \tag{IA.130}$$

[7]The constant is $C_0 = \frac{1}{2\tilde{\lambda}} \big(\Sigma_0 + \mu^2 \sigma_e^2 + \sum_{j=\ell}^{j=m} \tilde{\lambda}^2 \tilde{\sigma}_u^2 \big)$.

Since we are interested only in the existence of an equilibrium, we focus on strategies with constant coefficients: $\beta_j > 0$ for $j = 1, \dots, m$, and $\gamma_j > 0$ for $j = 1, \dots, \ell$.

From equation (IA.95), we have $\mathrm{d}x_{t,j} = B_{t,j}(v_t - p_t)\mathrm{d}t + G_{t,j}\mathrm{d}v_t + \sum_{i=1}^{j-1} A^i_{t,j}\mathrm{d}u_{t,i}$ which implies that, for $j = 1, \dots, \ell$,

$$\begin{aligned} \rho_{t,j} &= \frac{\mathsf{Cov}(\mathrm{d}v_t, \mathrm{d}y_{t,j})}{\mathsf{Var}(\mathrm{d}y_{t,j})} = \frac{G_{t,j}}{G^2_{t,j} + \left(1 + \sum_{i=1}^{j-1}\left(A^i_{t,j}\right)^2\right)\tilde{a}} \\ \lambda_{t,j} &= \frac{\mathsf{Cov}_t(v_1, \mathrm{d}y_{t,j})}{\mathsf{Var}_t(\mathrm{d}y_{t,j})} = \frac{\phi_{t,j} + G_{t,j}}{G^2_{t,j} + \left(1 + \sum_{i=1}^{j-1}\left(A^i_{t,j}\right)^2\right)\tilde{a}}, \end{aligned} \tag{IA.131}$$

where

$$\phi_{t,j} = \frac{B_{t,j}\Sigma_t}{\sigma^2_v}, \quad j = 1, \dots, m. \tag{IA.132}$$

For $j = \ell + 1, \dots, m$,

$$\tilde{\lambda} = \lambda_{t,j} = \frac{\mathsf{Cov}_t(v_1, \mathrm{d}y_{t,j})}{\mathsf{Var}_t(\mathrm{d}y_{t,j})} = \frac{B_{t,j}\Sigma_t}{\tilde{\sigma}^2_u}, \tag{IA.133}$$

which implies

$$\phi_{t,j} = \tilde{\lambda}\,\tilde{a}, \quad j = \ell + 1, \dots, m. \tag{IA.134}$$

We search for an equilibrium in which $\lambda_{t,j}$, $\rho_{t,j}$, μ_t, and $\phi_{t,j}$ are all time-independent. Hence, the speculator's expected profit at $t = 0$ given by (IA.129) has time-independent coefficients, and the optimum $\gamma_{t,j}$ should also be time-independent. Thus, in the rest of this appendix we ignore the time-dependence for all variables, except $B_{t,j}$ and Σ_t (although their product is constant).

Define

$$\widetilde{\mathrm{d}z_t} = \mathrm{d}z_t - \mathrm{d}w^+_{t,\ell} = \mathrm{d}z_t - \sum_{j=1}^{\ell}\rho_j \mathrm{d}y_{t,j}, \tag{IA.135}$$

where the second equality follows from (IA.91). Equation (IA.95) implies that

$$\widetilde{\mathrm{d}z_t} = -\left(\sum_{j=1}^{\ell}\rho_j B_{t,j}\right)(p_t - v_t)\mathrm{d}t + \left(1 - \sum_{j=1}^{\ell}\rho_j G_j\right)\mathrm{d}v_t - \sum_{j=1}^{\ell}\left(\rho_j + \sum_{k=j+1}^{\ell}\left(\rho_k A^j_k\right)\right)\mathrm{d}u_{t,j} + \mathrm{d}e_t. \tag{IA.136}$$

We then have

$$\mu \;=\; \frac{\mathsf{Cov}_t(v_1, \widetilde{\mathrm{d}z_t})}{\mathsf{Var}_t(\widetilde{\mathrm{d}z_t})} \;=\; \frac{-\sum_{j=1}^{\ell}\rho_j B_j \Sigma + \left(1-\sum_{j=1}^{\ell}\rho_j G_j\right)\sigma_v^2}{\left(1-\sum_{j=1}^{\ell}\rho_j G_j\right)^2\sigma_v^2 + \sum_{j=1}^{\ell}\left(\rho_j + \sum_{k=j+1}^{\ell}\left(\rho_k A_k^j\right)\right)^2\tilde{\sigma}_u^2 + \sigma_e^2}. \tag{IA.137}$$

In general, $1-\sum_{j=1}^{\ell}\rho_j G_j = 1-\sum_{j=1}^{\ell}\rho_j\gamma_j C_{j-1}^1 = 1-\sum_{j=1}^{\ell}(C_{j-1}^1 - C_j^1) = 1-(1-C_\ell^1) = C_\ell^1$. Similarly, $\rho_j + \sum_{k=j+1}^{\ell}\left(\rho_k A_k^j\right) = \rho_j\left(1-\sum_{k=j+1}^{\ell}\rho_k\gamma_k C_{k-1}^{j+1}\right) = \rho_j C_\ell^{j+1}$. If we define $C_\ell^{\ell+1} = 1$, we get

$$\mu \;=\; \frac{-\sum_{j=1}^{\ell}\rho_j(B_{t,j}\Sigma_t) + \left(C_\ell^1\right)\sigma_v^2}{\left(C_\ell^1\right)^2\sigma_v^2 + \sum_{j=1}^{\ell}\left(\rho_j C_\ell^{j+1}\right)^2\tilde{\sigma}_u^2 + \sigma_e^2} \;=\; \frac{-\sum_{j=1}^{\ell}\rho_j\phi_j + \left(C_\ell^1\right)}{\left(C_\ell^1\right)^2 + \tilde{a}\sum_{j=1}^{\ell}\left(\rho_j C_\ell^{j+1}\right)^2 + b}. \tag{IA.138}$$

C.3. Equilibrium Formulas

We now put together the equations derived in Sections III.C.1 and III.C.2, to derive the equations satisfied by the equilibrium coefficients. This is a system of equations in the variables $\gamma_j, \rho_j, \lambda_j, \phi_j, \mu$, and $\tilde{\lambda}$, for $j = 1, \ldots, \ell$—these are the coefficients used in the speculator's optimal strategy and the dealer's pricing functions. The dealer's pricing functions are given by the formulas

$$\mathrm{d}w_{t,j} \;=\; \sum_{i=1}^{j-1}\rho_i \mathrm{d}y_{t,i}, \quad j = 1, \ldots, \ell, \qquad \mathrm{d}w_{t,\ell}^{+} \;=\; \sum_{i=1}^{\ell}\rho_i \mathrm{d}y_{t,i}, \tag{IA.139}$$

and, for $j = 1, \ldots, m$,

$$\begin{aligned} p_{t,j} - p_{t,j-1} &= \lambda_j \mathrm{d}y_{t,j}, \quad j \neq \ell+1, \\ p_{t,\ell+1} - p_{t,\ell} &= \lambda_{\ell+1}\mathrm{d}y_{t,\ell+1} + \mu \mathrm{d}z_t, \quad j = \ell+1. \end{aligned} \tag{IA.140}$$

To describe the speculator's optimal strategy at (t, j), note that in equilibrium the coefficient E_j on $\mathrm{d}e$ is zero for all $j = 1, \ldots, m$. Equations (IA.88) and (IA.95) then become, for $j = 1, \ldots, m$,

$$\begin{aligned} \mathrm{d}x_{t,j} &= \frac{\beta_j}{1-t}(v_t - p_t)\,\mathrm{d}t + \gamma_j(\mathrm{d}v_t - \mathrm{d}w_{t,j}), \\ &= \frac{B_j}{1-t}(v_t - p_t)\,\mathrm{d}t + G_j\,\mathrm{d}v_t + \sum_{i=1}^{j-1}A_j^i\,\mathrm{d}u_{i,t}, \end{aligned} \tag{IA.141}$$

where $\gamma_j = 0$ and $A^i_j = 0$ for $i < j = \ell+1,\ldots,m$. From (IA.122), the equilibrium expected profit at $t=0$ is

$$\begin{aligned}\pi_0 \;&=\; \frac{\Sigma_0}{2\tilde{\lambda}} + \frac{1}{2\tilde{\lambda}}\int_0^1 \left((1-Y_m)^2\sigma_v^2 + \sum_{j=1}^m (Z^j_m)^2\tilde{\sigma}_u^2 + \mu^2\sigma_e^2\right)\mathrm{d}t \\ &\quad + \sum_{j=1}^{\ell}\int_0^1 \left(G_j(1-Y_j)\sigma_v^2 - \sum_{i=1}^{j-1} A^i_j Z^i_j \tilde{\sigma}_u^2\right)\mathrm{d}t,\end{aligned} \tag{IA.142}$$

where, as in equations (IA.109) and (IA.110),

$$\begin{aligned} Y_k \;&=\; \sum_{j=1}^k \lambda_j G_j, \qquad Z^i_k \;=\; \lambda_i + \sum_{j=i+1}^k \lambda_j A^i_j, \qquad k=1,\ldots,\ell, \quad i=1,\ldots,k \\ Y_m \;&=\; \mu + \tilde{\lambda}\sum_{j=1}^{\ell} G_j, \qquad Z^i_m \;=\; \tilde{\lambda}\left(1+\sum_{j=i+1}^{\ell} A^i_j\right). \end{aligned} \tag{IA.143}$$

We now derive the formulas for the equilibrium coefficients. Using equations (IA.102) and (IA.103), we derive the following formulas:

$$\begin{aligned} \rho_j \;&=\; \frac{\gamma_j}{\tilde{a}+\gamma_1^2+\cdots+\gamma_j^2}, \\ C^i_j \;&=\; \prod_{k=i}^j (1-\rho_k\gamma_k) \;=\; \frac{\tilde{a}+\gamma_1^2+\cdots+\gamma_{i-1}^2}{\tilde{a}+\gamma_1^2+\cdots+\gamma_j^2}, \\ G_j \;&=\; \gamma_j C^1_{j-1} \;=\; \frac{\tilde{a}\gamma_j}{\tilde{a}+\gamma_1^2+\cdots+\gamma_{j-1}^2} \;=\; \frac{\tilde{a}\rho_j}{1-\rho_j\gamma_j}, \\ A^i_j \;&=\; -\rho_i\gamma_j C^{i+1}_{j-1} \;=\; -\frac{\gamma_i\gamma_j}{\tilde{a}+\gamma_1^2+\cdots+\gamma_{j-1}^2} \;=\; -\frac{\gamma_i}{\tilde{a}}G_j. \end{aligned} \tag{IA.144}$$

Rewrite equation (IA.119) as $\Sigma'_t = -2\tilde{\lambda}\sum_{j=1}^m (B_{t,j}\Sigma_t) + (1-Y_m)^2\sigma_v^2 + \sum_{j=1}^m (Z^j_m)^2\tilde{\sigma}_u^2 + \mu^2\sigma_e^2$. Using (IA.132) and (IA.134), we compute

$$\frac{\Sigma'_t}{\sigma_v^2} \;=\; -2\tilde{\lambda}\sum_{j=1}^{\ell}\phi_j - 2\tilde{\lambda}(m-\ell)\tilde{\lambda}\tilde{a} + (1-Y_m)^2 + \sum_{j=1}^m (Z^j_m)^2\tilde{a} + \mu^2 b. \tag{IA.145}$$

This implies that Σ'_t is a constant. The transversality condition (IA.123) then implies $\Sigma'_t = -\Sigma_0$, which after division by σ_v^2 implies

$$\frac{\Sigma'_t}{\sigma_v^2} \;=\; -c. \tag{IA.146}$$

We finally obtain

$$-c \;=\; -2\tilde{\lambda}\sum_{j=1}^{\ell}\phi_j-(m-\ell-1)\tilde{\lambda}^2\tilde{a}+(1-Y_m)^2+\sum_{j=1}^{\ell-1}(Z_m^j)^2\tilde{a}+\mu^2 b. \qquad \text{(IA.147)}$$

As explained at the end of Section III.C.1, the first-order conditions for the speculator's optimization problem with respect to γ are too complicated to be written explicitly. Therefore we simply summarize them by

$$\frac{\partial\pi_0}{\partial\gamma_j} \;=\; 0, \quad j=1,\ldots,\ell. \qquad \text{(IA.148)}$$

Equation (IA.105) implies that

$$\lambda_j \;=\; \tilde{\lambda}+\mu\rho_j, \quad j=1,\ldots,\ell. \qquad \text{(IA.149)}$$

Equation (IA.131) implies the following formulas for ϕ_j:

$$\phi_j \;=\; \frac{G_j(\lambda_j-\rho_j)}{\rho_j}, \quad j=1,\ldots,\ell. \qquad \text{(IA.150)}$$

The equation for μ is

$$\mu \;=\; \frac{-\sum_{j=1}^{\ell}\rho_j\phi_j+\left(C_\ell^1\right)}{\left(C_\ell^1\right)^2+\tilde{a}\sum_{j=1}^{\ell}\left(\rho_j C_\ell^{j+1}\right)^2+b}. \qquad \text{(IA.151)}$$

and the equation for ρ_j is

$$\rho_j \;=\; \frac{\gamma_j}{\tilde{a}+\gamma_1^2+\cdots+\gamma_j^2}, \quad j=1,\ldots,\ell. \qquad \text{(IA.152)}$$

Equations (IA.147) to (IA.152) provide $4\ell+2$ equations with $4\ell+2$ unknowns: $\gamma_j,\rho_j,\lambda_j,\phi_j,\mu$, and $\tilde{\lambda}$.

D. Generalization of the Slow Model: $m\geq 1$, $\ell=0$

Because $\ell=0$ in this section, most of the equations in the previous section become trivial. In particular, according to the analysis in the previous section, $\gamma_j=0$ for $j=1,\ldots,m$. Extra care, however, is required in obtaining the equivalent equations for (IA.138) and (IA.147).

To obtain the equivalent for (IA.138), we note that, unlike the case $\ell > 0$, the news $\mathrm{d}z_t = \mathrm{d}v_t + \mathrm{d}e_t$ is not predictable from the previous order flow. Therefore, $\widetilde{\mathrm{d}z_t} = \mathrm{d}z_t$, and equation (IA.137) becomes

$$\mu \;=\; \frac{\mathsf{Cov}_t(v_1, \widetilde{\mathrm{d}z_t})}{\mathsf{Var}_t(\widetilde{\mathrm{d}z_t})} \;=\; \frac{\sigma_v^2}{\sigma_v^2+\sigma_e^2} \;=\; \frac{1}{1+b}. \tag{IA.153}$$

To obtain the equivalent for (IA.145), we first compute Y_m and Z_m^j, as in (IA.143):

$$Y_m \;=\; \mu, \quad Z_m^j \;=\; \tilde{\lambda}, \quad j = 1, \dots, m. \tag{IA.154}$$

Equation (IA.145) becomes

$$-c \;=\; -m\tilde{\lambda}^2\tilde{a} + (1-\mu)^2 + \mu^2 b. \tag{IA.155}$$

Since $a = m\tilde{a}$ and $(1-\mu)^2 + \mu^2 b = \frac{b}{1+b} = (1-\mu)$, we obtain

$$\tilde{\lambda}^2 \;=\; \frac{c + (1-\mu)}{a} \;=\; \frac{\Sigma_0 + \frac{\sigma_v^2\sigma_e^2}{\sigma_v^2+\sigma_e^2}}{\sigma_u^2}. \tag{IA.156}$$

The price impact coefficient is the same in all periods: $\lambda_j = \tilde{\lambda}$ for all $j = 1, \dots, m$. Moreover, from (IA.133) we obtain $\beta_{t,j} = B_{t,j} = \frac{\tilde{\lambda}\tilde{\sigma}_u^2}{\Sigma_t} = \frac{\tilde{\lambda}\tilde{\sigma}_u^2}{\Sigma_0(1-t)} = \frac{\tilde{\lambda}\tilde{a}}{c(1-t)}$. We then obtain the following equations:

$$\mu \;=\; \frac{1}{1+b} \;=\; \frac{\sigma_v^2}{\sigma_v^2+\sigma_e^2}, \tag{IA.157}$$

$$\lambda_j \;=\; \left(\frac{c+(1-\mu)}{a}\right)^{1/2} \;=\; \frac{\Sigma_0^{1/2}}{\sigma_u}\left(1 + \frac{\sigma_v^2\sigma_e^2}{\Sigma_0(\sigma_v^2+\sigma_e^2)}\right)^{1/2}, \tag{IA.158}$$

$$\beta_{t,j} \;=\; \frac{1}{m}\,\frac{1}{1-t}\,\frac{a}{c}\left(\frac{c+(1-\mu)}{a}\right)^{1/2} \;=\; \frac{1}{m}\,\frac{1}{1-t}\,\frac{\sigma_u}{\Sigma_0^{1/2}}\left(1 + \frac{\sigma_v^2\sigma_e^2}{\Sigma_0(\sigma_v^2+\sigma_e^2)}\right)^{1/2}. \tag{IA.159}$$

Note that these are the same equations as in the slow model in the main paper ($m = 1$), except that $\beta_{t,j}$ is now $\frac{1}{m}$ of the β_t in the main paper. This is to be expected, because when the speculator has m trading periods in the interval $[t, t+\mathrm{d}t]$, he divides his trades equally so that on aggregate he trades in the same way.

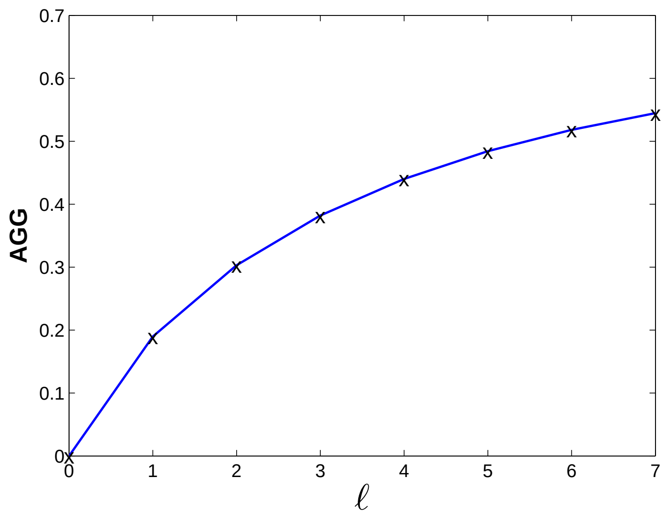

Figure IA.3. Effect of the dealer's latency on the speculator's news trading aggressiveness. We plot the speculator's news trading aggressiveness, AGG, against the dealer's latency, ℓ. The news frequency parameter is $m = 7$, while the dealer's latency varies from $\ell = 0$ to $\ell = 7$. The other parameter values are $\sigma_u = 1$ (standard deviation of noise traders' order flow), $\sigma_v = 1$ (standard deviation of innovations in the asset value), and $\Sigma_0 = 1$ (variance of asset value conditional on information available at date 0).

E. News Trading Aggressiveness

We define the speculator's news trading aggressiveness to be the covariance of the speculator's aggregate order flow with the news:

$$AGG_t \;=\; \mathsf{Cov}\left(\sum_{j=1}^{m} \mathrm{d}x_{t,j}\;,\;\mathrm{d}z_t\right). \tag{IA.160}$$

Intuitively, the larger this aggressiveness measure, the greater the overall weight with which the speculator trades on the news. Using equation (IA.141), we have $\mathrm{d}x_{t,j} = \frac{B_j}{1-t}(v_t - p_t)\,\mathrm{d}t + G_j\,\mathrm{d}v_t + \sum_{i=1}^{j-1} A_j^i\,\mathrm{d}u_{i,t}$, and thus it follows that AGG_t is constant and equal to

$$AGG \;=\; \sum_{j=1}^{m} G_j \sigma_v^2. \tag{IA.161}$$

We now let the parameter ℓ vary, holding the news frequency parameter m constant. The results are plotted in Figure IA.3 in the case $m = 7$. We see that the speculator's news trading aggressiveness is a concave function of the dealer's latency ℓ.

IV. Generalized Information Structure

A. Model

In this section, we extend our baseline model to a more general information structure. Specifically, we relax two assumptions. First, we relax the assumption that the speculator observes $\mathrm{d}v_t$ perfectly and allow for noise in the speculator's signal on $\mathrm{d}v_t$. Second, in addition to the signal on $\mathrm{d}v_t$, we allow the speculator to receive another (possibly noisy) signal about the incoming news $\mathrm{d}z_t$. Formally, at $t = 0$ the speculator observes v_0, and at each subsequent time t, he receives two signals

$$\mathrm{d}s_{1,t} = \mathrm{d}v_t + \mathrm{d}\varepsilon_{1,t}, \quad \text{with} \quad \mathrm{d}\varepsilon_{1,t} = \sigma_1 \mathrm{d}B^{\varepsilon}_{1,t}, \tag{IA.162}$$

$$\mathrm{d}s_{2,t} = \mathrm{d}z_t + \mathrm{d}\varepsilon_{2,t}, \quad \text{with} \quad \mathrm{d}\varepsilon_{2,t} = \sigma_2 \mathrm{d}B^{\varepsilon}_{2,t}, \tag{IA.163}$$

where $B^{\varepsilon}_{1,t}$ and $B^{\varepsilon}_{2,t}$ are Brownian motions independent from all other variables and from each other. We do not make any assumption about the relative size of σ_e and σ_1, that is, the speculator's signal about the innovation in the fundamental may be more or less informative than the public news. Note that this extension nests our baseline model as the limit case where $\sigma_1 = 0$ and $\sigma_2 \to \infty$. Moreover, this extension nests the particular case in which the speculator receives only a signal on $\mathrm{d}v_t$ as $\sigma_2 \to \infty$. All the findings hold in this special case. However, the case in which the speculator receives two signals is more general.

Compared to the baseline model, the speculator no longer knows v_t perfectly. Instead, the speculator's expectation of the asset value at t just before trading at t is

$$w_t = \mathsf{E}\big(v_1 \mid \mathcal{J}_t\big), \tag{IA.164}$$

where $\mathcal{J}_t = \{v_0\} \cup \{s_{1,\tau}\}_{\tau \le t} \cup \{s_{2,\tau}\}_{\tau \le t} \cup \{p_\tau\}_{\tau \le t} \cup \{z_\tau\}_{\tau \le t} \cup \{\mathrm{d}s_{1,t}\} \cup \{\mathrm{d}s_{2,t}\}$. Note that, as in the baseline model, the speculator's information set $\mathcal{J}_t$, when he chooses his order $\mathrm{d}x_t$, includes his signals, prices, and news up to date t, as well as his signals $\mathrm{d}s_{1,t}$ and $\mathrm{d}s_{2,t}$.

An important difference between the current setup and the baseline model is that the speculator no longer knows v_t perfectly. Instead, he forms a forecast w_t that evolves according to the following rules. Initially, $w_0 = v_0$, as the speculator starts with a perfect signal about v_0. Subsequently, if $t \in [0,1)$, in each interval $[t, t+\mathrm{d}t]$ there is the folowing sequence of events:

- The speculator receives signals $\mathrm{d}s_{t,1}$ and $\mathrm{d}s_{t,2}$, and submits a market order for $\mathrm{d}x_t$;
- The total order flow $\mathrm{d}y_t = \mathrm{d}x_t + \mathrm{d}u_t$ executes at the price $p_{t+\mathrm{d}t} = q_t + \lambda_t \mathrm{d}y_t$;
- The speculator observes (or infers) the dealer's news $\mathrm{d}z_t$.

Thus, at the end of $[t, t+\mathrm{d}t]$, the speculator updates his forecast by using not only his signals $\mathrm{d}s_{t,1}$ and $\mathrm{d}s_{t,2}$, but also the news $\mathrm{d}z_t$. Since $\mathrm{d}s_{t,2} = \mathrm{d}z_t + \mathrm{d}\varepsilon_{t,2}$ is an imperfect signal of the news, the speculator's forecast w_t evolves according to

$$\mathrm{d}w_t \;=\; \mathsf{E}\big(\mathrm{d}v_t \mid \mathrm{d}s_{1,t} \cup \mathrm{d}s_{2,t} \cup \mathrm{d}z_t\big) \;=\; \omega_1 \mathrm{d}s_{1,t} + \omega_e \mathrm{d}z_t, \tag{IA.165}$$

where

$$\omega_1 \;=\; \frac{\sigma_1^{-2}}{\sigma_1^{-2} + \sigma_e^{-2} + \sigma_v^{-2}}, \qquad \omega_e \;=\; \frac{\sigma_e^{-2}}{\sigma_1^{-2} + \sigma_e^{-2} + \sigma_v^{-2}}. \tag{IA.166}$$

By analogy with the baseline model, we define a linear equilibrium as one in which the speculator's strategy is of the form

$$\mathrm{d}x_t \;=\; \beta_t (w_t - q_t)\mathrm{d}t + \gamma_{1,t}\mathrm{d}s_{1,t} + \gamma_{2,t}\mathrm{d}s_{2,t}. \tag{IA.167}$$

As in the baseline model, the drift component of the strategy ($\beta_t(w_t - q_t)\mathrm{d}t$) is called the *value-trading component*, while the volatility component ($\gamma_{1,t}\mathrm{d}s_{1,t} + \gamma_{2,t}\mathrm{d}s_{2,t}$) is called the *news-trading component*.

B. Equilibrium

We rewrite the speculator's news-trading component by replacing the two signals $\mathrm{d}s_{1,t}$ and $\mathrm{d}s_{2,t}$ with two linear combinations of the signals,

$$\begin{aligned} \widehat{\mathrm{d}z_t} &= \theta_1 \mathrm{d}s_{1,t} + \theta_2 \mathrm{d}s_{2,t}, \\ \mathrm{d}z_t^{\perp} &= \psi_1 \mathrm{d}s_{1,t} + \psi_2 \mathrm{d}s_{2,t}, \end{aligned} \tag{IA.168}$$

where θ_1, θ_2, ψ_1 and ψ_2 are given by equations (IA.177) and (IA.179) below. In the proof of Theorem IA.4, we show that $\widehat{\mathrm{d}z_t}$ is the speculator's expectation of the news given his signals, and $\mathrm{d}z_t^{\perp}$ is orthognal to $\widehat{\mathrm{d}z_t}$, which we write formally as follows:

$$\begin{aligned} \widehat{\mathrm{d}z_t} &= \mathsf{E}\Big(\mathrm{d}z_t \,\Big|\, \mathrm{d}s_{1,t} \cup \mathrm{d}s_{2,t}\Big), \\ \mathrm{d}z_t^{\perp} &\perp \widehat{\mathrm{d}z_t}. \end{aligned} \tag{IA.169}$$

Moreover, the linear space generated by $\widehat{\mathrm{d}z_t}$ and $\mathrm{d}z_t^{\perp}$ is the same as the linear space generated by the signals $\mathrm{d}s_{1,t}$ and $\mathrm{d}s_{2,t}$.[8] Thus, the speculator's trading strategy is of the form

$$\mathrm{d}x_t \;=\; \beta_t(w_t - q_t)\mathrm{d}t + \gamma_t\widehat{\mathrm{d}z_t} + \alpha_t \mathrm{d}z_t^{\perp}. \tag{IA.170}$$

In Theorem IA.4, we prove two main results. First, we show that the optimal strategy of the speculator has a news-trading component that involves $\widehat{\mathrm{d}z_t}$, the speculator's forecast of the news, but *not* $\mathrm{d}z_t^{\perp}$, the part of the signal orthogonal to his news forecast. Second, we prove that a linear equilibrium of the model exists if a certain system of nonlinear equations has a solution.

THEOREM IA.4: *Consider the version of the fast model in which the speculator observes two signals at $t \in [0, 1)$: (i) a signal about the value change, $\mathrm{d}s_{1,t} = \mathrm{d}v_t + \mathrm{d}\varepsilon_{1,t}$, and (ii) a signal about the incoming news, $\mathrm{d}s_{2,t} = \mathrm{d}z_t + \mathrm{d}\varepsilon_{2,t}$. As above, denote by $\widehat{\mathrm{d}z_t}$ the speculator's expectation of news given his signals, by $\mathrm{d}z_t^{\perp}$ a combination of the signals which is orthogonal to $\widehat{\mathrm{d}z_t}$, and by α_t the speculator's trading weight on $\widehat{\mathrm{d}z_t}$ from equation (IA.170). Then the speculator's optimal strategy must have $\alpha_t = 0$ for all t.*

Furthermore, consider equation (IA.196) derived in the proof, which is a cubic equation in g. Then, if this equation admits a solution $g \in (0, 1)$, there exists an equilibrium of the model of the form

$$\mathrm{d}x_t \;=\; \beta_t(w_t - q_t)\mathrm{d}t + \gamma\widehat{\mathrm{d}z_t}, \tag{IA.171}$$

$$p_{t+\mathrm{d}t} \;=\; q_t + \lambda \mathrm{d}y_t, \tag{IA.172}$$

$$\mathrm{d}q_t \;=\; \lambda \mathrm{d}y_t + \mu(\mathrm{d}z_t - \rho \mathrm{d}y_t), \tag{IA.173}$$

where $\gamma, \rho, \lambda, \mu,$ and ϕ are given by equations (IA.194) and (IA.195) derived in the proof.

As in the baseline model, the equilibrium reduces to a cubic equation in $g \in (0, 1)$, but here the coefficients are much more complicated. Numerically, the cubic equation

[8] Note that in the baseline fast model, $\widehat{\mathrm{d}z_t}$ coincides with $\mathrm{d}v_t$. Formally, we have $\sigma_1 = 0$ (precise signal about $\mathrm{d}v_t$) and $\sigma_2 = \infty$ (no signal about $\mathrm{d}z_t$). Then $\theta_1 = 1$ and $\theta_2 = 0$, which implies $\widehat{\mathrm{d}z_t} = \mathrm{d}v_t$.

has the same properties as in the baseline model. Indeed, for all the parameter values we have checked, we find that there is a unique solution $g \in (0,1)$.[9]

C. Proof of Theorem IA.4

We start by computing the various instantaneous covariances among the signals $\mathrm{d}s_{t,1}$, $\mathrm{d}s_{t,2}$, and $\mathrm{d}z_t$:

$$\begin{aligned} \sigma_z^2 &= \sigma_v^2 + \sigma_e^2, \quad \sigma_{zs_1} = \sigma_v^2, \quad \sigma_{zs_2} = \sigma_z^2, \\ \sigma_{s_1}^2 &= \sigma_v^2 + \sigma_1^2, \quad \sigma_{s_2}^2 = \sigma_z^2 + \sigma_2^2, \quad \sigma_{s_1 s_2} = \sigma_v^2. \end{aligned} \tag{IA.174}$$

Denote by A the instantaneous covariance matrix of the signals $\mathrm{d}s_{t,1}$ and $\mathrm{d}s_{t,2}$:

$$A = \begin{bmatrix} \sigma_{s_1}^2 & \sigma_{s_1 s_2} \\ \sigma_{s_1 s_2} & \sigma_{s_2}^2 \end{bmatrix}. \tag{IA.175}$$

The dealer's expectation of the news given his signals, $\widehat{\mathrm{d}z_t}$, is the projection of the news, $\mathrm{d}z_t$, on the linear space generated by the signals $\mathrm{d}s_{t,1}$ and $\mathrm{d}s_{t,2}$:

$$\widehat{\mathrm{d}z_t} = \theta_1 \mathrm{d}s_{1,t} + \theta_2 \mathrm{d}s_{2,t}, \quad \text{with} \quad \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = A^{-1} \begin{bmatrix} \sigma_{zs_1} \\ \sigma_{zs_2} \end{bmatrix}. \tag{IA.176}$$

We compute

$$\begin{aligned} \theta_1 &= \frac{\sigma_v^2 \sigma_2^2}{\sigma_v^2\sigma_e^2 + \sigma_v^2\sigma_1^2 + \sigma_v^2\sigma_2^2 + \sigma_e^2\sigma_1^2 + \sigma_1^2\sigma_2^2}, \\ \theta_2 &= \frac{\sigma_v^2\sigma_e^2 + \sigma_v^2\sigma_1^2 + \sigma_e^2\sigma_1^2}{\sigma_v^2\sigma_e^2 + \sigma_v^2\sigma_1^2 + \sigma_v^2\sigma_2^2 + \sigma_e^2\sigma_1^2 + \sigma_1^2\sigma_2^2}. \end{aligned} \tag{IA.177}$$

Moreover, we want $\mathrm{d}z_t^{\perp} = \psi_1 \mathrm{d}s_{t,1} + \psi_2 \mathrm{d}s_{t,2}$ to be orthogonal to $\widehat{\mathrm{d}z_t} = \theta_1 \mathrm{d}s_{1,t} + \theta_2 \mathrm{d}s_{2,t}$. In matrix notation, if we denote the matrix transpose by T, this translates to

$$\frac{1}{\mathrm{d}t} \mathsf{Cov}\big(\mathrm{d}z_t^{\perp}, \widehat{\mathrm{d}z_t}\big) = \begin{bmatrix}\psi_1 \ \psi_2\end{bmatrix} A \begin{bmatrix}\theta_1 \ \theta_2\end{bmatrix}^T = \begin{bmatrix}\psi_1 \ \psi_2\end{bmatrix} \begin{bmatrix}\sigma_{zs_1} \ \sigma_{zs_2}\end{bmatrix}^T = 0. \tag{IA.178}$$

[9] We normalized $\sigma_v = 1$, and for the other volatilities (σ_u, σ_e, $\sigma_0 = (\Sigma_0)^{1/2}$, σ_1, σ_2), we chose random values from the exponential distribution with mean one.

Thus, we can simply take

$$\psi_1 \;=\; -1, \qquad \psi_2 \;=\; \frac{\sigma_{zs_1}}{\sigma_{zs_2}} \;=\; \frac{\sigma_v^2}{\sigma_z^2} \;=\; \frac{\sigma_v^2}{\sigma_v^2+\sigma_e^2}. \tag{IA.179}$$

Because $\mathrm{d}z_t^{\perp}$ is orthogonal to $\widehat{\mathrm{d}z}_t$ and is nonzero, it follows that the linear space generated by $\mathrm{d}z_t^{\perp}$ and $\widehat{\mathrm{d}z}_t$ is the same as the linear space generated by the signals $\mathrm{d}s_{t,1}$ and $\mathrm{d}s_{t,2}$.

For the rest of the appendix, we use the following notation:

$$\begin{aligned}
\sigma_h^2 &= \frac{1}{\mathrm{d}t}\mathsf{Var}(\widehat{\mathrm{d}z}_t), & \sigma_{hv} &= \frac{1}{\mathrm{d}t}\mathsf{Cov}(\widehat{\mathrm{d}z}_t,\mathrm{d}v_t), & \sigma_{hz} &= \frac{1}{\mathrm{d}t}\mathsf{Cov}(\widehat{\mathrm{d}z}_t,\mathrm{d}z_t),\\
\sigma_o^2 &= \frac{1}{\mathrm{d}t}\mathsf{Var}(\mathrm{d}z_t^{\perp}), & \sigma_{ov} &= \frac{1}{\mathrm{d}t}\mathsf{Cov}(\mathrm{d}z_t^{\perp},\mathrm{d}v_t), & \sigma_{oz} &= \frac{1}{\mathrm{d}t}\mathsf{Cov}(\mathrm{d}z_t^{\perp},\mathrm{d}z_t),\\
\sigma_w^2 &= \frac{1}{\mathrm{d}t}\mathsf{Var}(\mathrm{d}w_t), & \sigma_{hw} &= \frac{1}{\mathrm{d}t}\mathsf{Cov}(\mathrm{d}w_t,\widehat{\mathrm{d}z}_t), & \sigma_{ow} &= \frac{1}{\mathrm{d}t}\mathsf{Cov}(\mathrm{d}w_t,\mathrm{d}z_t^{\perp}),
\end{aligned} \tag{IA.180}$$

and so on. For future reference, we compute some of the more important variables as follows. First, note that $\widehat{\mathrm{d}z}_t$ is the speculator's expectation of $\mathrm{d}z_t$, and therefore $\mathsf{Cov}(\widehat{\mathrm{d}z}_t,\mathrm{d}z_t)=\mathsf{Cov}(\widehat{\mathrm{d}z}_t,\widehat{\mathrm{d}z}_t)=\mathsf{Var}(\widehat{\mathrm{d}z}_t)$. Similarly, $\mathsf{Cov}(\mathrm{d}z_t^{\perp},\mathrm{d}z_t)=\mathsf{Cov}(\mathrm{d}z_t^{\perp},\widehat{\mathrm{d}z}_t)=0$. Also, $\mathsf{Cov}(\mathrm{d}s_{t,i},\widehat{\mathrm{d}z}_t)=\mathsf{Cov}(\mathrm{d}s_{t,i},\mathrm{d}z_t)$ for $i=1,2$. We obtain

$$\begin{aligned}
\sigma_h^2 \;=\; \sigma_{hz} &= \frac{1}{\mathrm{d}t}\mathsf{Cov}(\theta_1\mathrm{d}s_{t,1}+\theta_2\mathrm{d}s_{t,2},\mathrm{d}z_t) \;=\; \theta_1\sigma_v^2+\theta_2\sigma_z^2,\\
\sigma_{hv} &= \frac{1}{\mathrm{d}t}\mathsf{Cov}(\theta_1\mathrm{d}s_{t,1}+\theta_2\mathrm{d}s_{t,2},\mathrm{d}v_t) \;=\; \theta_1\sigma_v^2+\theta_2\sigma_v^2,\\
\sigma_{oh} \;=\; \sigma_{oz} &= 0,\\
\sigma_{wz} &= \frac{1}{\mathrm{d}t}\mathsf{Cov}(\omega_1\mathrm{d}s_{t,1}+\omega_e\mathrm{d}z_t,\mathrm{d}z_t) \;=\; \omega_1\sigma_v^2+\omega_e\sigma_z^2 \;=\; \sigma_v^2,\\
\sigma_{hw} &= \frac{1}{\mathrm{d}t}\mathsf{Cov}(\omega_1\mathrm{d}s_{t,1}+\omega_e\mathrm{d}z_t,\widehat{\mathrm{d}z}_t) \;=\; \omega_1\sigma_v^2+\omega_e\sigma_h^2,\\
\sigma_w^2 &= \frac{1}{\mathrm{d}t}\mathsf{Var}(\omega_1\mathrm{d}s_{t,1}+\omega_e\mathrm{d}z_t) \;=\; \omega_1^2\sigma_{s_1}^2+2\omega_1\omega_e\sigma_v^2+\omega_e^2\sigma_z^2,
\end{aligned} \tag{IA.181}$$

We now proceed as in the proof of Theorem 2 in the paper. First, we compute the optimal trading strategy of the speculator, while taking as given the dealer's pricing rule.

For $t\in[0,1)$, the speculator's expected profit is $\pi_t=\mathsf{E}_t\left(\int_t^1(v_1-p_{\tau+\mathrm{d}\tau})\mathrm{d}x_\tau\right)$, where the expectation is conditional on the speculator's information set $\mathcal{J}_t$. We compute[10]

$$\begin{aligned}
\pi_\tau &= \mathsf{E}_t\left(\int_t^1 (w_{\tau+\mathrm{d}\tau} - p_{\tau+\mathrm{d}\tau})\mathrm{d}x_\tau\right) \\
&= \mathsf{E}_t\left(\int_t^1 \Big((w_\tau + \mathrm{d}w_\tau) - (q_\tau + \lambda \mathrm{d}y_\tau)\Big)\mathrm{d}x_\tau\right) \\
&= \mathsf{E}_t\left(\int_t^1 \Big(w_\tau - q_\tau + \mathrm{d}w_\tau - \lambda \mathrm{d}x_\tau\Big)\mathrm{d}x_\tau\right),
\end{aligned} \tag{IA.182}$$

where the first equality follows from the law of iterated expectations, and the second equality follows from $\mathrm{d}x_\tau$ being orthogonal to $\mathrm{d}u_\tau$. Since $\mathrm{d}x_t = \beta_t(w_t - q_t)\mathrm{d}t + \gamma_t\widehat{\mathrm{d}z}_t + \alpha_t\mathrm{d}z_t^\perp$, we compute

$$\begin{aligned}
\pi_t &= \mathsf{E}_t\left(\int_t^1 \Big(w_\tau - q_\tau + \mathrm{d}w_\tau - \lambda_\tau \mathrm{d}x_\tau\Big)\mathrm{d}x_\tau\right) \\
&= \int_t^1 \Big(\beta_\tau V_{t,\tau} + \gamma_\tau \operatorname{\mathsf{Cov}}\big(\mathrm{d}w_\tau - \lambda\gamma_\tau\widehat{\mathrm{d}z}_\tau, \widehat{\mathrm{d}z}_\tau\big) + \alpha_\tau \operatorname{\mathsf{Cov}}\big(\mathrm{d}w_\tau - \lambda\alpha_\tau\mathrm{d}z_\tau^\perp, \mathrm{d}z_\tau^\perp\big)\Big) \\
&= \int_t^1 \Big(\beta_\tau V_{t,\tau} + \gamma_\tau\sigma_{hw} - \lambda\gamma_\tau^2\sigma_h^2 + \alpha_\tau\sigma_{ow} - \lambda\alpha_\tau^2\sigma_o^2\Big),
\end{aligned} \tag{IA.183}$$

where

$$V_{t,\tau} = \mathsf{E}_t\big((w_\tau - q_\tau)^2\big), \quad \tau \in [t, 1). \tag{IA.184}$$

The dealer's quote q_τ evolves according to

$$\mathrm{d}q_\tau = \mu\mathrm{d}z_\tau + \Lambda\mathrm{d}y_\tau, \tag{IA.185}$$

where

$$\Lambda = \lambda - \mu\rho. \tag{IA.186}$$

We then have that $V_{t,\tau}$ evolves as follows:

$$\begin{aligned}
V_{t,\tau+\mathrm{d}\tau} &= \mathsf{E}_t\Big((w_\tau + \mathrm{d}w_\tau - q_\tau - \mathrm{d}q_\tau)^2\Big) \\
&= V_{t,\tau} - \mathsf{E}_t\Big((w_\tau - q_\tau)\mathrm{d}q_\tau\Big) + \mathsf{E}_t\Big((\mathrm{d}w_\tau - \mathrm{d}q_\tau)^2\Big) \\
&= V_{t,\tau} - 2\ell\beta_\tau V_{t,\tau}\mathrm{d}\tau + \mathsf{E}_t\Big(\big(\mathrm{d}w_\tau - \Lambda\gamma_\tau\widehat{\mathrm{d}z}_\tau - \Lambda\alpha_\tau\mathrm{d}z_\tau^\perp - \Lambda\mathrm{d}u_\tau - \mu\mathrm{d}z_\tau\big)^2\Big).
\end{aligned} \tag{IA.187}$$

[10] We are only interested in the existence of an equilibrium, conjectured to have constant coefficients. Thus, the speculator assumes that the dealer uses a pricing rule with constant coefficients.

We now use equation (IA.181), which implies $\sigma_{oz} = \sigma_{oh} = 0$, and hence, $V_{t,\tau}$ satisfies the first-order linear ODE:

$$\begin{aligned}\frac{\mathrm{d}V_{t,\tau}}{\mathrm{d}\tau} &= -2\Lambda\beta_t V_{t,\tau} + \sigma_w^2 + \Lambda^2\gamma_\tau^2\sigma_h^2 + \Lambda^2\alpha_\tau^2\sigma_o^2 + \Lambda^2\sigma_u^2 + \mu^2\sigma_z^2 \\ &\quad - 2\Lambda\gamma_\tau\sigma_{hw} - 2\Lambda\alpha_\tau\sigma_{ow} - 2\mu\sigma_{wz} + 2\Lambda\gamma_\tau\mu\sigma_{hz}.\end{aligned} \tag{IA.188}$$

Substituting this into (IA.183) yields

$$\begin{aligned}\pi_t &= -\int_t^1 \frac{\mathrm{d}V_{t,\tau}}{2\Lambda} + \int_t^1 \Bigg(\gamma_\tau\sigma_{hw} - \lambda\gamma_\tau^2\sigma_h^2 + \alpha_\tau\sigma_{ow} - \lambda\alpha_\tau^2\sigma_o^2 \\ &\quad + \frac{\sigma_w^2 + \Lambda^2\gamma_\tau^2\sigma_h^2 + \Lambda^2\alpha_\tau^2\sigma_o^2 + \Lambda^2\sigma_u^2 + \mu^2\sigma_z^2}{2\Lambda} \\ &\quad - \gamma_\tau\sigma_{hw} - \alpha_\tau\sigma_{ow} - \frac{\mu\sigma_{wz}}{\Lambda} + \gamma_\tau\mu\sigma_{hz}\Bigg)\,\mathrm{d}\tau.\end{aligned} \tag{IA.189}$$

As usual, since $V_{t,\tau} \geq 0$ can be arbitrarily chosen for $\tau > t$, the transversality condition $V_{t,1} = 0$ must be satisfied.

We now turn to the choice of γ_τ and α_τ. The first-order conditions with respect to γ_τ and α_τ in (IA.189) are, respectively,

$$\begin{aligned}0 &= \sigma_{hw} - 2\lambda\gamma_\tau\sigma_h^2 + \Lambda\gamma_\tau\sigma_h^2 - \sigma_{hw} + \mu\sigma_{hz}, \\ 0 &= \sigma_{ow} - 2\lambda\alpha_\tau\sigma_o^2 + \Lambda\alpha_\tau\sigma_o^2 - \sigma_{ow},\end{aligned} \tag{IA.190}$$

which yields

$$\begin{aligned}\gamma_\tau &= \frac{\mu}{\lambda + \mu\rho}\,\frac{\sigma_{hz}}{\sigma_h^2} = \frac{\mu}{\lambda + \mu\rho}, \\ \alpha_\tau &= 0,\end{aligned} \tag{IA.191}$$

since equation (IA.181) implies that $\sigma_{hz} = \sigma_h^2$.

Next, we derive the pricing rule from the dealer's zero-profit condition as in the baseline model:

$$
\begin{aligned}
\lambda_t &= \frac{\mathsf{Cov}_t(v_1, \mathrm{d}y_t)}{\mathsf{Var}_t(\mathrm{d}y_t)} = \frac{\beta_t \Sigma_t + \gamma_t \sigma_{hv}}{\gamma_t^2 \sigma_h^2 + \sigma_u^2}, \\
\rho_t &= \frac{\mathsf{Cov}_t(\mathrm{d}z_t, \mathrm{d}y_t)}{\mathsf{Var}_t(\mathrm{d}y_t)} = \frac{\gamma_t \sigma_{hz}}{\gamma_t^2 \sigma_h^2 + \sigma_u^2}, \\
\mu_t &= \frac{\mathsf{Cov}_t(v_1, \mathrm{d}z_t - \rho_t \mathrm{d}y_t)}{\mathsf{Var}_t(\mathrm{d}z_t - \rho_t \mathrm{d}y_t)} = \frac{-\rho_t \beta_t \Sigma_t + \sigma_v^2 - \rho_t \gamma_t \sigma_{hv}}{\sigma_z^2 + \rho_t^2 (\gamma_t^2 \sigma_h^2 + \sigma_u^2) - 2\rho_t \gamma_t \sigma_{hz}} \\
&= \frac{-\rho_t \beta_t \Sigma_t + \sigma_v^2 - \rho_t \gamma_t \sigma_{hv}}{\sigma_z^2 - \sigma_h^2 + (1 - \rho_t \gamma_t)^2 \sigma_h^2 + \rho_t^2 \sigma_u^2},
\end{aligned}
\tag{IA.192}
$$

where the last equality follows from the formula $\mathrm{d}z_t - \rho_t \mathrm{d}y_t = \mathrm{d}z_t - \widehat{\mathrm{d}z}_t + (1 - \rho_t \gamma_t)\widehat{\mathrm{d}z}_t - \rho_t \mathrm{d}u_t$, which implies $\frac{1}{\mathrm{d}t} \mathsf{Var}_t(\mathrm{d}z_t - \rho_t \mathrm{d}y_t) = \sigma_z^2 - \sigma_h^2 + (1 - \rho_t \gamma_t)^2 \sigma_h^2 + \rho_t^2 \sigma_u^2$.

We search for an equilibrium in which $\beta_t \Sigma_t$, γ_t , λ_t, ρ_t, and μ_t are constant. Since Σ_t satisfies the same ODE (IA.188) as $V_{0,t}$, the transversality condition implies $\Sigma_1 = V_{0,1} = 0$. Therefore, $\Sigma_t = (1-t)\Sigma_0$, and $\beta_t = \frac{\beta_0}{1-t}$. Also, we adapt equation (IA.188) to Σ_t, and note that $\frac{\mathrm{d}\Sigma_t}{\mathrm{d}t} = -\Sigma_0$. Since $\sigma_{wz} = \sigma_v^2$ and $\sigma_{hz} = \sigma_h^2$, we get

$$
-\Sigma_0 = -2\Lambda \beta_0 \Sigma_0 + \sigma_w^2 + \Lambda^2 \sigma_u^2 + \mu^2 \sigma_z^2 - 2\mu + \Lambda^2 \gamma^2 \sigma_h^2 - 2\Lambda \gamma \sigma_{hw} + 2\Lambda \gamma \mu \sigma_h^2. \tag{IA.193}
$$

Formulas for the constants involved are given in (IA.166), (IA.177), and (IA.181).

We now use a similar method as in the baseline fast model, and define the following variables:

$$
\begin{aligned}
a &= \frac{\sigma_u^2}{\sigma_h^2}, \quad b = \frac{\sigma_z^2 - \sigma_h^2}{\sigma_h^2}, \quad c = \frac{\Sigma_0}{\sigma_v^2}, \\
A_h &= \frac{\sigma_h^2}{\sigma_v^2}, \quad A_{hv} = \frac{\sigma_{hv}}{\sigma_v^2}, \quad \text{etc.}, \\
g &= \frac{\gamma^2}{a}, \quad \tilde{\lambda} = A_h \lambda \gamma, \quad \tilde{\rho} = \rho \gamma, \quad \tilde{\mu} = A_h \mu, \quad \tilde{\Lambda} = A_h \Lambda \gamma, \quad \psi = \frac{\beta_0 \Sigma_0}{a \sigma_v^2} \gamma.
\end{aligned}
\tag{IA.194}
$$

A similar computation as for the baseline model then produces

$$
\begin{aligned}
\tilde{\lambda} &= \frac{1}{2 + b(1+g)}, \qquad \tilde{\rho} = \frac{g}{1+g}, \qquad \tilde{\mu} = \frac{1+g}{2 + b(1+g)}, \\
\psi &= \frac{1+g}{2 + b(1+g)} - A_{hv} g, \qquad \tilde{\Lambda} = \frac{1-g}{2 + b(1+g)}.
\end{aligned}
\tag{IA.195}
$$

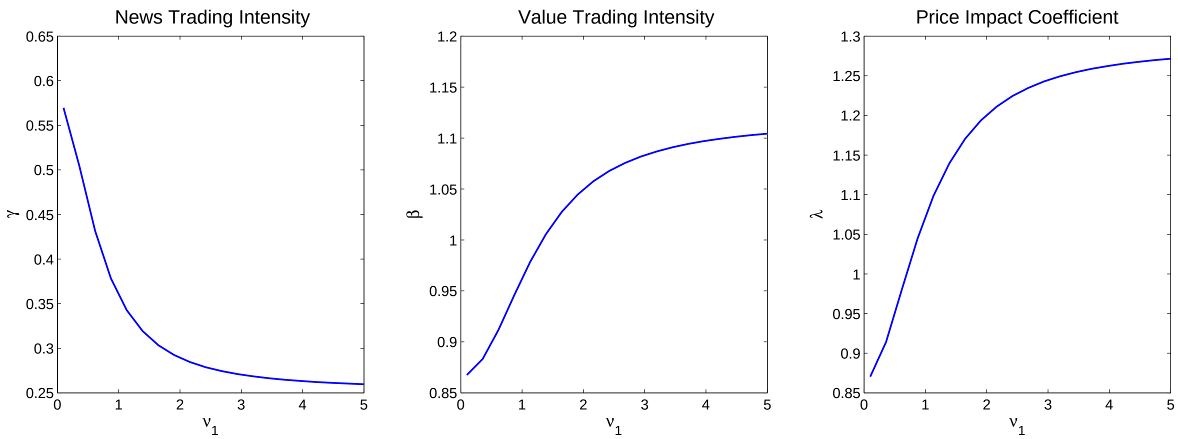

Figure IA.4. Effect of speculator's signal informativeness on γ, β, λ. The figure compares the effect of the speculator's signal informativeness $\nu_1 = \frac{1}{\sigma_1}$ in a fast model with generalized signal structure. The graphs plot the following variables in the model against the ν_1 parameter: (i) the news trading intensity γ, (ii) the value trading intensity β_0, and (iii) the price impact coefficient λ. The other parameter values are $\sigma_u = 1$ (standard deviation of noise traders' order flow), $\sigma_v = 1$ (standard deviation of innovations in the asset value), $\sigma_e = 1$ (standard deviation of noise in news), $\Sigma_0 = 1$ (variance of asset value conditional on information available at date 0), and $\sigma_2 = +\infty$ (standard deviation of noise in speculator's signal about the news).

With this notation, equation (IA.193) (after dividing by σ_v^2) becomes

$$-c \;=\; -\frac{2}{A_h}\frac{\tilde{\Lambda}\psi}{g} + A_w + \frac{\tilde{\Lambda}^2}{A_h g} + \frac{A_z}{A_h^2}\tilde{\mu}^2 - 2\frac{1}{A_h}\tilde{\mu} + \frac{\tilde{\Lambda}^2}{A_h} - 2\frac{A_{hw}}{A_h}\tilde{\Lambda} + \frac{2}{A_h}\tilde{\Lambda}\tilde{\mu}. \qquad \text{(IA.196)}$$

As in the baseline model, this is a cubic equation in $g \in (0,1)$, but the coefficients are more complicated. Numerically, however, there appears to be a unique solution $g \in (0,1)$, as in the baseline model.

D. *Some Comparative Statics*

We consider the case in which the speculator observes only a noisy signal of $\mathrm{d}v_t$. Thus, we assume that at t (i) the speculator receives a noisy signal about the fundamental value $\mathrm{d}s_{1,t} = \mathrm{d}v_t + \mathrm{d}\varepsilon_{1,t}$, that is, $\sigma_1 > 0$, and (ii) the speculator's signal about the news $\mathrm{d}s_{2,t} = \mathrm{d}v_t + \mathrm{d}\varepsilon_{2,t}$ is uninformative, that is, $\sigma_2 = +\infty$. (Note that the baseline model corresponds to $\sigma_1 = 0$ and $\sigma_2 = +\infty$.)

We first perform some comparative statics with respect to the informativeness of the speculator's signal, $\nu_1 = \frac{1}{\sigma_1}$. Figure IA.4 studies the dependence of the news trading intensity (γ), the value trading intensity (β), and the price impact coefficient (λ) on $\nu_1 =$

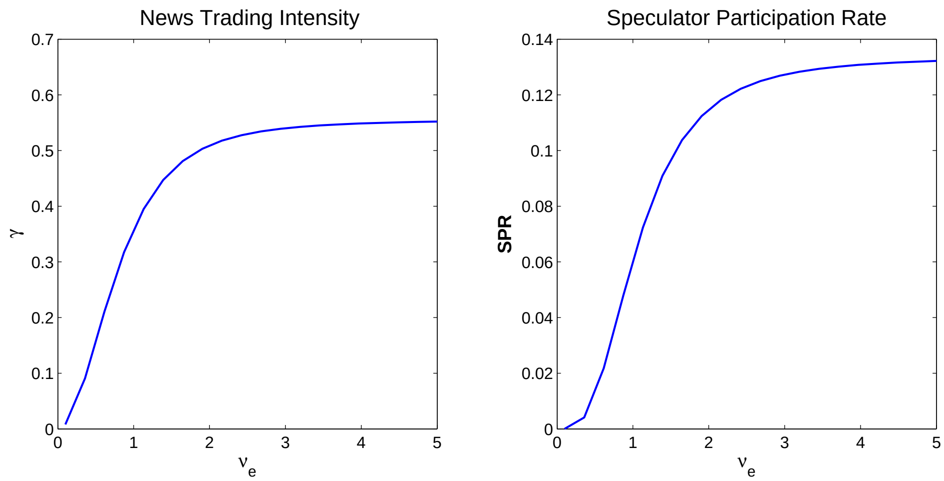

Figure IA.5. Effect of news informativeness on γ and SPR. The figure compares the effect of the news informativeness $\nu_e = \frac{1}{\sigma_e}$ in a fast model with generalized signal structure. The graphs plot four variables in the model against the news informativeness parameter: (i) the news trading intensity γ, (ii) the value trading intensity β_0, (iii) the price impact coefficient λ, and (iv) the speculator participation rate $\frac{\mathsf{Var}(dx_t)}{\mathsf{Var}(dy_t)} = \frac{g}{1+g}$. The other parameter values are $\sigma_u = 1$ (standard deviation of noise traders' order flow), $\sigma_v = 1$ (standard deviation of innovations in the asset value), $\sigma_1 = 1$ (standard deviation of noise in speculator's signal about the asset value), $\Sigma_0 = 1$ (variance of asset value conditional on information available at date 0), and $\sigma_2 = +\infty$ (standard deviation of noise in speculator's signal about the news).

$\frac{1}{\sigma_1}$. We observe that the speculator's news trading intensity (γ) decreases and the value trading intensity increases when his signal is more precise (σ_1 is higher). This is a result of the substitution effect between value trading and news trading. Indeed, when the signal is more precise, the speculator trades more on his long-term signal (value trading), and therefore trades less on his short-term signal (news trading). Furthermore, when the speculator's signal is more precise, the dealer is subject to more adverse selection, and therefore the price impact coefficient λ is higher.

Next, we perform some comparative statics with respect to the news informativeness parameter, $\nu_e = \frac{1}{\sigma_e}$. Figure IA.5 studies the dependence of the news trading intensity (γ) and the speculator participation rate (SPR) on the news informativeness parameter $\nu_e = \frac{1}{\sigma_e}$. As expected, the speculator's news trading intensity (γ) is higher when his signal is more precise, and so is the speculator participation rate $SPR = \frac{g}{1+g}$.

V. Closed-Form Solution When $\sigma_e = 0$

In this section we provide a closed-form solution for the equilibrium coefficients in the fast model in the special case $\sigma_e = 0$. Note that the proof of Theorem 2 in the paper also works when $\sigma_e = 0$. Thus, it is still the case that g is the unique solution in $(0,1)$ to equation (23). However, when $\sigma_e = 0$, the equation becomes quadratic. One can now check that the unique solution in $(0,1)$ to this equation is

$$g = \left(\left(1 + \frac{\Sigma_0}{\sigma_v^2}\right)^{1/2} - \left(\frac{\Sigma_0}{\sigma_v^2}\right)^{1/2} \right)^2 . \tag{IA.197}$$

The formulas for the other coefficients then follow from equations (18) to (22) in Theorem 2:

$$\beta_t^F = \frac{1}{1-t} \frac{\sigma_u}{(\Sigma_0 + \sigma_v^2)^{1/2}} \left(1 + \frac{(1-g)\sigma_v^2}{2\Sigma_0}\right), \tag{IA.198}$$

$$\gamma^F = \frac{\sigma_u}{2(\Sigma_0 + \sigma_v^2)^{1/2}}(1+g), \tag{IA.199}$$

$$\lambda^F = \frac{(\Sigma_0 + \sigma_v^2)^{1/2}}{\sigma_u(1+g)}, \tag{IA.200}$$

$$\mu^F = \frac{1+g}{2}, \tag{IA.201}$$

$$\rho^F = \frac{\sigma_v^2}{2\sigma_u(\Sigma_0 + \sigma_v^2)^{1/2}}. \tag{IA.202}$$

References

[1] Kyle, Albert, 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315–1335.