

## News Trading and Speed

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### ABSTRACT

We compare the optimal trading strategy of an informed speculator when he can trade ahead of incoming news (is “fast”), versus when he cannot (is “slow”). We find that speed matters: the fast speculator’s trades account for a larger fraction of trading volume, and are more correlated with short-run price changes. Nevertheless, he realizes a large fraction of his profits from trading on long-term price changes. The fast speculator’s behavior matches evidence about high-frequency traders. We predict that stocks with more informative news are more liquid even though they attract more activity from informed high-frequency traders.

*High-frequency traders do not care if information is accurate or inaccurate. They just want to know what is coming out on the market that might sway public sentiment. So this is very different than traditional insider trading [...]. This is all just about what might move the market, because they are in and out in milliseconds. They don't really care about the long-term effects of the information.*

Atty. Gen. Schneiderman’s speech, “High-Frequency Trading and Insider Trading 2.0.”<sup>1</sup>

TODAY’S FINANCIAL MARKETS are characterized by an almost continuous flow of “news.” Every quote update or trade in one asset (e.g., a stock index futures

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<sup>1</sup> March 18, 2014 (available at: [http://www.ag.ny.gov/pdfs/HFT\\_and\\_market\\_structure.pdf](http://www.ag.ny.gov/pdfs/HFT_and_market_structure.pdf)).

or an exchange-traded fund) is a source of information for pricing other assets. Furthermore, traders increasingly rely on machine-readable text in tweets, Facebook pages, blogs, newswires, economic and corporate reports, company websites, etc., which greatly expands their information set because the arrival rate of such news is very high.<sup>2</sup>

News thus plays an increasing role in shaping trade and price patterns in financial markets. High-frequency trading is a case in point. High-frequency traders' (HFTs) strategies are diverse (see SEC (2014)): some specialize in market making whereas others follow directional strategies, establishing positions in anticipation of future price movements, mainly using aggressive (i.e., marketable) orders.<sup>3</sup>

Academic evidence suggests that high-frequency news plays an important role in directional HFTs' strategies.<sup>4</sup> First, HFTs' aggressive orders anticipate short-term price movements and contribute significantly to trading volume. For instance, Brogaard, Hendershott, and Riordan (2014) find that HFTs' aggressive orders predict price changes over very short horizons and account for 25% to 42% of trading volume depending on market capitalization (see also Baron, Brogaard, and Kirilenko (2014), Benos and Sagade (2013), and Kirilenko et al. (2014) for similar evidence). Second, HFTs' aggressive orders are correlated with news such as market-wide returns, quote updates, macroeconomic announcements, E-mini price changes, and newswires items (see Brogaard, Hendershott, and Riordan (2014) and Zhang (2012)). These observations suggest that directional HFTs trade on soon-to-be-released information. However, directional HFTs realize a large fraction of their profits on aggressive orders over relatively long horizons (e.g., over the day; see Carrion (2013), table 5, and Baron, Brogaard, and Kirilenko (2014), table 6). This last finding is difficult to reconcile with the view that directional HFTs trade only on short-term price reactions to news. Carrion (2013, p. 710) thus concludes that "models where HFTs solely profit from very short-term activities [...] may be incomplete."

In this paper, we propose a model of trading on news that explains the aforementioned facts and generates new predictions, especially about the effect of news informativeness on HFTs' trading strategy, the sources of their profitability (speculation on short-term versus long-term price movements), and liquidity. We therefore contribute to the theoretical literature on high-frequency trading, which thus far has not considered dynamic models of trading on news.

<sup>2</sup> See "Trading via Twitter", *Traders Magazine*, June 2014. This article takes the example of a prop trading firm that "everyday scans 400 to 500 million tweets looking for a breaking news event."

<sup>3</sup> SEC (2014) provides a survey of empirical findings on HFTs. This survey notes, on page 9, that: "Perhaps the most noteworthy finding of the HFT dataset papers is that HFT is not a monolithic phenomenon, but rather encompasses a diverse range of trading strategies. In particular, HFT is not solely, or even primarily, characterized by passive market making strategies [...]. For example, Carrion (2013) and Brogaard, Hendershott, and Riordan (2014) [...] find that more than 50% of HFT activity is attributable to aggressive, liquidity taking orders." See also Hagströmer and Nordén (2013) and Benos and Sagade (2013) for evidence that HFTs' strategies are diverse.

<sup>4</sup> Numerous media articles also emphasize the importance of news in HFTs' strategies. See, for instance, "Computers that Trade on the News", the *New York Times*, May 22, 2012 or "Speed Traders Get an Edge", the *Wall Street Journal*, February 7, 2014.

Our model builds on Kyle (1985). One speculator and one competitive dealer continuously trade while receiving a flow of signals about the payoff of a risky asset (its “long-run” value).<sup>5</sup> The dealer’s signals are public information. We interpret these signals as high-frequency news. In contrast, the speculator’s signals are private and informative about the long-run value of the asset. Since news is also informative about the long-run value of the asset, the speculator’s signals can also be used to predict short-run price reactions to (the surprise component of) news.<sup>6</sup>

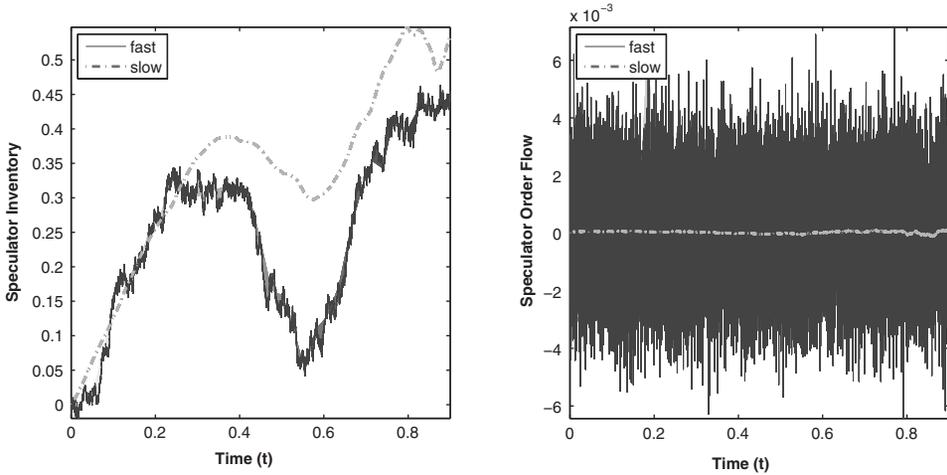
We say that there is *news trading* if the speculator’s signals affect his trades above and beyond their effects on the speculator’s estimate of the long-run change in the asset. We show that news trading arises in equilibrium only when the speculator is *fast* relative to the dealer, that is, if he can trade on his forecast of short-run price movements *before* the dealer reacts to (or receives) news. In this case, the speculator’s optimal position in the risky asset follows a stochastic process with a drift proportional to the speculator’s forecast of the long-run change in the asset value (as in Kyle (1985) and others) and an instantaneous volatility proportional to the speculator’s forecast of news. This volatility component is a novel feature of our model and is key for our predictions. This component drives short-run changes in the speculator’s position while the drift component determines the long-run change in this position.

To develop intuition, suppose that the speculator’s latest signal is positive, and yet his forecast of the asset payoff (which depends on his history of signals, not just the latest signal) is *lower* than the asset price. In this case, the speculator expects the price to *increase* in the short-run, due to news arrival (because the speculator’s signal is positively correlated with news), but to *decrease* in the long run. This calls for two different trades: a buy in anticipation of the short-run price increase and a sell in anticipation of the longer-run price decline. The drift component of the speculator’s position is his desired trade given his estimate of the long-run price change, while the volatility component is his desired trade given his forecast of impending news. The speculator’s actual trade is the sum of these two—possibly conflicting—desired trades.

The volatility component always swamps the drift component in explaining short-term variations in the speculator’s position. Thus, short-run changes in the speculator’s position are driven by news, that is, the speculator trades in the direction of incoming news. However, over a longer period of time, the speculator’s position changes in the direction of his long-run forecast of the asset value. Hence, in the previous example, the speculator buys the asset just ahead of news arrival, even though he estimates the asset to be overpriced relative to its long-run value; then, in the longer run, he sells the asset to exploit this mispricing. Hence, when he is fast, the speculator trades on what moves prices in the short run but he also cares about the long-run implications of his information.

<sup>5</sup> “Long-run” in our model should be interpreted as, say, an hourly or daily horizon. Forecasts at this horizon are long-run relative to forecasts of price changes over the next second.

<sup>6</sup> The model nests the particular case in which the speculator can perfectly forecast news. This corresponds to the case of advance access to news content.



**Figure 1. Fast and slow speculation.** The figure plots the evolution over time of the speculator's position (left graph) and the change in this position—the speculator's trade—(right graph) when he is fast (plain line) and when he is slow (dot-dashed line) using the characterization of his equilibrium trading strategy in each case for the following parameter values (see Section II):  $\sigma_u = 1$  (standard deviation of noise traders' order flow),  $\sigma_v = 1$  (standard deviation of innovations in the asset value),  $\sigma_e = 1$  (standard deviation of noise in news) and  $\sum_0 = 1$  (variance of asset value conditional on information available at date 0). The path for the signals received by the speculator and the dealer is the same in each case. News and trades are assumed to take place every second. The liquidation time  $t = 1$  corresponds to one trading day = 23,400 seconds.

In contrast, there is no news trading when the speculator is *slow* relative to the dealer, that is, when prices react to news *before* the speculator can trade on his forecast of this reaction. In this case, the instantaneous volatility of the speculator's position is zero and his trades are unrelated to news arrival. Hence, it is crucial that the speculator reacts to news arrival faster than the dealer to obtain a strong correlation between his trades and news, as found empirically for directional HFTs.

Figure 1 (left graph) shows the dynamics of the speculator's position in the asset when he is slow (dot-dashed line) and when he is fast (solid line), for a given realization of information flow. The speculator's position is much more volatile when he is fast. In this case, the speculator optimally deviates from his long-run desired position to exploit his anticipation of short-run price reactions to news. As news is frequent, the speculator is “in and out in milliseconds,” that is, carries out many round-trip trades (buys/sells) within short time intervals (right graph), as HFTs do. Furthermore, these trades anticipate short-run price movements, as found empirically for HFTs' aggressive orders. The reason is that, in the short-run, the speculator trades in the direction of incoming news and news affects prices. None of these properties obtain if the speculator is slow. Thus, speed and news anticipation are important to understand stylized facts about HFTs' aggressive orders.

When the speculator is fast, his total expected profit can be split into two components: (i) his expected profit from the drift component of his strategy and (ii) his expected profit from the volatility component. The former is obtained from speculating smoothly on the long-run value of the asset while the latter is obtained from speculating aggressively on short-run price reactions to news. We refer to these components as, respectively, the value-trading and news-trading components of the speculator's strategy. The contribution of the value-trading component of the speculator's strategy to his overall profit increases as news informativeness decreases (see below), even though short-run changes in his position remain mostly explained by news. Thus, as Carrion (2013) finds for HFTs, a fast speculator in our model does not solely (or even mainly) profit from very short-term trading activities, even though his trades are highly correlated with short-term price movements.

The speculator obtains larger expected profits when he is fast than when he is slow. However, his expected profit from value trading is smaller when he is fast. Indeed, the fast speculator trades much more aggressively on his signals, which are therefore incorporated into prices more quickly. To offset this effect, the fast speculator trades less aggressively on his estimate of long-run price changes.

The market is less liquid (i.e., trades impact prices more) when the speculator is fast. Indeed, the dealer is then at risk of selling (buying) the asset just before good (bad) news, which increases adverse selection. This increase comes entirely from the speculator's ability to anticipate short-term price movements since, as we just explained, the speculator makes less profits from betting on long-run price changes when he is fast. This fits well with the perception that HFTs' aggressive orders expose liquidity suppliers to losses on short-run price moves. Moreover, when the speculator is fast, his trades are more informative about short-term price changes and less informative about long-run price changes than when he is slow. In our model, these two effects exactly offset each other so that the speed of price discovery (i.e., the rate at which the dealer's pricing error decays) is identical, regardless of whether the speculator is fast or slow.<sup>7</sup>

The model has many testable implications. Consider an econometrician with data on trades for prop trading firms using directional strategies before and after these firms become fast.<sup>8</sup> Our model predicts that the "footprints" of these firms should change around this event. First, their trades should be

<sup>7</sup> Interestingly, Chaboud et al. (2014) show empirically that algorithmic trading causes a reduction in the frequency of triangular arbitrage opportunities in the foreign exchange market. This finding is consistent with the possibility that HFTs correct short-term inefficiencies (e.g., slow reaction of prices to news) faster, as our model predicts, while leaving overall efficiency unchanged (the absence of arbitrage does not mean that prices fully reflect all available information about long-term payoffs). Chaboud et al. (2014) also find that algorithmic trading causes a reduction in the autocorrelation of high-frequency returns. Our model cannot explain this finding since returns are uncorrelated in our model, as is usual in models à la Kyle (1985).

<sup>8</sup> For papers with account-level data for HFTs, see Hagströmer and Nordén (2013) and Benos and Sagade (2013).

more correlated with short-term price reactions to news *after* they become fast. Furthermore, their share of total trading volume should increase because their optimal position becomes much more volatile. Last, the autocorrelation in their trades should decrease after they become fast. Indeed, the speculator's trades are positively autocorrelated (as in Kyle (1985)), regardless of whether he is fast or slow.<sup>9</sup> However, when the speculator is fast, short-term changes in his position are determined mainly by his forecast of short-term price reactions to the surprise component of news. His trades are therefore less autocorrelated when he is fast.

The most surprising predictions are those regarding the effect of news informativeness on high-frequency trading. When news is more informative, the speculator expects prices to react more to news. Thus, he trades more aggressively on short-term price reactions and, for this reason, his share of total trading volume increases. This increases the risk to the dealer of selling (buying) the asset just before good (bad) news. Yet, because news is more informative, the dealer is less at risk of losing money on long-run price changes. The latter effect dominates so that liquidity improves when news is more informative. Thus, when the speculator is fast, the model predicts a joint increase in informed trading and liquidity when news informativeness increases. In contrast, when the speculator is slow, an increase in news informativeness improves liquidity but reduces the speculator's share of trading volume, as is usual in models of informed trading (e.g., Kim and Verrecchia (1994)).

Second, the fraction of directional HFTs' profits coming from speculation on very short-run price movements should be higher in stocks with more informative news. Indeed, the speculator's expected profit from news trading increases when prices are more sensitive to news, and hence with news informativeness. In contrast, his expected profit from value trading declines with news informativeness because the speculator's long-run informational advantage is then reduced. Thus, the contribution of news trading to the speculator's profit increases with news informativeness.

Furthermore, the speculator's total expected profit decreases with news informativeness regardless of whether he is slow or fast, but at a lower rate when he is fast. Thus, the net gain of being fast (i.e., the difference between the profit of a fast and a slow speculator) increases with news informativeness. This yields two additional predictions: (i) the profitability of directional HFTs should be inversely related to news informativeness, but (ii) stocks with more informative news are more likely to attract directional HFTs.

In the baseline version of our model, news arrives at each trading opportunity. We also consider the more general case in which trading opportunities are more frequent than news. In this case, the equilibrium of the slow model is identical to that obtained in the baseline model. In the fast model, a new effect arises: illiquidity and the speculator's share of total trading volume are higher just before news arrival than after. The reason is that the speculator aggressively

<sup>9</sup> Benos and Sagade (2013) find that the (signed) trades of HFTs who mainly use aggressive orders are positively autocorrelated.

trades on his expectation of short-term price reactions to news just before news arrival but not after. This pattern is stronger for a stock with more informative news. However, as obtained in the baseline version of the model and for the same reasons, the average illiquidity (i.e., average price impact of all trades, before and after news) for such a stock should be smaller.

Our paper contributes to the growing theoretical literature on high-frequency trading.<sup>10</sup> Existing papers on this topic do not consider dynamic models of trading with news and private information. This feature, which is unique to our model, helps explain stylized facts about HFTs' aggressive orders and generates new predictions. Our model is designed to analyze HFTs' directional strategies and the effect of news on these strategies. It is therefore silent on high-frequency market-making (studied, for instance, by Ait-Sahalia and Saglam (2013) and Weller (2014)). Our modeling approach is related to dynamic extensions of Kyle (1985) (in particular, Back and Pedersen (1998), Chau and Vayanos (2008), Li (2013), Martinez and Roşu (2013), and Cao, Ma, and Ye (2013)). We discuss this relationship in depth in Section VI.

The paper is organized as follows. Section I describes the model. In Section II, we derive the equilibrium when the speculator is fast and the equilibrium when he is slow. Section III shows that the speculator's footprints are significantly different in each case. Section IV studies the effects of news informativeness. In Section V, we relax the assumption that the news arrival rate is identical to the trading rate. Section VI discusses the relationship between our model and dynamic extensions of Kyle (1985), and Section VII concludes. Proofs of the main results are in the Appendix. A companion Internet Appendix contains additional results and robustness checks.<sup>11</sup>

## I. Model

Trading for a risky asset takes place continuously over the time interval  $[0, 1]$ . The liquidation value of the asset is

$$v_1 = v_0 + \int_0^1 dv_t, \quad \text{with} \quad dv_t = \sigma_v dB_t^v, \quad (1)$$

where  $B_t^v$  is a Brownian motion,  $\sigma_v > 0$ , and  $v_0 \sim N(0, \Sigma_0)$ , with  $\Sigma_0 > 0$ . We interpret  $v_1$  as the "long-run" value of the asset, which here means the value of the asset at, say, the end of the trading day (this is long-run relative to short-run price movements due to news; see below). The risk-free rate is assumed to be zero. There are three types of market participants: (i) one

<sup>10</sup> See Cartea and Penalva (2012), Jovanovic and Menkveld (2012), Pagnotta and Philippon (2012), Ait-Sahalia and Saglam (2013), Budish, Cramton, and Shim (2013), Biais, Foucault, and Moinas (2015), Du and Zhu (2014), Hoffmann (2014), and Weller (2014), among others.

<sup>11</sup> The Internet Appendix is available in the online version of this article on the *Journal of Finance* website.

risk-neutral speculator (“he”), (ii) noise traders, and (iii) one competitive risk-neutral dealer (“she”), who sets the price at which trades take place.<sup>12</sup>

*News.* At date 0, the speculator receives the signal  $v_0$  about the liquidation value of the asset. The variance of this signal,  $\Sigma_0$ , represents the initial information advantage of the speculator. Then, new information arrives continuously. Specifically, in  $[t, t + dt]$ , the speculator privately observes the innovation in the asset value,  $dv_t$ , and the dealer receives the signal

$$dz_t = dv_t + de_t, \quad \text{with} \quad de_t = \sigma_e dB_t^e, \quad (2)$$

where  $B_t^e$  is a Brownian motion independent of all other variables. We refer to the signal received by the dealer,  $dz_t$ , as *news*. News informativeness decreases with  $\sigma_e$ .<sup>13</sup>

For simplicity, we assume that the speculator’s signal about the innovation in the asset value is perfect. This assumption, however, is not key for our findings and can be relaxed (see Internet Appendix Section IV). The important point is that the speculator’s signal is correlated with news. Thus, the speculator’s signal in  $[t, t + dt]$  can be used to forecast both the long-run value of the asset *and* incoming news. Specifically, the speculator expects the dealer’s news to be equal to his signal on average since  $E(dz_t | dv_t) = dv_t$ . However, the speculator does not perfectly know the news received by the dealer, unless  $\sigma_e = 0$ .

As explained in the introduction, in today’s markets, traders use a wide variety of high-frequency signals to predict future returns, for example, stock index returns, limit order books, order flows, and machine readable text (newswires, tweets, Facebook pages, blogs, firms’ websites, etc.). Thus, information (the speculator’s signals and news) in our model should be interpreted very broadly. The exact source of information does not matter for our results. Rather, our results rely only on news moving prices and the speculator’s signal being correlated with news.

*Trades, Prices, and Speed.* We denote by  $dx_t$  and  $du_t$  the market orders submitted by the speculator and noise traders, respectively, over  $[t, t + dt]$ . As in Kyle (1985),  $du_t = \sigma_u dB_t^u$ , where  $B_t^u$  is a Brownian motion independent of  $B_t^v$ . Thus, the order flow executed by the dealer is

$$dy_t = dx_t + du_t. \quad (3)$$

As the dealer is competitive and risk neutral, she continuously posts a price equal to her expectation of the asset liquidation value conditional on her

<sup>12</sup> For tractability, we only analyze the case in which there is a single speculator. Extending the continuous-time version of the model to the case with multiple speculators is challenging—see, for instance, Holden and Subrahmanyam (1992) or Back, Cao, and Willard (2000) for treatment without news. The generalization to multiple speculators is therefore left for future work.

<sup>13</sup> News is serially uncorrelated in our model. If the news were serially correlated, the dealer would react only to the innovation in news. Thus,  $dz_t$  should be interpreted as the innovation in news.

information. At date  $t$ , let  $I_t = \{y_\tau\}_{\tau \leq t} \cup \{z_\tau\}_{\tau \leq t}$  be the dealer's information set, and let  $q_t$  be the dealer's valuation for the asset,

$$q_t = E(v_1 | I_t). \tag{4}$$

We refer to  $q_t$  as the dealer's *quote* at date  $t$ .

In  $[t, t + dt]$ , the dealer receives two signals about the asset value: (i) the news,  $dz_t$ , and (ii) the order flow realized in this interval,  $dy_t$ , which contains information because the speculator's trade is informative. If the dealer is fast relative to the speculator, she updates her quote given the news *before* executing the order flow,  $dy_t$ . If instead she is slow relative to the speculator, she updates her quote only *after* executing the order flow  $dy_t$ . This formulation captures the notion that fast trading enables speculators to trade just ahead of news. In either case, the dealer's price accounts for the information contained in the order flow,  $dy_t$ . Thus, the order flow at date  $t$ ,  $dy_t$ , executes at

$$p_{t+dt} = \begin{cases} E(v_1 | I_t \cup dy_t) & \text{in the fast speculator model,} \\ E(v_1 | I_t \cup dz_t \cup dy_t) & \text{in the slow speculator model,} \end{cases} \tag{5}$$

where the "fast speculator model" ("slow speculator model") refers to the case in which the speculator places his market order,  $dx_t$ , before (after) the dealer updates her quote to reflect news,  $dz_t$ .

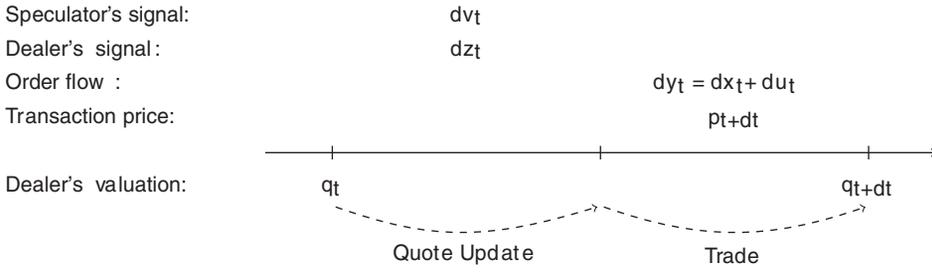
By comparing the properties of the slow and the fast speculator models (henceforth, slow and fast models), we can analyze how the speculator's ability to trade slightly ahead of news affects equilibrium trades and prices, everything else held equal.<sup>14</sup> Figure 2 summarizes the information structure and the timing of actions in the fast and slow models. In Section V, we consider a more general version of the model that allows for (i) a news arrival rate lower than the trading rate and (ii) longer delays in the dealer's reaction to news.

*Equilibrium Definition.* We assume that the speculator perfectly observes the news after it has been released to the dealer because news is public information. For instance, after an economic report has been publicly released, its content ( $dz_t$ ) is known to all. However, this assumption is not necessary for our findings because, in equilibrium, the speculator can infer the dealer's news from the history of prices and trades.<sup>15</sup> In electronic markets, this history is readily available in real time for sophisticated traders. In sum, the speculator's information set,  $\mathcal{I}_t$ , when he chooses his order,  $dx_t$ , includes (i) his signals, prices, and news up to date  $t$ , and (ii) his signal  $dv_t$ :  $\mathcal{I}_t = \{v_\tau\}_{\tau \leq t} \cup \{p_\tau\}_{\tau \leq t} \cup \{z_\tau\}_{\tau \leq t} \cup dv_t$ .

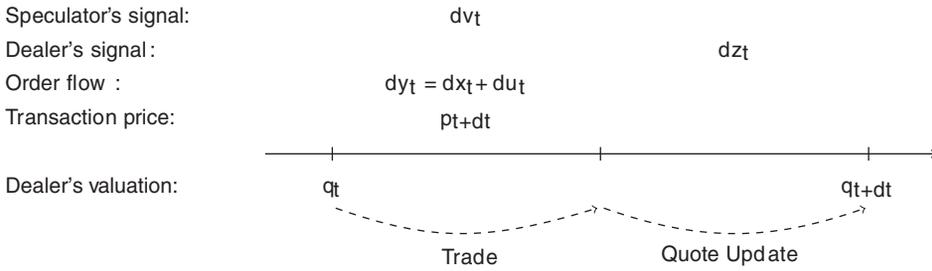
<sup>14</sup> Existing literature focuses on the slow speculator scenario. To our knowledge, we are the first to consider the fast speculator scenario in a dynamic trading model à la Kyle (1985). See Section VI for a comparison of our results with the existing literature.

<sup>15</sup> For instance, consider the equilibrium of the fast model described in Theorem 2. After observing the order flow at date  $t$ ,  $dy_t$ , and the transaction price,  $p_{t+dt}$ , at this date, the speculator can infer the dealer's quote  $q_t$  (see (16)). Now, as shown by (17), observing  $q_t$  and the trade at date  $t$  enables the speculator to learn the news released at this date, even if he does not directly observe it. Thus, the speculator can reconstruct the history of news from the history of trades and transaction prices.

Slow model



Fast model



**Figure 2.** Timing of events during  $[t, t + dt]$ .

A trading strategy for the speculator is a process for his position in the risky asset,  $x_t$ , measurable with respect to  $\mathcal{J}_t$ . For a given trading strategy, the speculator's expected profit  $\pi_\tau$  from date  $\tau$  onwards is

$$\pi_\tau = E \left( \int_\tau^1 (v_1 - p_{t+dt}) dx_t \mid \mathcal{J}_\tau \right). \tag{6}$$

An equilibrium is such that (i) at every date  $\tau$ , the speculator's trading strategy maximizes his expected trading profit (6) given the dealer's pricing policy, and (ii) the dealer's pricing policy is given by (5) and is consistent with the equilibrium speculator's trading strategy. As in Kyle (1985), we focus on the linear equilibria of the fast or the slow models. Specifically, we consider equilibria in which the speculator's strategy has the form

$$dx_t = \underbrace{\beta_t^k (v_t - q_t) dt}_{\text{Value-trading component}} + \underbrace{\gamma_t^k dv_t}_{\text{News-trading component}} \quad \text{for } k \in \{S, F\}, \tag{7}$$

where  $\beta_t^k$  and  $\gamma_t^k$  are smooth (i.e., continuously differentiable) functions of  $t \in [0, 1]$ . The superscripts  $S$  and  $F$  refer to the slow model and the fast model, respectively.

The speculator's trade at a given point can be decomposed into two distinct trades. The first, given by  $\beta_t^k (v_t - q_t) dt$ , exploits the speculator's forecast of the

long-run change in the value of the asset,  $(v_t - q_t)$ . Hence, we refer to this trade as the *value-trading component* of the speculator's trading strategy. The second trade, given by  $\gamma_t^k dv_t$ , exploits the speculator's ability to forecast news. It is therefore proportional to this forecast, which as explained before is  $dv_t$ . We refer to this trade as the *news-trading component* of the speculator's trading strategy.

When  $\gamma_t^k = 0$ , the speculator's trading strategy has no news-trading component, as in Kyle (1985) and extensions of this model allowing for incremental information (e.g., Back and Pedersen (1998), Chau and Vayanos (2008), Li (2013), and Cao, Ma, and Ye (2013)). In this case, the speculator's signals affect his position only because they affect his forecast of the long-run value of the asset,  $v_t$ . When  $\gamma_t^k > 0$ , the speculator's signals affect his trade at a given date above and beyond their effect on his forecast of the long-run value of the asset. As explained below, this is because the speculator uses his signal to speculate on short-run the price reaction to news. Hence, we say that there is news trading when  $\gamma_t^k > 0$ . In the next section, we show that speed is a prerequisite for news trading:  $\gamma_t^k > 0$  only when the speculator is fast in equilibrium. In this case, the speculator's trading strategy features a nonzero volatility component.

The speculator's trade at a given date could linearly depend on his past signals according to many forms other than (7). However, in the discrete-time formulation of the model, the unique linear equilibrium of the fast or the slow model has the form specified by (7) (see Internet Appendix Section I). It is therefore natural to focus on equilibria of this form in continuous time. Analytical solutions for the equilibrium are easily obtained in continuous time (which is the reason we focus on this case), regardless whether the speculator is fast or slow. In contrast, in discrete time, one must solve for the equilibrium numerically (even with only two trading rounds), which obfuscates economic intuition. All of our findings also hold in discrete time as long as there are at least two trading rounds. With only one trading round, one must specify exogenously the reaction of prices to news (the coefficients  $\mu^k$  in Theorems 1 and 2; see below). This is restrictive because the new insights provided by the model (e.g., those regarding the effects of news informativeness) come from the endogeneity of price reactions to news.

## II. Equilibrium News Trading

In this section, we derive the equilibria of the slow model (Theorem 1) and the fast model (Theorem 2). We then compare equilibrium trades and prices in each case. We also analyze how speed affects the speculator's expected profit, price discovery, and the contribution of news to price volatility.

**THEOREM 1 (Benchmark: Slow Equilibrium):** *In the slow model, there is a unique linear equilibrium of the form*

$$dx_t = \beta_t^S (v_t - q_t) dt + \gamma^S dv_t, \quad (8)$$

$$p_{t+dt} = q_t + \mu^S dz_t + \lambda^S dy_t, \tag{9}$$

$$dq_t = \mu^S dz_t + \lambda^S dy_t, \tag{10}$$

with

$$\beta_t^S = \frac{1}{1-t} \frac{\sigma_u}{\sum_0^{1/2}} \left( 1 + \frac{\sigma_v^2 \sigma_e^2}{\sum_0 (\sigma_v^2 + \sigma_e^2)} \right)^{1/2}, \tag{11}$$

$$\gamma^S = 0, \tag{12}$$

$$\lambda^S = \frac{\sum_0^{1/2}}{\sigma_u} \left( 1 + \frac{\sigma_v^2 \sigma_e^2}{\sum_0 (\sigma_v^2 + \sigma_e^2)} \right)^{1/2}, \tag{13}$$

$$\mu^S = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_e^2}. \tag{14}$$

Hence, when the speculator is slow, there is no news trading ( $\gamma^S = 0$ ) and the equilibrium is similar to that in Kyle (1985). The only difference is that the dealer’s price at each date is affected by the news ( $dz_t$ ), in addition to the information contained in trades ( $dy_t$ ). The sensitivity of the dealer’s price to news is measured by  $\mu^S$  and the sensitivity of her price to the order flow is measured by  $\lambda^S$ .<sup>16</sup>

**THEOREM 2 (Fast Equilibrium):** *In the fast model, there is a unique linear equilibrium of the form*

$$dx_t = \beta_t^F (v_t - q_t)dt + \gamma^F dv_t, \tag{15}$$

$$p_{t+dt} = q_t + \lambda^F dy_t, \tag{16}$$

$$dq_t = \lambda^F dy_t + \mu^F (dz_t - \rho^F dy_t), \tag{17}$$

with

$$\beta_t^F = \frac{1}{1-t} \frac{\sigma_u}{(\Sigma_0 + \sigma_v^2)^{1/2}} \frac{1}{\left( 1 + \frac{\sigma_e^2}{\sigma_v^2} g \right)^{1/2}} \left( 1 + \frac{(1-g)\sigma_v^2}{\Sigma_0} \frac{1 + \frac{\sigma_e^2}{\sigma_v^2} + \frac{\sigma_e^2}{\sigma_v^2} g}{2 + \frac{\sigma_e^2}{\sigma_v^2} + \frac{\sigma_e^2}{\sigma_v^2} g} \right), \tag{18}$$

<sup>16</sup>When  $\sigma_v$  goes to zero, the equilibrium of the slow model converges to the unique linear equilibrium in the continuous-time version of Kyle (1985). Indeed, in this case, there is no news and therefore the model is identical to Kyle (1985). For the same reason, this is also the case for the equilibrium of the fast model.

$$\gamma^F = \frac{\sigma_u}{\sigma_v} g^{1/2} = \frac{\sigma_u}{(\Sigma_0 + \sigma_v^2)^{1/2}} \frac{\left(1 + \frac{\sigma_e^2}{\sigma_v^2} g\right)^{1/2} (1 + g)}{2 + \frac{\sigma_e^2}{\sigma_v^2} + \frac{\sigma_e^2}{\sigma_v^2} g}, \quad (19)$$

$$\lambda^F = \frac{(\Sigma_0 + \sigma_v^2)^{1/2}}{\sigma_u} \frac{1}{\left(1 + \frac{\sigma_e^2}{\sigma_v^2} g\right)^{1/2} (1 + g)}, \quad (20)$$

$$\mu^F = \frac{1 + g}{2 + \frac{\sigma_e^2}{\sigma_v^2} + \frac{\sigma_e^2}{\sigma_v^2} g}, \quad (21)$$

$$\rho^F = \frac{\sigma_v}{\sigma_u} \frac{g^{1/2}}{1 + g} = \frac{\sigma_v^2}{\sigma_u (\Sigma_0 + \sigma_v^2)^{1/2}} \frac{\left(1 + \frac{\sigma_e^2}{\sigma_v^2} g\right)^{1/2}}{2 + \frac{\sigma_e^2}{\sigma_v^2} + \frac{\sigma_e^2}{\sigma_v^2} g}, \quad (22)$$

where  $g$  is the unique root in  $(0, 1)$  of the cubic equation

$$g = \frac{\left(1 + \frac{\sigma_e^2}{\sigma_v^2} g\right) (1 + g)^2}{\left(2 + \frac{\sigma_e^2}{\sigma_v^2} + \frac{\sigma_e^2}{\sigma_v^2} g\right)^2} \frac{\sigma_v^2}{\sigma_v^2 + \Sigma_0}. \quad (23)$$

Theorem 2 characterizes the equilibrium of the fast model for any level of news informativeness,  $\sigma_e$ . The equilibrium is almost in closed form because all coefficients are a function of exogenous parameters and  $g$ , which solves the cubic polynomial (23). When  $\sigma_e = 0$ , one can easily solve for  $g$  analytically and obtain a closed form solution for the expressions of equilibrium coefficients  $\beta_t$ ,  $\gamma_t$ , etc. (see Internet Appendix Section V). However, all of our findings hold for any values of  $\sigma_e$  and do not specifically require a closed form solution for  $g$ .

Comparison of the speculator's trading strategy when he is slow and fast directly yields the following result, which is central for the novel implications of our paper.

PROPOSITION 1: *There is news trading only when the speculator is fast, that is,  $\gamma^F > 0$  while  $\gamma^S = 0$ , for all parameter values.*

The economic intuition is as follows. As explained previously, the speculator's signal can be used to forecast news. As news affects prices ( $\mu^k > 0$ ), the speculator's signal can be used to forecast price reactions to news. For instance, suppose that the speculator receives signal  $dv_t$  and does not trade ( $dx_t = 0$ ). Then he expects the dealer's quote to change by  $\mu^k E(dz_t \mid dv_t) = \mu^k dv_t$  (see (10) and (17)): when the speculator receives a positive (negative) signal, he expects the dealer to mark her quote up (down) upon receiving the news. The speculator can exploit this short-run predictability in price changes only if he acts before the dealer's quote reflects the news, that is, only if he is fast. For this reason, there is news trading only in the fast model.

Thus, when he is fast, the speculator's trades are driven by two different bets: one on the long-run change in the value of the asset (the value-trading bet) and one on the short-run price reaction to news (the news-trading bet). As he trades on news, the speculator dissipates his long-run information advantage (his knowledge of  $v_t$ ) more quickly because he trades more aggressively on each new incoming signal. To offset this effect, the speculator optimally scales down the value-trading component of his strategy, that is,  $\beta_t^F < \beta_t^S$ , as the next corollary shows.

PROPOSITION 2: *For all parameter values and at each date,  $\beta_t^F < \beta_t^S$ .*

We now study the contribution of each speculator's bet to his expected profit. To this end, we separately compute the ex ante expected profits on trades due to each component of the speculator's trading strategy. For  $k \in \{S, F\}$ , we denote by  $\pi_\beta^k$  the ex ante expected profit on value trading and by  $\pi_\gamma^k$  the expected profit on news trading.<sup>17</sup> The speculator's ex ante expected profit is  $\pi_0^k = \pi_\beta^k + \pi_\gamma^k$ .

PROPOSITION 3: *The speculator earns a strictly positive expected profit from value trading, equal to  $\pi_\beta^k = \beta_0^k \sum_0$ , regardless of whether he is fast or slow. However, his profit from value trading is smaller when he is fast ( $\pi_\beta^S > \pi_\beta^F > 0$ ). The speculator earns profit from news trading if and only if he is fast ( $\pi_\gamma^F > \pi_\gamma^S = 0$ ). In net, the speculator's total expected profit is larger when he is fast, that is,  $\pi_0^F > \pi_0^S$ .*

A fast speculator optimally trades less aggressively on his long-run estimate of the fundamental value than a slow speculator (Proposition 2). For this reason, the speculator's expected profit from value trading is smaller when he is fast. However, he still earns profits from value trading, that is, from relatively long-run market timing. Consistent with this implication, Carrion (2013) finds that high-frequency traders in his sample realize most of their profits on aggressive orders at relatively long (i.e., daily) horizons (see table 5 in Carrion (2013)). Similarly, Baron, Brogaard, and Kirilenko (2014, p. 5) find that "Aggressive HFTs [...] gain money by predicting price movements on longer (but still intraday) time scales." The relative contribution of each type of trade to total profit depends on the parameters. In particular, it should vary according to news informativeness (see Section IV).

As in Kyle (1985), the speculator's expected profit is equal to liquidity traders' expected trading costs (or equivalently, the dealer's expected loss on trades with the speculator), that is,  $\lambda^k \sigma_u^2$ . Hence, the sensitivity of prices to trades,  $\lambda^k$ , is a measure of market illiquidity. As the speculator earns larger profit when he is fast (Proposition 3), we obtain the following result.

PROPOSITION 4: *Illiquidity is higher when the speculator reacts to news faster than the dealer, that is,  $\lambda^F > \lambda^S$ .*

<sup>17</sup> In  $[t, t + dt]$ , the trade due to the value-trading component is  $dx_{val,t} \equiv \beta_t^k(v_t - q_t)dt$ . Hence,  $\pi_\beta^k = E(\int_0^1 (v_1 - p_t) dx_{val,t} | \mathcal{J}_0)$  and  $\pi_\gamma^k = \pi_0^k - \pi_\beta^k$  for  $k \in \{S, F\}$ .

The negative effect of speed on liquidity comes entirely from the fact that speed allows the speculator to make profits on short-run price reactions to news at the expense of other market participants. Indeed, as explained previously, the speculator earns a smaller expected profit from the value-trading component of his strategy when he is fast. This effect partially mitigates the deleterious effect of speed on trading costs.

PROPOSITION 5: *The dealer’s valuation of the asset,  $q_t$ , is less sensitive to the surprise component of news when the speculator is fast, that is,  $\mu^F < \mu^S$ .*

When the speculator is fast, his trades, and therefore the order flow  $dy_t$ , contain information on incoming news. Accordingly, the surprise component of news (i.e.,  $dz_t - E(dz_t|dy_t)$ ) is less informative when the speculator is fast, and for this reason the dealer’s valuation is less sensitive to the surprise component of news in the fast model.

Thus, prices are more sensitive to trades (Proposition 4) and less sensitive to news when the speculator is fast. One testable implication of these results is that speed should affect the relative contribution of trades and news to volatility. To see this, observe that the instantaneous volatility of the dealer’s quote for the asset ( $\text{Var}(dq_t)$ ) comes from news and trades since these are the two sources of information for the dealer. Using Theorems 1 and 2, we have

$$\text{Var}(dq_t) = \underbrace{\text{Var}(dq_{\text{trades},t})}_{\text{Trade component}} + \underbrace{\text{Var}(dq_{\text{news},t})}_{\text{News component}}, \tag{24}$$

where  $\text{Var}(dq_{\text{trades},t}) = \text{Var}(\lambda^k dy_t)$  for  $k \in \{S, F\}$ ,  $\text{Var}(dq_{\text{news},t}^S) = \text{Var}(\mu^S dz_t)$ , and  $\text{Var}(dq_{\text{quotes},t}^F) = \text{Var}(\mu^F (dz_t - \rho^F dy_t))$ .

COROLLARY 1: *Regardless of whether the speculator has a speed advantage, the instantaneous volatility of prices is constant and equal to  $\frac{1}{dt} \text{Var}(dq_t) = \sigma_v^2 + \sum_0$ . However, trades contribute to a greater fraction of this volatility when the speculator reacts faster to news.*

Hasbrouck (1991) shows how to estimate the relative contributions of trades and public information to price volatility. Using this methodology, one could test whether an increase in speculators’ speed of reaction to news (e.g., the introduction of colocation as in Boehmer, Fong, and Wu (2014) or Brogaard et al. (2014)) lowers the contribution of news to volatility, as predicted by Corollary 1.

When the speculator is fast, trades are more informative about innovations in the asset value ( $dv_t$ ). However, they are less informative about the long-run value of the asset ( $v_t$ ) because the speculator optimally trades less aggressively on his forecast of the long-run price change (Proposition 2). The next corollary shows that these two effects offset each other exactly so that speed has no effect on pricing efficiency (as measured by the average squared pricing error).

COROLLARY 2: Let  $\Sigma_t$  be the average squared pricing error at date  $t$  ( $\Sigma_t = E((v_t - q_t)^2)$ ). In equilibrium,

$$d\Sigma_t = \left[ -2\text{Cov}(dq_{u_t}, v_t - q_t) - 2\text{Cov}(dq_{u_t}, dv_t) + (2\sigma_v^2 + \Sigma_0) \right] dt. \quad (25)$$

When the speculator reacts faster to news, short-run price changes are more correlated with innovations in the asset value (i.e.,  $\text{Cov}(dq_{u_t}, dv_t)$  is higher in the fast model), but less correlated with the dealer's pricing error (i.e.,  $\text{Cov}(dq_{u_t}, v_t - q_t)$  is smaller in the fast model). The first effect strengthens price discovery while the second weakens it. In equilibrium, they just offset each other and the question of who is faster is irrelevant for price discovery, that is,  $\Sigma_t = (1 - t)\Sigma_0$  in both the fast and the slow models.

At any date, the average pricing error,  $\Sigma_t$ , depends only on the speculator's initial information advantage,  $\Sigma_0$ , since  $\Sigma_t = (1 - t)\Sigma_0$ . As  $\Sigma_0$  goes to zero,  $\Sigma_t$  goes to zero as well, no matter how large the subsequent innovations in the asset value ( $\sigma_v$ ) or how imprecise the news received by the dealer ( $\sigma_e$ ). However, the speculator's expected profits remain strictly positive, even when  $\Sigma_0$  goes to zero, as the next corollary shows.

COROLLARY 3: When  $\Sigma_0$  goes to zero, the speculator earns strictly positive expected profit on the value-trading component of his trades iff  $\sigma_e > 0$ :  $\lim_{\Sigma_0 \rightarrow 0} \pi_\beta^k > 0$  for  $\sigma_e > 0$ , for  $k \in \{S, F\}$ . Moreover, when he is fast, he earns strictly positive expected profit on news-trading for all values of  $\sigma_e$ :  $\lim_{\Sigma_0 \rightarrow 0} \pi_\gamma^F > 0$  for  $k \in \{S, F\}$ .

First consider the case in which the speculator is slow. The speculator's expected profit on his trade at date  $t$  is then  $\beta_t^S \Sigma_t = \lambda^S \sigma_u^2$ . When  $\Sigma_0$  goes to zero,  $\beta_t^S$  goes to infinity (see (11)). Hence, in equilibrium, the speculator trades very fast on each new bit of information,  $dv_t$  (his trading rate becomes infinite), which dissipates his information advantage very quickly ( $\Sigma_t$  goes to zero). In other words, the speculator's profit per trade becomes very small but the speculator's number of trades per unit of time becomes very large. These two opposite forces net out so that the speculator's expected profit ( $\beta_t^S \Sigma_t$ ) converges to a finite limit ( $\lambda^S \sigma_u^2$ ), which is strictly positive when  $\sigma_e > 0$ , that is, the speculator's signals are more informative than news. This result is identical to that in Chau and Vayanos (2008). The same mechanism holds when the speculator is fast. However, in this case and in contrast to Chau and Vayanos (2008), the speculator can sustain strictly positive expected profits even when  $\sigma_e = 0$  (in this case,  $\lim_{\Sigma_0 \rightarrow 0} \pi_\gamma^F = \frac{\sigma_v \sigma_u}{2}$ ; see the proof of the corollary). The reason is that, by moving slightly ahead of the dealer, the speculator is able to earn a profit on the price reaction to news, even though his information advantage is very short lived.<sup>18</sup>

<sup>18</sup> When  $\Sigma_0 \rightarrow 0$  and  $\sigma_e = 0$ , the speculator has only short-lived information, that is, information that will soon be observed perfectly by the dealers, as in Admati and Pfleiderer (1988). This case is rather special, however, because, no matter how small  $\sigma_e$  is, the speculator will derive trading profits from the  $\beta$  and  $\gamma$  components of his trading strategy when  $\sigma_e > 0$ .

### III. News Trading's Footprints

In this section, we derive predictions about the effect of speed on a speculator's "footprints" (e.g., the autocorrelation of his trades or the correlation between his trades and returns). To test these predictions, one could collect data on proprietary trading firms' trades around the date these firms become fast and test whether their footprints change as predicted in this section. Several empirical studies make inferences about HFTs' strategies from both patterns in their trades and the association between these trades and returns. Our results in this section also provide guidance for such "reverse engineering" exercises.

#### A. Fast Speculator Has a Higher Participation Rate

The speculator slowly exploits his long-run information advantage. In contrast, the speculator trades aggressively on news when he is fast. Thus, over short time intervals, the news-trading component of the speculator's strategy explains most of the variation in his position when he is fast. As a result, over short-term intervals, changes in the speculator's position are more correlated with news ( $\text{Cov}(dx_t, dz_t) = \gamma^F \sigma_v^2$  in the fast model, while this covariance is zero in the slow model) and the speculator's position is much more volatile when he is fast (see Figure 1 in the introduction). Consequently, the speculator accounts for a much larger fraction of total trading volume when he is fast. To see this formally, let the speculator's participation rate ( $SPR_t$ ) be the instantaneous contribution of the speculator's trade to total trading volume:

$$SPR_t = \frac{\text{Var}(dx_t)}{\text{Var}(dy_t)} = \frac{\text{Var}(dx_t)}{\text{Var}(du_t) + \text{Var}(dx_t)}. \quad (26)$$

COROLLARY 4: *The speculator's participation rate is higher when the speculator is fast. Specifically,*

$$SPR^S = 0, \quad SPR^F = \frac{g}{1+g} > 0, \quad (27)$$

where  $g \in (0, 1)$  is defined in Theorem 2.

Corollary 4 is derived for trades measured over an infinitesimal time interval. Over a longer time interval, the contribution of a slow speculator to total trading volume is not negligible because he slowly builds up a position in the direction of his estimate of the asset mispricing. However, the speculator's participation rate remains higher when he is fast, even when trades are aggregated over longer time intervals (see Internet Appendix Section II). This difference becomes smaller as these intervals become longer (see Table I) because trades due to news trading are uncorrelated over time (see Section III.B) and therefore cancel out when they are aggregated over long intervals.

Brogaard et al. (2014) define fast traders as those using colocation services. Using data from NASDAQ OMX Stockholm, they find that colocated traders account for about 41% of the trading volume for the stocks in their sample,

**Table I**  
**Speculator's Participation Rate at Various Sampling Frequencies**

The table compares the speculator participation rate ( $SPR$ ) in the slow ( $SPR_{\text{slow}}$ ) and fast ( $SPR_{\text{fast}}$ ) models when data are sampled at various frequencies, as explained in Internet Appendix II. The order flow is thus aggregated—for the speculator and the noise traders—over time intervals of various lengths: one millisecond ( $10^{-3}$  seconds),  $10^{-1}$  seconds, one second, and one minute. The parameter values are  $\sigma_u = 1$  (standard deviation of noise traders' order flow),  $\sigma_v = 1$  (standard deviation of innovations in the asset value),  $\sigma_e = 0.5$  (standard deviation of noise in news), and  $\Sigma_0 = 0.25$  (variance of asset value conditional on information available at date 0). The liquidation time  $t = 1$  corresponds to one trading day = 23,400 seconds.

Sampling Interval	$10^{-3}$ second	$10^{-1}$ second	One second	One minute
$SPR_{\text{slow}}$	0.000008%	0.0008%	0.0077%	0.4615%
$SPR_{\text{fast}}$	19.8238%	19.8242%	19.8280%	20.0776%
$SPR_{\text{fast}}/SPR_{\text{slow}}$	2,577,096	25,771.51	2,577.65	43.50

despite the fact that they represent only a third of all trading accounts in their sample. They also find that those buying colocation upgrades (faster connections to NASDAQ OMX servers) experience a significant increase in their share of trading volume (see their table 4). If news trading plays a role in this increase, then, according to our model, the correlation between the trading volume of traders buying colocation upgrades and news should increase after these traders become fast. In line with this prediction, Brogaard et al. (2014) find that the trades of the fastest traders on NASDAQ OMX Stockholm are positively related to lagged index futures returns and that this correlation increases after colocation upgrades (see their table 8).

### *B. Fast Speculator's Trades Are Less Autocorrelated*

The value-trading component of the speculator's trading strategy ( $\beta_t(v_t - q_t)dt$ ) calls for repeated trades in the same direction because the speculator's forecast of the long-run change in the asset value,  $v_t - q_t$ , changes slowly. This feature generates a positive autocorrelation in the speculator's trades regardless of whether he is fast or slow. However, when the speculator is fast, his trades are less autocorrelated because, over short time intervals, they are mainly driven by the news-trading component of his strategy ( $\gamma dv_t$ ).<sup>19</sup> When changes in the fast speculator's position are observed over shorter and shorter time intervals, the autocorrelation of these changes goes to zero while it remains strictly positive for a slow speculator, as the next corollary shows.

<sup>19</sup> Trades due to this component are uncorrelated because the speculator's signals are uncorrelated.

COROLLARY 5: *The autocorrelation of the speculator’s trades over short time intervals is lower when he is fast. More specifically, for  $\tau \in (0, 1 - t)$ ,*

$$\begin{aligned} \text{Corr}(dx_t^S, dx_{t+\tau}^S) &= \left( \frac{1 - t - \tau}{1 - t} \right)^{\lambda^S \beta_0^S - \frac{1}{2}} > 0, \\ \text{Corr}(dx_t^F, dx_{t+\tau}^F) &= 0. \end{aligned} \tag{28}$$

The autocorrelation of changes in a fast speculator’s position remains smaller than that for a slow speculator, even when these changes are measured over noninfinitesimal time intervals (see Internet Appendix Section II). However, this autocorrelation increases with the length of these intervals because changes in the speculator’s position become increasingly driven by the value component of his trading strategy as these changes are measured over longer and longer time periods.

Some papers (e.g., Benos and Sagade (2013) and Hirschey (2013)) find that aggressive orders placed by HFTs are positively autocorrelated. This finding is consistent with both the slow and the fast models. Our new prediction is that the autocorrelation in a speculator’s aggressive orders should fall after he becomes fast.<sup>20</sup>

C. *Fast Speculator’s Trades Anticipate Short-Term Price Movements*

As explained previously, a fast speculator trades on short-run price reactions to news while a slow speculator does not. Hence, the information content of the speculator’s trades about subsequent price changes at very short horizons should be much stronger when he is fast. To formalize this conjecture, let  $CPI_t^k$  be the covariance between the speculator’s trade per unit of time and the subsequent cumulative price change over  $[t, t + \tau]$  for  $\tau > 0$ :

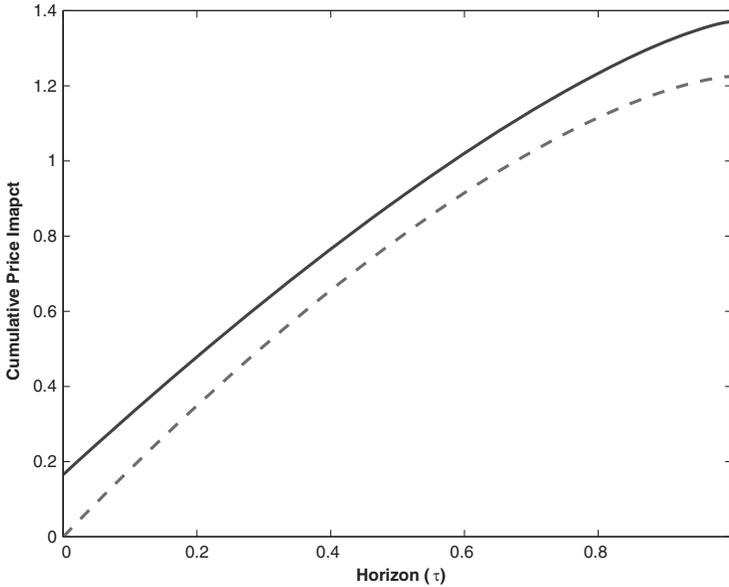
$$CPI_t^k(\tau) = \text{Cov}(dx_t, q_{t+\tau} - q_t) / dt \quad \text{for } k \in \{S, F\}. \tag{29}$$

This covariance can be seen as a measure of the cumulative price impact (CPI) over time interval  $\tau$  of the speculator’s trade at a given point in time. It is a measure of the information content of the speculator’s trade at a given point in time.

We show in the proof of Corollary 6 that in the slow model

$$CPI_t^S(\tau) = C_1^S \left[ 1 - \left( 1 - \frac{\tau}{1 - t} \right)^{\lambda^B \beta_0^S} \right], \tag{30}$$

<sup>20</sup> Some papers (for example, Menkveld (2013) or Kirilenko et al. (2014)) find evidence of mean-reverting inventories for HFTs. Mean reversion in inventories might be a characteristic of high-frequency market making, a strategy that our model does not intend to describe. For instance, Menkveld (2013) shows that the HFT in his data set behaves more like a market maker than an informed investor.



**Figure 3. Cumulative price impact at different horizons.** The figure plots the cumulative price impact,  $CPI_0(\tau)$  over an interval of length  $\tau \in (0, 1]$  that starts at time  $t = 0$  in the slow model (dotted line) and the fast model (solid line). The parameter values are  $\sigma_u = 1$  (standard deviation of noise traders' order flow),  $\sigma_v = 1$  (standard deviation of innovations in the asset value),  $\sigma_e = 1$  (standard deviation of noise in news), and  $\Sigma_0 = 1$  (variance of asset value conditional on information available at date 0).

while in the fast model

$$CPI_t^F(\tau) = C_0^F + C_1^F \left[ 1 - \left( 1 - \frac{\tau}{1-t} \right)^{(\lambda^F - \mu^F \rho^F) \beta_0^F} \right], \tag{31}$$

where  $C_0^F$ ,  $C_1^S$ , and  $C_1^F$  are positive coefficients. We deduce the following result.

**COROLLARY 6:** *As  $\tau$  approaches zero, the CPI of the speculator's trade goes to zero when he is slow but remains strictly positive (and equal to  $C_0^F$ ) when he is fast.*

Figure 3 illustrates Corollary 6 for specific values of the parameters. The covariance between the speculator's trade and the subsequent cumulative price change is positive and, at short horizons (small  $\tau$ ), is much larger when the speculator reacts faster to news. As Figure 3 shows, empiricists could assess the existence of news trading by estimating CPI for various values of  $\tau$ . Indeed, a large value of CPI over short horizons is indicative of news trading, since  $CPI_t^F(\tau) \approx C_0^F > 0$ , while  $CPI_t^S(\tau) \approx 0$  for  $\tau$  small.

Brogaard, Hendershott, and Riordan (2014) and Kirilenko et al. (2014) find that HFTs' aggressive orders have a positive correlation with subsequent returns over a very short horizon. They interpret this finding as reflecting HFTs'

ability to anticipate short-term price movements. In our model, this is indeed the reason why  $CPI(\tau)$  is much higher at short horizons when the speculator is fast. This pattern is obtained even though the speculator also trades on his long-run forecast of the asset value. For instance, for the parameter values used to produce Figure 3, the speculator earns most (86.75%) of his profit from the value-trading component of his trading strategy. Hence, a strong short-run correlation between HFTs' aggressive orders and returns is insufficient to conclude that HFTs only gain from trading on short-run information.

#### IV. News Informativeness and Fast Trading

In this section, we derive predictions about the effects of news informativeness on patterns in liquidity and trading volume (Section IV.A). We also study how news informativeness affects the speculator's expected profit and his incentive to be fast (Section IV.B). We measure news informativeness by the square root of the precision of the signal received by the dealer, that is,  $\nu = \frac{1}{\sigma_e}$  (news is more informative when  $\sigma_e$  is smaller). Our predictions in this section relate to the effects of cross-sectional variation in  $\nu$ . Recent advances in textual analysis offer ways to build firm-level proxies for  $\nu$ . For instance, news vendors (Reuters, Bloomberg, and Dow Jones) now report firm-specific news in real time, assigning a direction and a relevance score to each news item (see, for instance, Gross-Klussmann and Hautsch (2011)). Firms with more relevant news should be firms for which dealers receive more informative news (higher  $\nu$ ).<sup>21</sup>

##### A. News Informativeness, Volume, and Liquidity

We first study how news informativeness affects the speculator's trading strategy.<sup>22</sup>

**PROPOSITION 6:** *When the speculator is fast, he trades more aggressively on news when news is more informative, that is,  $\frac{\partial \gamma^F}{\partial \nu} > 0$ . In contrast, regardless of whether he is fast or slow, the speculator trades less aggressively on his forecast of the long-run price change when news is more informative, that is,  $\frac{\partial \beta_t^k}{\partial \nu} < 0$  for  $k \in \{S, F\}$ . If the dealer receives uninformative news ( $\nu = 0$ ), then there is no news trading ( $\gamma^F = 0$ ), and  $\beta_t^F = \beta_t^S$ .*

<sup>21</sup> HFT firms are less likely to rely on relevance scores provided by data vendors, as these are provided with a delay relative to source news. In line with this conjecture, Gross-Klussmann and Hautsch (2011) find that a large fraction of the cumulative abnormal price change around news arrivals in their sample takes place *before* news arrival.

<sup>22</sup> All the formulas of the model are homogeneous in the following ratios: (i)  $a = \frac{\sigma_u^2}{\sigma_v^2}$ , (ii)  $b = \frac{\sigma_e^2}{\sigma_v^2}$ , and (iii)  $c = \frac{\sum_0}{\sigma_v^2}$ . This means that, if we hold ratios  $a$  and  $c$  constant, then an increase in  $\sigma_e$  (a decrease in news informativeness) has the same effect as a decrease in  $\sigma_v$ . Both increase the noise-to-signal ratio,  $b$ , in the signal received by the dealer. Thus, comparative statics results for  $\nu = \frac{1}{\sigma_e}$  are identical to comparative statics results for the signal-to-noise ratio,  $1/b = \frac{\sigma_v^2}{\sigma_e^2}$ .

When news informativeness increases, price reacts more to news ( $\mu^k$  increases with  $\nu$ ). Thus, trading on short-term price reactions to news becomes more profitable, and therefore a fast speculator trades on his expectation of short-term price movements more aggressively ( $\gamma^F$  increases with  $\nu$ ). Doing so, however, dissipates the speculator's long-run information advantage more quickly. This effect is reinforced by the fact that the dealer obtains more precise information when news informativeness increases. To mitigate these effects, the speculator optimally trades less aggressively on his forecast of the long-run price change in the asset when  $\nu$  increases, regardless of whether he is slow or fast ( $\beta_t^k$  declines with  $\nu$ ).<sup>23</sup>

When  $\nu = 0$ , news is uninformative. Hence, in this polar case prices do not react to news and therefore there is no news trading even when the speculator is fast ( $\gamma^F = 0$  if  $\nu = 0$ ). This observation highlights that both speed and predictability of short-run price reactions to news are required for news trading.

The positive effect of news informativeness on the intensity of news trading when the speculator is fast has several testable implications, summarized in the next corollary.

**COROLLARY 7:** *When the speculator is fast, an increase in the dealer's news informativeness triggers an increase in (i) the speculator's participation rate ( $SPR^F$ ), (ii) trading volume per unit of time (i.e.,  $TV^F = \frac{\text{Var}(dy)}{dt}$ ), and (iii) liquidity ( $1/\lambda^F$ ).*

When news informativeness increases, the speculator trades more aggressively on news. Accordingly, trading volume increases and the speculator accounts for a larger share of trading volume. However, liquidity increases because the dealer can better forecast the long-run value of the asset, in which case she loses less on the speculator's long-run bets. Hence, in equilibrium, trading volume, informed trading, and liquidity jointly increase with news informativeness.

This prediction differs sharply from that of other models analyzing the effects of public information. In these models, an increase in the precision of public signals induces informed investors to trade less, not more. Accordingly, these models predict that an increase in news informativeness should result in higher liquidity but less volume and informed trading (see, for instance, Propositions 1 and 2 in Kim and Verrecchia (1994)). The reason is that these models do not consider the possibility of informed investors trading ahead of news, that is, of being fast in our terminology. In this case, more informative news induces the speculator to trade more because he expects news to move prices more, which generates larger profits from speculating on short-term price movements.

<sup>23</sup> Furthermore,  $\beta_t^k$  increases in  $\sigma_v$  and  $\sigma_u$  for  $k \in \{S, F\}$ . When  $\sigma_v$  increases, uncertainty on the final value of the asset is higher for the dealer, other things equal. This is also the case when  $\sigma_u$  increases, because the order flow becomes noisier. In either case, the speculator optimally reacts by trading more aggressively on his forecast of the long-run price change. These effects are standard.

*B. News Informativeness, Trading Profits, and the Decision to Be Fast*

When news informativeness increases, the speculator trades less aggressively on his forecast of the long-run price change for the asset and more on his forecasts of short-term price reactions to news (Proposition 6). This result has the following implication.

**COROLLARY 8:** *Suppose that the speculator is fast. The news-trading component of his expected profit increases with news informativeness ( $\frac{\partial \pi_0^F}{\partial \nu} > 0$ ) while the value trading component of his expected profit decreases with news informativeness ( $\frac{\partial \pi_\beta^F}{\partial \nu} < 0$ ). In net, the speculator’s total expected profit decreases with news informativeness ( $\frac{\partial \pi_0^F}{\partial \nu} < 0$ ).*

Thus, profits from trades on short-term price reactions to news should contribute relatively more to fast traders’ profits in stocks with more informative news. Corollary 8 suggests that variations in directional HFTs’ profits across stocks can be explained by variations in news informativeness. According to this corollary, directional HFTs should (i) earn smaller total profits and (ii) realize a larger fraction of their total profits over short time intervals in stocks with more informative news. To our knowledge, these predictions have not been tested so far.

The model also has implications for the effect of news informativeness on the decision to trade fast. To analyze these, suppose that a speculator must pay a cost  $C_f$  to be fast, otherwise he remains slow.<sup>24</sup> Cost  $C_f$  represents the marginal cost of setting up a fast connection with an exchange for one particular stock (e.g., a colocation fee or the opportunity cost of using computing capacity to react fast to news in this stock). If the speculator becomes fast, his expected trading profit increases by  $\pi_0^F(\nu) - \pi_0^S(\nu)$  (Proposition 3). Thus, the speculator chooses to be fast if and only if

$$\pi_0^F(\nu) - \pi_0^S(\nu) \geq C_f. \tag{32}$$

The speculator’s expected profit declines with news informativeness regardless of whether he is fast or slow because he earns a smaller expected profit from value trading. However, when the speculator is fast, this effect is partially offset by a larger profit from news trading when news informativeness is higher (Corollary 8). Thus,  $\pi_0^F(\nu)$  declines with  $\nu$  at a smaller rate than  $\pi_0^S(\nu)$ . Accordingly, the net gain of becoming fast,  $\pi_0^F(\nu) - \pi_0^S(\nu)$ , increases with news informativeness, as the next corollary shows.

**COROLLARY 9:** *The net gain of becoming fast,  $\pi_0^F(\nu) - \pi_0^S(\nu)$ , increases with news informativeness,  $\nu$ .*

<sup>24</sup> This cost is in addition to the cost paid by the speculator to obtain information. To focus the analysis on the decision to be fast or slow, we assume that the information acquisition cost is always low enough that buying information is always optimal, even if the speculator is slow. As  $\pi_0^S(\nu)$  decreases with  $\nu$ , this is the case if the cost of information is less than  $\pi_0^S(\infty) = \sigma_u(\Sigma_0)^{1/2}$ .

News trading has no value if news is not informative, since in this case  $\gamma^F = 0$ . Thus,  $\pi_0^F(0) - \pi_0^S(0) = 0$ . Hence, Corollary 9 implies that, if  $0 < C_f < \pi_0^F(\infty) - \pi_0^S(\infty)$ , there is a cutoff value  $\widehat{\nu}(C_f) > 0$  such that the speculator becomes fast if  $\nu > \widehat{\nu}(C_f)$  and remains slow if  $\nu \leq \widehat{\nu}(C_f)$ . This means that the speculator is more likely to be fast in stocks with high news informativeness than in stocks with low news informativeness.<sup>25</sup>

Hence, directional HFTs should be more active stocks with more informative news (there is cross-sectional variation in HFTs' activity; see Brogaard, Hendershott, and Riordan (2014), table I) because being fast has more value in these stocks. To our knowledge, this prediction is new. In existing theories (e.g., Biais, Foucault, and Moinas (2015)), news informativeness plays no role in traders' decision to be fast.

## V. Infrequent News and Latency

In this section, we extend the model in two directions. First, we relax the assumption that trades and news occur at the same rate. Second, we allow the delay with which the dealer reacts to news to be longer than one trading round. Our goal is to show the robustness of the results obtained in the baseline case. For brevity, here we simply outline the modeling approach for this extension and its main properties. We formally derive the equilibrium obtained in this case in Internet Appendix Section III.

To preserve the continuous-time formulation, we proceed as follows. As in the baseline model, during the interval  $[t, t + dt]$  the speculator receives the signal,  $dv_t$ , and the dealer observes the news,  $dz_t$ . However, each interval  $[t, t + dt]$  is partitioned into  $m$  equal intervals and there is one trading round in each interval. Hence, the ratio of the trading rate to the news rate is  $m$ , and as  $m$  increases news becomes less frequent relative to trading opportunities. Moreover, the dealer receives news  $dz_t$  after  $\ell \leq m$  trading rounds.<sup>26</sup> Therefore, the speculator has  $\ell$  opportunities to trade on the price reaction to news in  $[t, t + dt]$ . We refer to  $\ell$  as the *dealer's latency*. The higher is this latency, the greater is the speculator's speed advantage. For any  $m$ , if  $\ell \geq 1$ , the speculator is fast, while, if  $\ell = 0$ , the speculator is slow. The fast model considered in the previous sections is the particular case in which  $m = \ell = 1$ , while the slow model is the particular case in which  $m = 1$  and  $\ell = 0$ .

Figure 4 describes the timing of the model for arbitrary values of  $m$  and  $\ell$ . Let  $(t, j)$  denote the trading round at the end of the  $j$ th interval in  $[t, t + dt]$ , where  $j = 1, \dots, m$ . Just before  $(t, 1)$ , the speculator privately observes  $dv_t$ . Then, at  $(t, j)$  the speculator submits a market order  $dx_{t,j}$ , and the noise traders submit an aggregate market order  $du_{t,j}$ . We set  $\text{Var}(du_{t,j}) = \frac{1}{m}\sigma_u^2 dt$  so that the variance

<sup>25</sup> If  $C_f > \pi_0^F(\infty) - \pi_0^S(\infty)$ , the speculator chooses to be slow for all levels of news informativeness.

<sup>26</sup> Thus, the time at which news arrives is known with certainty. This is required for tractability. An extension of our model with stochastic news arrival dates is challenging and left for future research.

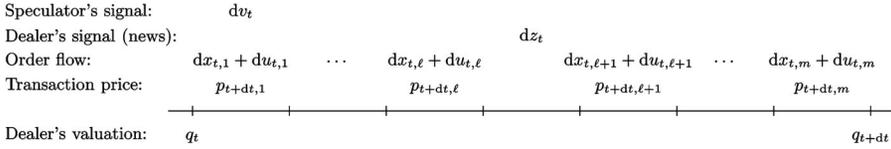


Figure 4. Timing of events during  $[t, t + dt]$  for arbitrary news frequency,  $m$ , and dealer's latency,  $\ell$ .

of noise trading per unit of time remains  $\sigma_u^2$  as in the baseline model. The total order flow at  $(t, j)$  is  $dy_{t,j} = dx_{t,j} + du_{t,j}$ . It is executed by the dealer at price  $p_{t+dt,j}$ , which is equal to her expectation of the asset payoff conditional on her information in trading round  $(t, j)$ :

$$p_{t+dt,j} = \begin{cases} E(v_1 | I_t \cup dy_{t,1} \cup \dots \cup dy_{t,j}) & \text{for } 1 \leq j \leq \ell, \\ E(v_1 | I_t \cup dy_{t,1} \cup \dots \cup dy_{t,j} \cup dz_t) & \text{for } \ell + 1 \leq j \leq m, \end{cases} \quad (33)$$

where  $I_t = \{y_{\tau,1}\}_{\tau \leq t} \cup \dots \cup \{y_{\tau,m}\}_{\tau \leq t} \cup \{z_\tau\}_{\tau \leq t}$  is the dealer's information set at date  $t$ . As in the baseline model,  $q_t = E(v_1 | I_t)$  is the valuation of the asset for the dealer at date  $t$ .

An equilibrium is a trading strategy of the speculator, and a pricing policy of the dealer, such that (i) the speculator's trading strategy maximizes his expected trading profit, given the dealer's pricing policy, and (ii) the dealer's pricing policy satisfies (33) given the equilibrium speculator's trading strategy. A linear equilibrium is such that the speculator's trading strategy is of the form

$$dx_{t,j} = \beta_{t,j}(v_t - q_t)dt + \gamma_{t,j}(dv_t - dw_{t,j}), \quad j = 1, \dots, m, \quad (34)$$

where  $dw_{t,j}$  is the dealer's expectation of the incoming news,  $dz_t$ , given her information until trading round  $(t, j)$ .<sup>27</sup> From  $(t, 2)$  onwards, this expectation is not zero because the dealer can infer information about incoming news from the speculator's trades in past trading rounds. This is not the case at  $(t, 1)$  since this is the first time at which the speculator observes  $dv_t$  and starts trading on it. Thus,  $dw_{t,1} = 0$  and, when  $m = 1$ , the speculator's strategy in (34) is identical to that in the baseline case.<sup>28</sup>

In Internet Appendix Section III, we show that, when the speculator is slow, the equilibrium is similar to that provided in Theorem 1. In particular, the speculator's strategy has no news-trading component ( $\gamma_{t,j}^S = 0$  for all  $j \in [1, m]$ ). The only difference with the baseline case is that the sensitivity,  $\beta_{t,j}^S$ , of the speculator's trade in each trading round to his forecast of the long-run change in price is  $(\frac{1}{m})^{\text{th}}$  of its value in the baseline slow model (i.e.,  $\beta_t^S$  in Theorem 1).

<sup>27</sup> That is, at  $(t, j)$ , the news trading component is proportional to the component of the speculator's forecast of the news that is orthogonal to the dealer's forecast of this news. This is intuitive since the dealer's forecast of the news is already incorporated into the price at  $(t, j)$ .

<sup>28</sup> As for the baseline case, it is possible to show that, in any discrete-time linear equilibrium, the speculator's optimal trading strategy must be the discrete-time analog of (34).

Intuitively, in each interval  $[t, t + dt]$ , the speculator divides his trade over the  $m$  trading rounds so that his aggregate trade is identical to that obtained when there is only one trading round.

When the speculator is fast ( $\ell \geq 1$ ), the optimal linear trading strategy for the speculator features a news trading component until the dealer observes the news  $dz_t$ , that is,  $\gamma_{t,j} > 0$  for  $1 \leq j \leq \ell$  and  $\gamma_{t,j} = 0$  for  $\ell + 1 \leq j \leq m$ . This is intuitive. In a given interval,  $[t, t + dt]$ , the speculator's signal  $dv_t$  is informative about the price reaction to news in this interval. Exactly as in the baseline model, the speculator exploits this short-run price predictability by conditioning his trades on his forecast ( $dv_t$ ) of the news until it arrives.<sup>29</sup> After news arrival, the speculator cannot forecast short-run price movements until he receives a new signal and therefore his optimal trading strategy features just a value-trading component.

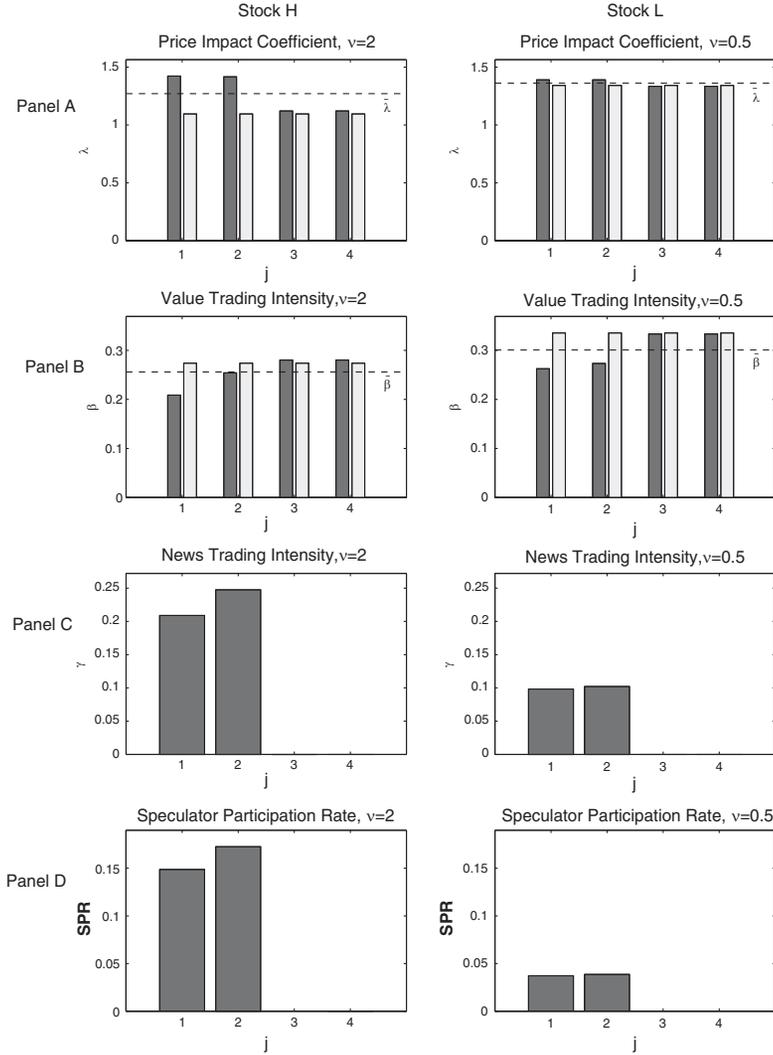
Thus, with infrequent news, the news-trading component (given by the  $\gamma$  coefficients) varies around news arrival. In turn, this generates variation in the value-trading component (the  $\beta$  coefficients) and illiquidity (the  $\lambda$  coefficients) around news. In particular, illiquidity is higher before news arrival than after because the speculator trades in the direction of the news before news arrival. Hence, the dealer is more likely to sell (buy) just ahead of good (bad) news. Consequently, the market maker is more exposed to adverse selection before news arrival than after.

As in the baseline case, we cannot characterize the equilibrium in closed form when the speculator is fast. However, as shown in Internet Appendix Section III the coefficients that characterize the equilibrium (including the constant  $\gamma$ ,  $\beta$ , and  $\lambda$  coefficients) are solutions to a system of  $(4\ell + 2)$  nonlinear equations. This system can be solved numerically for given parameter values ( $m$ ,  $\ell$ ,  $\sigma_v$ , etc.). One can then check numerically that the implications obtained in the baseline model are still valid in the extended version of this model.

For instance, Figure 5 compares equilibrium values of various variables of interest (namely,  $\lambda$ ,  $\gamma$ ,  $\beta$ , and  $SPR$ ) for two different stocks, labeled H and L. The news frequency is identical for each stock: news arrives every four trading rounds ( $m = 4$ ). However, for stock H, news informativeness is high ( $\nu = \frac{1}{\sigma_e} = 2$ ), while for stock L, news informativeness is low ( $\nu = \frac{1}{\sigma_e} = 0.5$ ). Furthermore, for each stock, we consider the effect of varying latency from  $\ell = 0$  (the slow model) to  $\ell = 2$  (the fast model). Thus, when the speculator is fast, he observes a signal correlated with incoming news two periods before the dealer receives the news. For a given stock, equilibrium values when the speculator is fast (slow) are displayed using dark (light) bars when different from zero.

When the speculator is fast,  $\lambda$ ,  $\gamma$ ,  $\beta$ , and  $SPR$  vary across trading rounds. Hence, we also show their average values (dashed line) over the four trading rounds. The predictions of the baseline model do indeed correspond to

<sup>29</sup> In the Internet Appendix, we also show numerically that the sensitivity of the speculator's aggregate trades before news arrival to the news increases with the dealer's latency,  $\ell$  (see Figure IA.5 in the Internet Appendix). In this sense, higher dealer latency induces the speculator to trade more aggressively on news.



**Figure 5. News informativeness and speed.** We plot equilibrium values of various variables of interest in a given interval  $[t, t + dt]$  in each trading round  $j$  for two different stocks, H (plots on the left) and L (plots on the right). The news frequency is  $m = 4$  for each stock. News informativeness is higher for stock H than for stock L ( $v = \frac{1}{\sigma_e} = 2$  versus  $v = \frac{1}{\sigma_e} = 0.5$ ). For each stock, we show equilibrium values of  $\lambda_{t,j}$  (illiquidity),  $\beta_{t,j}$  (value trading intensity),  $\gamma_{t,j}$  (news trading intensity), and  $SPR_{t,j} = \frac{\text{Var}(dx_{t,j})}{\text{Var}(dx_{t,j}) + \text{Var}(du_{t,j})}$  in each trading round  $j \in \{1, 2, 3, 4\}$  when the dealer's latency is  $\ell = 2$  (left dark bars) or  $\ell = 0$  (right light bars). The horizontal dotted lines correspond to the average value of the relevant variable over the four trading rounds when the speculator is fast (e.g.,  $\bar{\lambda} = (\sum_{j=1}^4 \lambda_{t,j})/4$ ) in the fast model. The other parameter values are  $\sigma_u = 1$  (standard deviation of noise traders' order flow),  $\sigma_v = 1$  (standard deviation of innovations in the asset value), and  $\Sigma_0 = 1$  (variance of asset value conditional on information available at date 0).

cross-sectional effects of latency or news informativeness on the average values of these variables rather than their time-series variation around news.

Consider illiquidity (Panel A) first. When the speculator is slow, illiquidity ( $\lambda$ ) is constant over time. It is lower for stock H than for stock L (1.10 versus 1.34) because the dealer receives more informative news in the former stock. When the speculator is fast, illiquidity is higher *before* news arrival ( $j = 1, 2$ ) than *after* news arrival ( $j = 3, 4$ ) because, as previously explained, the dealer is more exposed to adverse selection before news. Furthermore, illiquidity declines over time as the news release approaches ( $\lambda_{t,1} > \lambda_{t,2}$  for stocks H and L) because the dealer accumulates information about the speculator's signal,  $dv_t$ , from order flows as the news release gets closer. This alleviates his exposure to adverse selection due to advance trading on news.

However, the cross-sectional implications of the baseline model for illiquidity (measured by the average value of  $\lambda$  over the four trading rounds) remain valid. Specifically, in line with Proposition 4, average illiquidity is higher in the fast model than in the slow model for both stocks and, regardless of whether the speculator is slow or fast, average illiquidity is higher for the stock with low news informativeness (as implied by Corollary 7). Interestingly, this is not necessarily the case if one measures illiquidity only *before* the news arrival. Indeed, when the speculator is fast, illiquidity before news arrival is higher in stock H than in stock L. The reason is that the speculator's advance information on incoming news exposes the dealer to greater adverse selection when the news is more informative.<sup>30</sup> Thus, in testing our predictions, one should measure illiquidity on average for a stock, not just before news or after news.

Now consider the value- and news-trading components (respectively, Panels B and C) of the speculator's strategy. For a given stock, the news-trading component is strictly positive only when the speculator is fast and before news arrival. Moreover, in line with Proposition 2, the average value of  $\beta$  is smaller when the speculator is fast than when he is slow, and, in line with Proposition 6, the average value of  $\gamma$  increases with news informativeness ( $\gamma$  is higher on average for stock H).

The economic intuition for these observations is exactly as in the baseline model. In particular, the speculator trades more aggressively on short-term price reactions when prices react more to news, that is, when the news is more informative. This explains why the average value of  $\gamma$  is higher for stock H. Consequently, to avoid dissipating his information advantage too quickly, the speculator trades less aggressively (on average) on his forecast of the long-run price change in stock H, which explains why the average  $\beta$  is lower for this stock. One implication is that trading on short-term price reactions contributes to a larger fraction of the speculator's trading profit in stock H than in stock L (33.38% versus 19.64% for the parameter values in Figure 5).

<sup>30</sup> In line with this implication, Gross-Kluschmann and Hautsch (2011) find that bid-ask spreads are larger before news arrival than after and that this effect is stronger for more relevant news (see Figure 5 in their paper).

Finally, as shown in the last panel of Figure 5, the average speculator's participation rate is higher when he is fast. As in the baseline model, the reason is that the speculator's position is much more volatile when he is fast because his trades are much more sensitive to incoming news. Furthermore, the speculator's participation rate ( $SPR_{i,j}$ ) is higher on average in stock H than in stock L when he is fast because he trades even more aggressively on short-term price reactions to news when news is more informative. Hence, when the speculator is fast, stock H attracts more informed trading and is less liquid just before news. However, it is more liquid on average than stock L, exactly as in the baseline model.

## VI. Related Models

Our modeling approach is related to dynamic extensions of Kyle (1985) with gradual release of information over time. These include Foster and Viswanathan (1990), Back and Pedersen (1998), Chau and Vayanos (2008), Li (2013), and Martinez and Roşu (2013). In Back and Pedersen (1998), the informed investor receives a continuous stream of signals but dealers receive no news. Thus, there is no news trading (the instantaneous volatility of the speculator's position is zero) in Back and Pedersen (1998) because the speculator's signals contain no information on short-term price movements. This situation arises as a special case of our model when dealers' news is completely uninformative ( $\nu = 0$ ; see Section IV.A).

Chau and Vayanos (2008) consider a situation in which dealers receive news, and Li (2013) extends their framework to allow for multiple informed investors. In these models, dealers always reflect news in their quotes before the informed investors can trade on their signals. That is, in our terminology, speculators are always slow in Chau and Vayanos (2008) and Li (2013). Thus, as in Back and Pedersen (1998) (but for a different reason), there is no news trading in these models as well.

In Foster and Viswanathan (1990), dealers receive news about the final payoff of an asset. However, between news arrivals, the informed investor can trade continuously, so the news arrival rate relative to the trading rate is zero. Hence, from the speculator's viewpoint, the time at which news affects prices is always infinitely far away and thus the speculator does not trade on news in Foster and Viswanathan (1990). In contrast, in our model, the news arrival rate relative to the trading rate is strictly positive (and equal to  $1/m$ ). As explained in Section V, this is sufficient for news trading to arise.

Martinez and Roşu's (2013) model is similar to that of Back and Pedersen (1998) (dealers do not receive any news) but informed investors are ambiguity averse. Ambiguity aversion induces informed investors to trade aggressively on their signals, so that their optimal trading strategy features a volatility component but for a reason different from that in our model, where the volatility component reflects the speculator's ability to anticipate short price reactions to incoming news. Accordingly, the two models are not observationally equivalent. For instance, our model predicts a positive correlation between

a speculator's trade and price reactions to the news (when the speculator is fast; see Section III.C). This correlation is zero in Martinez and Roşu (2013) because dealers never receive news in their model. For the same reason, our predictions about news informativeness (see Section IV) cannot be obtained in Martinez and Roşu (2013).

Huddart, Hughes, and Levine (2001) and Cao, Ma, and Ye (2013) consider extensions of Kyle (1985) in which insiders must disclose their trade after each trading round. Interestingly, in these models, insiders optimally play a mixed strategy (to avoid full revelation of their information after disclosure) by adding white noise to their trade. In continuous time, the instantaneous volatility of their position is therefore not zero. Thus, insiders' participation rate can be high in these models (e.g., it is equal to 50% in Cao, Ma, and Ye (2013)), as in our model (see Section III.A). However, other testable implications of our model are different because the volatility of the informed investor's position does not come from the same economic mechanism. For instance, in models with disclosure, the stochastic component of informed trades contains no information about the asset value (it is a white noise). In contrast, in our model it is driven by innovations in the asset value (the speculator's signals). Accordingly, our model implies that the speculator's trades are correlated with news and subsequent short-run returns (see Section III.C), while in Huddart, Hughes, and Levine (2001) and Cao, Ma, and Ye (2013) these correlations are zero.

## VII. Conclusion

We consider a model in which a speculator's private signals can be used to forecast both short-run price reactions to news arrival and long-term price changes. When the speculator is fast (i.e., can trade on his signals ahead of news arrival), the speculator's behavior in equilibrium matches well several stylized facts about directional HFTs. In particular, his trades forecast news, are highly correlated with short-run price changes, and significantly contribute to trading volume. However, the bulk of the speculator's profit does not necessarily derive from gains on short-term price movements because the speculator also exploits his superior information about long-run changes in prices.

Empirical studies show that HFTs' aggressive orders anticipate short-run price changes and news. One might therefore conclude that directional HFTs do not really contribute to price discovery since they seem to trade on soon-to-be-released information. Our results suggest that this conclusion is premature. Indeed, in our model the speculator's trades are strongly correlated with pending news and short-run price reactions to news. But the speculator also trades on his estimate of the long-run value of the asset and thereby contributes to price discovery. To make progress on this issue, future empirical research should study in more detail the nature of signals used by directional HFTs and the horizons over which they realize their profits.

Our model also suggests intriguing interactions between news informativeness and HFTs' strategies. In particular, it implies that directional HFTs should be more active in stocks with more informative news, and that gains on

short-term price movements should contribute more to HFTs' profits in these stocks. However, their overall profit should decline with news informativeness. These predictions are novel and offer one way to test whether the mechanisms described in our model help explain data on HFTs.

In our model, the speculator can forecast short-term price reactions due to news arrival but has no information on liquidity traders' future demands. This is consistent with Brogaard, Hendershott, and Riordan (2014), who find that HFTs' trades contain information above and beyond the information contained in other traders' liquidity demands. Future research could consider a case in which a speculator has signals on the asset payoff (as in our model) and future demands from liquidity traders (as suggested by Hirschey (2013) and Clark-Joseph (2013)). Predictions from such a model would enable empiricists to better assess the respective roles of each type of information in HFTs' strategies.

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### Appendix Proofs

PROOF OF THEOREM 1: First, we compute the optimal trading strategy of the speculator from the set of strategies of the form  $dx_\tau = \beta_\tau^S(v_\tau - q_\tau)d\tau + \gamma_\tau^S dv_\tau$ ,  $\tau \in [0, 1]$ , while taking as given the dealer's pricing rule,  $p_{\tau+d\tau} = q_\tau + \mu_\tau^S dz_\tau + \lambda_\tau^S dy_\tau$ , where the state variable  $q_\tau$  (the dealer's quote) evolves according to  $dq_\tau = \mu_\tau^S dz_\tau + \lambda_\tau^S dy_\tau$ . For convenience, in the rest of this proof we omit the superscript  $S$  on the coefficients  $\beta$ ,  $\gamma$ ,  $\mu$ , and  $\lambda$ .

For  $t \in [0, 1]$ , the speculator's expected profit is

$$\pi_t = E_t \left( \int_t^1 (v_1 - p_{\tau+d\tau}) dx_\tau \right), \tag{A1}$$

where the expectation is conditional on the speculator's information set  $J_t$ , defined in Section I. Recall that the dealer's quote satisfies  $q_\tau = E(v_1 | I_\tau) = E_\tau(v_1)$ . For any  $\tau \geq t$ , let:

$$V_{t,\tau} = E_t((v_\tau - q_\tau)^2). \tag{A2}$$

We compute

$$\begin{aligned} \pi_\tau &= E_t \left( \int_t^1 (v_{\tau+d\tau} - p_{\tau+d\tau}) dx_\tau \right) \\ &= E_t \left( \int_t^1 ((v_\tau + dv_\tau) - (q_\tau + \mu_\tau dz_\tau + \lambda_\tau dy_\tau)) dx_\tau \right) \\ &= E_t \left( \int_t^1 (v_\tau - q_\tau + (1 - \mu_\tau)dv_\tau - \lambda_\tau dx_\tau) dx_\tau \right), \end{aligned} \tag{A3}$$

where the first equality follows from the law of iterated expectations, and the second equality follows from  $dx_\tau$  being orthogonal to  $du_\tau$  and  $de_\tau$ . Since  $dx_\tau = \beta_\tau(v_\tau - q_\tau)d\tau + \gamma_\tau dv_\tau$ , we compute

$$\pi_t = \int_t^1 (\beta_\tau V_{t,\tau} + (1 - \mu_\tau - \lambda_\tau \gamma_\tau) \gamma_\tau \sigma_v^2) d\tau. \tag{A4}$$

We now omit the subscript  $t$  in  $V_{t,\tau}$ . Then  $V_\tau$  evolves according to

$$\begin{aligned} V_{\tau+d\tau} &= E_t((v_{\tau+d\tau} - q_{\tau+d\tau})^2) \\ &= E_t((v_\tau + dv_\tau - q_\tau - \mu_\tau dv_\tau - \mu_\tau de_\tau - \lambda_\tau dx_\tau - \lambda_\tau du_\tau)^2) \\ &= V_\tau + (1 - \mu_\tau - \lambda_\tau \gamma_\tau)^2 \sigma_v^2 d\tau + \mu_\tau^2 \sigma_e^2 d\tau + \lambda_\tau^2 \sigma_u^2 d\tau - 2\lambda_\tau \beta_\tau V_\tau d\tau, \end{aligned} \tag{A5}$$

which implies that  $V_\tau$  satisfies the first-order linear ordinary differential equation (ODE) in  $\tau \in [t, 1)$ ,

$$V'_\tau = -2\lambda_\tau \beta_\tau V_\tau + (1 - \mu_\tau - \lambda_\tau \gamma_\tau)^2 \sigma_v^2 + \mu_\tau^2 \sigma_e^2 + \lambda_\tau^2 \sigma_u^2, \tag{A6}$$

or equivalently,  $\beta_\tau V_\tau = \frac{-V'_\tau + (1 - \mu_\tau - \lambda_\tau \gamma_\tau)^2 \sigma_v^2 + \mu_\tau^2 \sigma_e^2 + \lambda_\tau^2 \sigma_u^2}{2\lambda_\tau}$ . We substitute this into (A4) and integrate by parts to obtain<sup>31</sup>

$$\begin{aligned} \pi_t &= -\frac{V_1}{2\lambda_1} + \frac{V_t}{2\lambda_t} + \int_t^1 V_\tau \left(\frac{1}{2\lambda_\tau}\right)' d\tau \\ &\quad + \int_t^1 \left(\frac{(1 - \mu_\tau - \lambda_\tau \gamma_\tau)^2 \sigma_v^2 + \mu_\tau^2 \sigma_e^2 + \lambda_\tau^2 \sigma_u^2}{2\lambda_\tau} + (1 - \mu_\tau - \lambda_\tau \gamma_\tau) \gamma_\tau \sigma_v^2\right) d\tau. \end{aligned} \tag{A7}$$

Thus, we have eliminated the choice variable  $\beta_\tau$  and replaced it by  $V_\tau \geq 0$ ,  $\tau > t$ .<sup>32</sup>

We now prove the existence of an equilibrium by assuming that  $\lambda_\tau$  is constant and showing that the speculator’s optimal strategy must be of the type described in Theorem 1. At the end of this proof, we show that for a linear equilibrium to exist,  $\lambda_\tau$  must indeed be constant.

Consider the case in which  $\lambda_\tau = \lambda$  is constant. First, observe that, in equilibrium,

$$\lambda > 0. \tag{A8}$$

Indeed, equation (A6) implies that, if the speculator chooses a very large  $\beta_\tau$ , the variable  $V_\tau > 0$  (i) does not depend on  $\beta_\tau$  if  $\lambda = 0$ , or (ii) becomes very large

<sup>31</sup> For this argument we need the function  $\lambda_\tau$  to be smooth (i.e., continuously differentiable) on  $[t, 1)$ . By (A12) below, it is enough to show that  $\beta_\tau$  and  $\Sigma_\tau$  are smooth. First,  $\beta_\tau$  is smooth by assumption: see the discussion after equation (7). Second,  $\Sigma_\tau$  is smooth because it is the solution of the ODE (A13).

<sup>32</sup> By convention, we allow the choice  $V_\tau = 0$ , with the understanding that corresponds to the limit case when  $\beta_\tau = +\infty$ . Note that  $V_t = (v_t - q_t)^2$  is not a choice variable, but  $V_\tau$  for  $\tau > t$  is.

if  $\lambda < 0$ . Thus, the speculator would be able to make arbitrarily large expected profits when  $\lambda \leq 0$ .

The assumption that  $\lambda_\tau = \lambda$  is constant on  $[0, 1)$  implies that  $(\frac{1}{2\lambda_\tau})' = 0$ . Equation (A7) then implies that the speculator must choose  $\frac{V_1}{\lambda_1} = 0$ . Since  $\lambda_1 = \lambda > 0$ , this translates into the following transversality condition:

$$V_1 = 0. \tag{A9}$$

We next turn to the choice of  $\gamma_\tau$ . The first-order condition with respect to  $\gamma_\tau$  in (A7) is

$$-(1 - \mu_\tau - \lambda_\tau \gamma_\tau) + (1 - \mu_\tau - \lambda_\tau \gamma_\tau) - \lambda_\tau \gamma_\tau = 0 \implies \gamma_\tau = 0. \tag{A10}$$

Thus, there is no news trading in the slow model, which proves (12). Note also that the second-order condition is  $\lambda_\tau > 0$  (regardless of whether  $\lambda_\tau > 0$  is constant or not).

Next, we derive the pricing rules from the dealer’s zero-profit conditions. Let  $t \in [0, 1)$ . In Section I we see that the dealer’s quote is initially set to  $q_t = E(v_1 | I_t)$ , where  $I_t$  is the dealer’s information set at the beginning of the interval  $[t, t + dt]$ . (See also Figure 2.) In the slow model, the first event is the arrival of the dealer’s signal  $dz_t$ , which is orthogonal to  $I_t$ . Denote by  $I_{t+}$  the information set generated by  $I_t$  and  $dz_t$ . Then,  $q_{t+} = E(v_1 | I_{t+}) = q_t + \mu_t dz_t$ , where

$$\mu_t = \frac{\text{Cov}(v_1, dz_t | I_t)}{\text{Var}(dz_t | I_t)} = \frac{\text{Cov}(v_0 + \int_0^1 dv_\tau, dv_t + de_t | I_t)}{\text{Var}(dv_t + de_t | I_t)} = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_e^2} = \mu. \tag{A11}$$

The dealer assumes that the speculator’s trading strategy is  $dx_t = \beta_t(v_t - q_t)dt$ , where  $\beta_t = \beta_t^S$  has the equilibrium value from Theorem 1, and  $\gamma_t = \gamma_t^S = 0$ . Since  $dz_t dt = 0$ , we can write  $dx_t = \beta_t(v_t - q_{t+})dt$ . But  $v_t - q_{t+}$  is orthogonal to  $I_{t+}$ , hence so is  $dy_t = dx_t + du_t$ . Therefore, the trading price is of the form  $p_{t+dt} = q_{t+} + \lambda_t dy_t$ , with

$$\lambda_t = \frac{\text{Cov}(v_1, dy_t | I_{t+})}{\text{Var}(dy_t | I_{t+})} = \frac{\text{Cov}(v_1, \beta_t(v_t - q_{t+})dt + du_t | I_{t+})}{\text{Var}(\beta_t(v_t - q_{t+})dt + du_t | I_{t+})} = \frac{\beta_t \Sigma_t}{\sigma_u^2}, \tag{A12}$$

where we use the equality  $\Sigma_t = E((v_t - q_t)^2) = E((v_t - q_{t+})^2)$ . Note that we have also proved that the trading price is of the form  $p_{t+dt} = q_{t+} + \lambda_t dy_t = q_t + \mu_t dz_t + \lambda_t dy_t$ , as specified in (9). Since the quote  $q_{t+dt}$  at the end of the interval  $[t, t + dt]$  is the same as the trading price, it follows that the quote evolves according to  $dq_{t+} = \mu_t dz_t + \lambda_t dy_t$ , as specified in (10).

Now, using the same derivation as for (A6), it is straightforward to check that  $\sum_t = E((v_t - q_t)^2)$  satisfies the first-order linear ODE:

$$\sum_t' = -2\lambda_t \beta_t \sum_t + (1 - \mu_t)^2 \sigma_v^2 + \mu_t^2 \sigma_e^2 + \lambda_t^2 \sigma_u^2. \tag{A13}$$

This is the same ODE as (A6), except that it has a different initial condition. When  $\lambda_t$  is constant, by explicitly solving (A6) and (A13), one sees that the

transversality condition  $V_1 = 0$  (from equation (A9)) is equivalent to  $\int_t^1 \beta_\tau d\tau = +\infty$ , which in turn is equivalent to

$$\Sigma_1 = 0. \tag{A14}$$

Next, from (A12) we get  $\beta_t \Sigma_t = \lambda_t \sigma_u^2$ ; therefore equation (A13) becomes

$$\Sigma'_t = -\lambda_t^2 \sigma_u^2 + (1 - \mu_t)^2 \sigma_v^2 + \mu_t^2 \sigma_e^2. \tag{A15}$$

Thus, as  $\lambda_t$  and  $\mu_t$  are constant,  $\Sigma'_t$  is constant as well. The condition  $\Sigma_1 = 0$  then implies  $\Sigma_t = (1 - t)\Sigma_0$ . Equation (A15) becomes  $-\Sigma_0 = -\lambda^2 \sigma_u^2 + (1 - \mu)^2 \sigma_v^2 + \mu^2 \sigma_e^2$ . From (A11),  $\mu = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_e^2}$ , hence  $\lambda^2 \sigma_u^2 = \Sigma_0 + \frac{\sigma_v^2 \sigma_e^2}{\sigma_v^2 + \sigma_e^2}$ . Hence, we have proved both (13) and (14). Equation (A12) implies that  $\beta_t \Sigma_t$  is constant, and thus  $\beta_t \Sigma_t = \beta_0 \Sigma_0 = \lambda \sigma_u^2$ . But  $\Sigma_t = \Sigma_0(1 - t)$ , so  $\beta_t = \frac{\beta_0}{1-t}$ . Substituting the formula for  $\lambda$  into  $\beta_0 = \frac{\lambda \sigma_u^2}{\Sigma_0}$ , we obtain (11).

Finally, we prove that in equilibrium  $\lambda_\tau$  is indeed constant for  $\tau \in [0, 1)$ . Given that the second-order condition  $\lambda_\tau > 0$  must hold, it is enough to show that  $(\frac{1}{2\lambda_\tau})' = 0$ . If this is not true, one possibility is that  $(\frac{1}{2\lambda_\tau})' > 0$  for  $\tau$  in a small interval  $I$ . Then, using equation (A7), one sees that the speculator can achieve an arbitrarily high expected profit by choosing  $V_\tau$  to be very large over the interval  $I$ , and zero elsewhere. This is inconsistent with equilibrium.

The other possibility is that  $(\frac{1}{2\lambda_\tau})' < 0$  for  $\tau$  in a small interval  $I$ . In this case, for a maximum expected profit, the speculator would choose  $V_\tau = 0$  for  $\tau \in I$ , which, by the law of iterated expectations, means that  $\sum_\tau = 0$  for  $\tau \in I$ . Hence, equation (A15) implies that  $0 = \sum'_\tau = -\lambda_\tau^2 \sigma_u^2 + (1 - \mu_\tau)^2 \sigma_v^2 + \mu_\tau^2 \sigma_e^2$ , which implies that  $\lambda_\tau = (\frac{(1-\mu_\tau)^2 \sigma_v^2 + \mu_\tau^2 \sigma_e^2}{\sigma_u^2})^{1/2}$  if  $\tau \in I$ . But since  $\mu_\tau = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_e^2}$  is constant,  $\lambda_\tau$  is also constant on  $I$ , which contradicts  $(\frac{1}{2\lambda_\tau})' < 0$ . □

**PROOF OF THEOREM 2:** As in the proof of Theorem 1, we first compute the optimal trading strategy of the speculator from the set of strategies of the form  $dx_\tau = \beta_\tau^F (v_\tau - q_\tau) d\tau + \gamma_\tau^F dv_\tau$ ,  $\tau \in [0, 1)$ , while taking as given the dealer's pricing rule,  $p_{\tau+d\tau} = q_\tau + \lambda_\tau^F dy_\tau$ , where the state variable  $q_\tau$  (the dealer's quote) evolves according to  $dq_\tau = \lambda_\tau^F dy_\tau + \mu_\tau^F (dz_\tau - \rho_\tau^F dy_\tau)$ . For convenience, in the rest of this proof we omit the superscript  $F$  on the coefficients  $\beta$ ,  $\gamma$ ,  $\lambda$ ,  $\mu$ , and  $\rho$ .

For  $t \in [0, 1)$ , the speculator's expected profit is  $\pi_t = E_t(\int_t^1 (v_1 - p_{\tau+d\tau}) dx_\tau)$ , where the expectation is conditional on the speculator's information set  $\mathcal{J}_t$  defined in Section I. We compute

$$\begin{aligned} \pi_t &= E_t \left( \int_t^1 (v_{\tau+d\tau} - p_{\tau+d\tau}) dx_\tau \right) \\ &= E_t \left( \int_t^1 ((v_\tau + dv_\tau) - (q_\tau + \lambda_\tau dy_\tau)) dx_\tau \right) \\ &= E_t \left( \int_t^1 (v_\tau - q_\tau + dv_\tau - \lambda_\tau dx_\tau) dx_\tau \right), \end{aligned} \tag{A16}$$

where the first equality follows from the law of iterated expectations, and the second equality follows from  $dx_\tau$  being orthogonal to  $du_\tau$ . Let  $V_{t,\tau} = E_t((v_\tau - q_\tau)^2)$ . Since  $dx_\tau = \beta_\tau(v_\tau - q_\tau)d\tau + \gamma_\tau dv_\tau$ , we compute

$$\pi_t = \int_t^1 (\beta_\tau V_{t,\tau} + (1 - \lambda_\tau \gamma_\tau) \gamma_\tau \sigma_v^2) d\tau. \tag{A17}$$

By comparing equations (A17) and (A4), we observe a key difference between the fast and slow models: in the slow model, an additional term,  $E_t((\mu_\tau dz_\tau) dx_\tau)$ , is subtracted from the speculator’s objective function. This term comes from the dealer’s quote adjustment of  $\mu_\tau dz_\tau$ , which in the slow model is included in the price paid by the speculator. This lowers the benefit of news trading in the slow model compared to the fast model. Since, as we have already proved, the optimal news trading is zero in the slow model, it is reasonable to expect that there is positive news trading in the fast model. Indeed, we will prove that, in the fast model,  $\gamma_\tau > 0$ .

Let

$$\Lambda_\tau = \lambda_\tau - \mu_\tau \rho_\tau, \quad \tau \in [0, 1). \tag{A18}$$

With this notation,  $dq_\tau = \lambda_\tau dy_\tau + \mu_\tau (dz_\tau - \rho_\tau dy_\tau) = \mu_\tau dz_\tau + \Lambda_\tau dy_\tau$ . Then, with a similar derivation as in the proof of Theorem 1 (where  $\lambda_\tau$  is replaced by  $\Lambda_\tau$ ), one can show that  $V_\tau = V_{t,\tau}$  satisfies the first-order linear ODE in  $\tau \in [t, 1)$ ,

$$V'_\tau = -2\Lambda_\tau \beta_\tau V_\tau + (1 - \mu_\tau - \Lambda_\tau \gamma_\tau)^2 \sigma_v^2 + \mu_\tau^2 \sigma_e^2 + \Lambda_\tau^2 \sigma_u^2, \tag{A19}$$

or equivalently,  $\beta_\tau V_\tau = \frac{-V'_\tau + (1 - \mu_\tau - \Lambda_\tau \gamma_\tau)^2 \sigma_v^2 + \mu_\tau^2 \sigma_e^2 + \Lambda_\tau^2 \sigma_u^2}{2\Lambda_\tau}$ . Substituting this into (A17) and integrating by parts, we obtain

$$\begin{aligned} \pi_t = & -\frac{V_1}{2\Lambda_1} + \frac{V_t}{2\Lambda_t} + \int_t^1 V_\tau \left( \frac{1}{2\Lambda_\tau} \right)' d\tau \\ & + \int_t^1 \left( \frac{(1 - \mu_\tau - \Lambda_\tau \gamma_\tau)^2 \sigma_v^2 + \mu_\tau^2 \sigma_e^2 + \Lambda_\tau^2 \sigma_u^2}{2\Lambda_\tau} + (1 - \lambda_\tau \gamma_\tau) \gamma_\tau \sigma_v^2 \right) d\tau. \end{aligned} \tag{A20}$$

We now prove the existence of an equilibrium by first assuming that  $\Lambda_\tau$  is constant and showing that the speculator’s optimal strategy must be of the type described in Theorem 2. Then, at the end of this proof, we show that, for a linear equilibrium to exist,  $\Lambda_\tau$  must indeed be constant.

Consider the case when  $\Lambda_\tau = \Lambda$  is constant. By the same argument that proves (A8) in Theorem 1, we have

$$\Lambda > 0. \tag{A21}$$

Also, the transversality condition  $V_1 = 0$  must hold.

The novel part is the choice of  $\gamma_\tau$ . The first-order condition with respect to  $\gamma_\tau$  in (A20) is

$$-(1 - \mu_\tau - \Lambda_\tau \gamma_\tau) + (1 - \lambda_\tau \gamma_\tau) - \lambda_\tau \gamma_\tau = 0 \Rightarrow \gamma_\tau = \frac{\mu_\tau}{2\lambda_\tau - \Lambda_\tau} = \frac{\mu_\tau}{\lambda_\tau + \mu_\tau \rho_\tau}. \tag{A22}$$

Thus, the news-trading component is in general different from zero when the speculator is fast. The second-order condition is  $\lambda_\tau + \mu_\tau \rho_\tau > 0$ .

Next, we derive the pricing rules from the dealer's zero-profit conditions. Let  $t \in [0, 1)$ , and let  $\Sigma_t = E_t((v_t - q_t)^2)$ , where the expectation is conditional on the dealer's information set  $I_t$ . In the fast model (see Figure 2), the first event in  $[t, t + dt]$  is the arrival of a market order  $dy_t = dx_t + du_t$ . Since  $du_t$  and the two components of  $dx_t = \beta_t(v_t - q_t)dt + \gamma_t dv_t$  are orthogonal to  $I_t$ , the trading price must be of the form  $p_{t+dt} = q_t + \lambda_t dy_t$ , as specified in (16), where

$$\lambda_t = \frac{\text{Cov}_t(v_1, dy_t)}{\text{Var}_t(dy_t)} = \frac{\text{Cov}_t(v_1, \beta_t(v_t - q_t)dt + \gamma_t dv_t + du_t)}{\text{Var}(\beta_t(v_t - q_t)dt + \gamma_t dv_t + du_t)} = \frac{\beta_t \Sigma_t + \gamma_t \sigma_v^2}{\gamma_t^2 \sigma_v^2 + \sigma_u^2}. \tag{A23}$$

Clearly, after trading the dealer sets the quote equal to the trading price  $p_{t+dt}$ . Then, after observing the signal  $dz_t$ , the dealer updates the quote by adding a multiple of the unexpected part of the signal,

$$q_{t+dt} = p_{t+dt} + \mu_t(dz_t - E(dz_t|I_t, dy_t)), \text{ with} \\ E(dz_t|I_t, dy_t) = E(dz_t|dy_t) = \rho_t dy_t,$$

where we use the fact that both  $dz_t$  and  $dy_t$  are orthogonal to  $I_t$ . We compute

$$\rho_t = \frac{\text{Cov}(dz_t, dy_t)}{\text{Var}(dy_t)} = \frac{\gamma_t \sigma_v^2}{\gamma_t^2 \sigma_v^2 + \sigma_u^2}, \tag{A24}$$

$$\mu_t = \frac{\text{Cov}_t(v_1, dz_t - \rho_t dy_t)}{\text{Var}(dz_t - \rho_t dy_t)} = \frac{-\rho_t \beta_t \Sigma_t + (1 - \rho_t \gamma_t) \sigma_v^2}{(1 - \rho_t \gamma_t)^2 \sigma_v^2 + \rho_t^2 \sigma_u^2 + \sigma_e^2}. \tag{A25}$$

We also obtain  $q_{t+dt} = p_{t+dt} + \mu_t(dz_t - \rho_t dy_t) = q_t + \lambda_t dy_t + \mu_t(dz_t - \rho_t dy_t)$ . This shows that  $dqu_t$  is of the form specified in (17).

Next, the same argument as in the proof of Theorem 1 implies that  $\Sigma_t$  satisfies the same ODE as (A19):

$$\Sigma'_t = -2\Lambda_t \beta_t \Sigma_t + (1 - \mu_t - \Lambda_t \gamma_t)^2 \sigma_v^2 + \mu_t^2 \sigma_e^2 + \Lambda_t^2 \sigma_u^2. \tag{A26}$$

Since  $\Lambda_t = \Lambda$  is assumed constant, the same argument as in the proof of Theorem 1 shows that the transversality condition implies  $\Sigma_1 = 0$ .

We now define the following variables:

$$a = \frac{\sigma_u^2}{\sigma_v^2} > 0, \quad b = \frac{\sigma_e^2}{\sigma_v^2} \geq 0, \quad c = \frac{\Sigma_0}{\sigma_v^2} > 0, \\ g_t = \frac{\gamma_t^2}{a}, \quad \tilde{\lambda}_t = \lambda_t \gamma_t, \quad \tilde{\rho}_t = \rho_t \gamma_t, \quad \tilde{\Lambda}_t = \Lambda_t \gamma_t, \quad \psi_t = \frac{\beta_t \Sigma_t}{\sigma_u^2} \gamma_t. \tag{A27}$$

Note that we cannot have  $g_t = 0$  in equilibrium. Indeed, in that case equation (A24) implies  $\rho_t = 0$ , and equation (A25) implies  $\mu_t = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_e^2} = \frac{1}{1+b}$ . Also, equation (A18) implies  $\lambda_t = \Lambda_t$ , which is a positive constant by assumption.

But then equation (A22) becomes  $0 = \gamma_t = \frac{\mu_t}{\lambda_t}$ , which implies  $\mu_t = 0$ , which contradicts  $\mu_t = \frac{1}{1+b} > 0$ . Therefore,  $g_t \neq 0$ , and since  $g_t = \frac{\gamma_t^2}{a}$ , it must be the case that  $g_t > 0$ .

For notational simplicity, we omit the  $t$  subscript. With the notation in (A27), equations (A23) to (A25) become

$$\tilde{\lambda} = \frac{\psi + g}{1 + g}, \quad \tilde{\rho} = \frac{g}{1 + g}, \quad \mu = \frac{1 - \psi}{1 + b(1 + g)}, \tag{A28}$$

and equation (A22) becomes  $\lambda\gamma = \mu(1 - \rho\gamma)$ , or  $\tilde{\lambda} = \mu(1 - \tilde{\rho})$ . Substituting  $\tilde{\lambda}$ ,  $\tilde{\rho}$ , and  $\mu$  from (A28) into  $\tilde{\lambda} = \mu(1 - \tilde{\rho})$  and solving for  $\psi$ , we obtain

$$\psi = \frac{1 - (1 + b)g - bg^2}{2 + b + bg} = \frac{1 + g}{2 + b + bg} - g. \tag{A29}$$

Equation (A28) then becomes

$$\tilde{\lambda} = \frac{1}{2 + b + bg}, \quad \tilde{\rho} = \frac{g}{1 + g}, \quad \mu = \frac{1 + g}{2 + b + bg}, \tag{A30}$$

and since equation (A18) implies  $\tilde{\Lambda} = \tilde{\lambda} - \mu\tilde{\rho}$ , we get

$$\tilde{\Lambda} = \frac{1 - g}{2 + b + bg}. \tag{A31}$$

The condition  $\Lambda > 0$  (from (A21)), together with  $g > 0$  (proved above), implies that  $\tilde{\Lambda} > 0$ , which implies in turn that  $g < 1$ . Hence,

$$g \in (0, 1). \tag{A32}$$

We compute  $\frac{\partial \tilde{\Lambda}}{\partial g} = \frac{-2-b(1+2g-g^2)}{g^2(2+b+bg)^2} < 0$  for  $g \in (0, 1)$ , and thus  $\tilde{\Lambda}$  is strictly decreasing in  $g$ . From (A27),  $\Lambda = (ag)^{-1/2}\tilde{\Lambda}$ , and thus  $\Lambda$  is also strictly decreasing in  $g$ . Therefore, given  $\Lambda$ , there is a unique  $g$  that satisfies the equilibrium conditions. But  $\Lambda$  is constant, and thus  $g$  and  $\gamma = (ag)^{1/2}$  are also constant. By (A30),  $\lambda$ ,  $\rho$ , and  $\mu$  are also constant. Equation (A23) then implies that  $\beta_t \Sigma_t$  is also constant, and equation (A26) implies that  $\Sigma'_t$  is constant. By the usual argument from the proof of Theorem 1, we thus obtain  $\Sigma_t = \Sigma_0(1 - t)$ ,  $\Sigma'_t = -\Sigma_0$ , and  $\beta_t = \frac{\beta_0}{1-t}$ . Equation (A26) then becomes  $-c = -\frac{2\psi\tilde{\Lambda}}{g} + (1 - \mu - \tilde{\Lambda})^2 + \mu^2b + \frac{\tilde{\Lambda}^2}{g}$ . Using equations (A29) to (A31), we compute

$$1 + c = \frac{(1 + bg)(1 + g)^2}{g(2 + b + bg)^2}. \tag{A33}$$

We show that this equation has a unique solution  $g \in (0, 1)$ . Define the function  $F_b(x) = \frac{(1+bx)(1+x)^2}{x(2+b+bx)^2}$ , and observe that with this notation equation (A33) becomes  $F_b(g) = 1 + c$ . One can verify that  $F'_b(x) = \frac{(x+1)(x-1)(2+b+3bx)}{x^2(2+b+bx)^3}$ , and thus  $F_b(x)$  is

strictly decreasing on  $[0, 1]$ . Since  $F_b(0) = +\infty$  and  $F_b(1) = \frac{1}{1+b} \leq 1$ , there is a unique  $g \in (0, 1)$  that solves  $F_b(g) = 1 + c$ .

Now, use equations (A27), (A30), and (A31) to write

$$\begin{aligned} \gamma &= a^{1/2}g^{1/2}, \quad \lambda = \frac{1}{\gamma} \frac{1}{2+b+bg}, \quad \rho = \frac{1}{\gamma} \frac{g}{1+g}, \quad \mu = \frac{1+g}{2+b+bg}, \\ \Lambda &= \frac{1}{\gamma} \frac{1-g}{2+b+bg}, \quad 1-\mu-\Lambda\gamma = \frac{b+bg}{2+b+bg}, \quad \psi = \frac{1+g}{2+b+bg} - g. \end{aligned} \tag{A34}$$

After some algebraic manipulations, one can show that equations (18) to (23) follow from equations (A33) and (A34). To illustrate how this works, we prove the formulas for  $\beta_t^F$  and  $\gamma^F$ , and leave the rest to the reader. To compute  $\beta_0^F$ , note that the definition of  $\psi$  from (A27) implies  $\beta_0^F = \frac{\sigma_u^2}{\Sigma_0\gamma} \psi = \frac{a}{c\gamma} \psi = \frac{a^{1/2}}{cg^{1/2}} \psi$ . We compute  $\psi = \frac{1+g}{2+b+bg} - g = \frac{g(2+b+bg)}{(1+g)(1+bg)} \left( \frac{(1+g)^2(1+bg)}{g(2+b+bg)^2} - \frac{(1+g)(1+bg)}{2+b+bg} \right) = \frac{g(2+b+bg)}{(1+g)(1+bg)} \left( c + 1 - \frac{(1+g)(1+bg)}{2+b+bg} \right) = \frac{g(2+b+bg)}{(1+g)(1+bg)} (c + (1-g) \frac{1+b+bg}{2+b+bg})$ . Hence,  $\beta_0^F = \frac{a^{1/2}g^{1/2}(2+b+bg)}{(1+g)(1+bg)} (1 + \frac{1-g}{c} \frac{1+b+bg}{2+b+bg})$ . Using (A33) again, we get  $\beta_0^F = \frac{a^{1/2}}{(1+c)^{1/2}(1+bg)^{1/2}} (1 + \frac{1-g}{c} \frac{1+b+bg}{2+b+bg})$ , which, together with the formula  $\beta_t^F = \frac{\beta_0^F}{1-t}$ , proves (18). To compute  $\lambda^F$ , note that (A33) implies  $g^{1/2}(2+b+bg) = \frac{(1+bg)^{1/2}(1+g)}{(1+c)^{1/2}}$ . Hence, from (A34),  $\lambda^F = \frac{1}{a^{1/2}g^{1/2}(2+b+bg)} = \frac{(1+c)^{1/2}}{a^{1/2}} \frac{1}{(1+bg)^{1/2}(1+g)}$ , which proves (20).

Finally, we prove that in equilibrium  $\Lambda_\tau$  is indeed constant for  $\tau \in [0, 1]$ . The argument is the same as in the proof of Theorem 1, except for the case in which  $(\frac{1}{2\Lambda_\tau})' < 0$  for  $\tau$  in a small interval  $I$ . Then, as in the proof of Theorem 1, the market is strong-form efficient for  $\tau \in I$ , and thus  $\Sigma_\tau = 0$  on that interval. Using the same argument that follows equation (A26), we arrive at the same conclusion, namely, that  $g_\tau$  satisfies equation (A33), except that now  $c_\tau$  is replaced by  $-\Sigma'_\tau = 0$  for all  $\tau \in I$ . But we have seen that the solution to (A33) is unique for  $g \in (0, 1)$ . Therefore,  $g_\tau$  is constant on  $I$ , and thus  $\Lambda_\tau$  is also constant on  $I$  (as a function of  $g_\tau$ ). This contradicts  $(\frac{1}{2\Lambda_\tau})' < 0$ .  $\square$

**PROOF OF PROPOSITION 2:** We use the notation from the proof of Theorems 1 and 2, in particular,  $a, b$ , and  $c$  defined in (A27). We have  $\beta_t^k = \frac{\beta_0^k}{1-t}$  for all  $t \in [0, 1]$  and for  $k \in \{F, S\}$ . Thus, we need to prove that  $\beta_0^F < \beta_0^S$ . The formula (18) for  $\beta_0^F$  is more difficult to work with, and thus we provide an alternate formula. By the definition of  $\psi$  from (A27),  $\beta_0^F = \frac{\sigma_u^2}{\Sigma_0\gamma} \psi = \frac{a}{c\gamma} \psi = \frac{a^{1/2}}{cg^{1/2}} \psi$ . The formula for  $\psi$  in equation (A34) then implies  $\beta_0^F = \frac{a^{1/2}}{cg^{1/2}} (\frac{1+g}{2+b+bg} - g)$ . We now show that

$$\beta_0^F = \frac{a^{1/2}}{cg^{1/2}} \left( \frac{1+g}{2+b+bg} - g \right) < \beta_0^S = \frac{a^{1/2}}{c} \left( c + \frac{b}{1+b} \right)^{1/2}. \tag{A35}$$

If we square both sides and multiply by  $\frac{c^2}{a}$ , the desired inequality is equivalent to  $\frac{1}{g} \frac{(1-g-bg-bg^2)^2}{(2+b+bg)^2} < c + 1 - \frac{1}{1+b}$ . If we replace  $c + 1$  by the formula in (A33), (A35) is equivalent to  $\frac{1}{1+b} < \frac{(1+bg)(1+g)^2}{g(2+b+bg)^2} - \frac{1}{g} \frac{(1-g-bg-bg^2)^2}{(2+b+bg)^2} = \frac{4+3b+bg(2-b)-bg^2(1+2b)-b^2g^3}{(2+b+bg)^2}$ .

After some algebra, this is equivalent to  $b(1+b)g(1+g)^2 + (1+b)g^2 + bg^2 + 2g < 3 + 2b$ . Sufficient conditions for this inequality are (i)  $b(1+b)g(1+g)^2 + (1+b)g^2 < 1 + b$ , and (ii)  $bg^2 + 2g < 2 + b$ . Inequality (ii) follows directly from  $g \in (0, 1)$ . Thus, we just need to prove (i). After dividing by  $1 + b$ , (i) becomes  $bg(1+g)^2 + g^2 < 1$ , which is equivalent to  $bg < \frac{1-g}{1+g}$ . To prove this inequality, note that equation (18) implies  $\beta_0^F > 0$ . The expression for  $\beta_0^F$  in (A35) thus shows that  $\frac{1+g}{2+b+bg} - g > 0$ . Some algebra then shows that the inequality  $\frac{1+g}{2+b+bg} > g$  implies  $bg < \frac{1-g}{1+g}$ , which finishes the proof.  $\square$

PROOF OF PROPOSITION 3: In both models,  $k \in \{S, F\}$ ,  $\beta_t^k \Sigma_t$ ,  $\gamma_t^k$ ,  $\lambda_t^k$ , and  $\mu_t^k$  are constant with respect to time. Equations (A4) and (A17) then imply

$$\begin{aligned} \pi_0^S &= \left[ \beta_0^S \Sigma_0 \right] + \left[ (1 - \mu^S - \lambda^S \gamma^S) \gamma^S \sigma_v^2 \right] = \pi_\beta^S + \pi_\gamma^S, \\ \pi_0^F &= \left[ \beta_0^F \Sigma_0 \right] + \left[ (1 - \lambda^F \gamma^F) \gamma^F \sigma_v^2 \right] = \pi_\beta^F + \pi_\gamma^F, \end{aligned} \tag{A36}$$

where in each case the first expression in brackets is  $\pi_\beta^k$  and the second is  $\pi_\gamma^k$ . In the slow model,  $\gamma^S = 0$ , and hence  $\pi_\gamma^S = 0$ . Moreover, equation (A12) implies that  $\lambda^S = \frac{\beta_0^S \Sigma_0}{\sigma_u^2}$ , and thus  $\pi_0^S = \lambda^S \sigma_u^2$ . In the fast model,  $\lambda^F = \frac{\beta_0^F \Sigma_0 + \gamma^F \sigma_v^2}{(\gamma^F)^2 \sigma_v^2 + \sigma_u^2}$  (equation (A23)), and thus  $\lambda^F \sigma_u^2 = \beta_0^F \Sigma_0 + \gamma^F (1 - \lambda^F \gamma^F) \sigma_v^2$ . But the right-hand side of this equation is exactly the formula for  $\pi_0^F$  in (A36). Thus, we have  $\pi_0^F = \lambda^F \sigma_u^2$ . In conclusion, we have proved the following formulas for the speculator's expected profit in the slow and the fast models:<sup>33</sup>

$$\pi_0^S = \lambda^S \sigma_u^2, \quad \pi_0^F = \lambda^F \sigma_u^2. \tag{A37}$$

Proposition 4 implies that  $\lambda^F > \lambda^S$ , and therefore the total expected profit in the fast model is higher than that in the slow model:  $\pi_0^F > \pi_0^S$ . At the same time, Proposition 2 implies that  $\beta_0^F < \beta_0^S$ , and hence  $\pi_\beta^F < \pi_\beta^S$ .

For future reference, we also express the various components of the speculator's expected profit when he is fast or slow as a function of exogenous parameters and  $g$  in (A38) below. We again use the notational shortcuts ( $a$ ,  $b$ , and  $c$ ) defined in (A27).

In the slow model, equation (13) implies that  $\pi_0^S = \lambda^S \sigma_u^2 = \sigma_u \sigma_v (c + \frac{b}{1+b})^{1/2}$ . In the fast model, equation (20) implies that  $\lambda^F = \frac{(c+1)^{1/2}}{a^{1/2}} \frac{1}{(1+bg)^{1/2}(1+g)} = \frac{1}{a^{1/2} g^{1/2(2+b+bg)}}$ , where the second equality follows from (23). From (19),  $\gamma^F = \alpha^{1/2} g^{1/2}$ , and thus  $1 - \lambda^F \gamma^F = \frac{1+b+bg}{2+b+bg}$ . We compute  $\pi_0^F = \lambda^F \sigma_u^2 = \sigma_u \sigma_v \frac{1}{g^{1/2(2+b+bg)}}$ ,

<sup>33</sup> These formulas are intuitive. Indeed, since the dealer sets prices at the expected value, she must break even on average. Hence, the expected profit of the speculator must be equal to the expected loss incurred by the noise traders, which over  $[0, 1]$  adds up to  $\lambda^k \sigma_u^2$ ,  $k \in \{S, F\}$ .

and  $\pi_\gamma^F = (1 - \lambda^F \gamma^F) \gamma^F \sigma_v^2 = \sigma_u \sigma_v \frac{g^{1/2}(1+b+bg)}{2+b+bg}$ . Their difference is  $\pi_\beta^F = \sigma_u \sigma_v \frac{(1+g)\left(\frac{1-g}{1+g} - bg\right)}{g^{1/2}(2+b+bg)}$ .<sup>34</sup> We now collect all the formulas:

$$\begin{aligned} \pi_0^S &= \sigma_u \sigma_v \left( c + \frac{b}{1+b} \right)^{1/2}, & \pi_\beta^S &= \pi_0^S, & \pi_\gamma^S &= 0, \\ \pi_0^F &= \sigma_u \sigma_v \frac{1}{g^{1/2}(2+b+bg)} = \pi_\beta^F + \pi_\gamma^F, & & & & (A38) \\ \pi_\beta^F &= \sigma_u \sigma_v \frac{(1+g)\left(\frac{1-g}{1+g} - bg\right)}{g^{1/2}(2+b+bg)}, & \pi_\gamma^F &= \sigma_u \sigma_v \frac{g^{1/2}(1+b+bg)}{2+b+bg}. & & \square \end{aligned}$$

PROOF OF PROPOSITION 4: We use the notation from the proof of Theorems 1 and 2, in particular, the notational shortcuts ( $a$ ,  $b$ , and  $c$ ) defined in (A27). Using this notation, we need to show that

$$\lambda^F = \frac{(1+c)^{1/2}}{a^{1/2}} \frac{1}{(1+bg)^{1/2}(1+g)} > \lambda^S = \frac{c^{1/2}}{a^{1/2}} \left( 1 + \frac{b}{c(b+1)} \right)^{1/2}. \quad (A39)$$

After squaring the two sides and using  $1+c = \frac{(1+bg)(1+g)^2}{g(2+b+bg)^2}$  (equation (A33)), we need to prove that  $\frac{1}{g(2+b+bg)^2} > c + 1 - \frac{1}{1+b}$ , or equivalently,  $\frac{1}{1+b} > \frac{(1+bg)(1+g)^2}{g(2+b+bg)^2} - \frac{1}{g(2+b+bg)^2} = \frac{2+b+g+2bg+bg^2}{(2+b+bg)^2}$ . Algebraic manipulation shows that this is equivalent to  $1+b+(1-g)(1+bg) > 0$ , which is true, since  $b > 0$  and  $g \in (0, 1)$ .  $\square$

PROOF OF PROPOSITION 5: With the notation from the proof of Theorems 1 and 2, we need to show that

$$\mu^F = \frac{1+g}{2+b+bg} < \mu^S = \frac{1}{1+b}. \quad (A40)$$

Algebraic manipulation shows that this is equivalent to  $g < 1$ , which is true since  $g \in (0, 1)$ .

PROOF OF COROLLARY 1: In both the slow model and the fast model ( $k \in \{S, F\}$ ), the quote change is of the form  $dq_t = \lambda^k dy_t + \mu^k (dz_t - \rho^k dy_t)$ , with the first component,  $\lambda^k dy_t = dq_{\text{trades},t}$ , caused by trading, and the second component,  $dq_{\text{news},t} = \mu^k (dz_t - \rho^k dy_t)$ , caused by news. Thus, we decompose the instantaneous quote variance into trading and news components:

$$\sigma_q^2 = \sigma_{q,\text{trades}}^2 + \sigma_{q,\text{news}}^2 \iff \frac{\text{Var}(dq_t)}{dt} = \frac{\text{Var}(dq_{\text{trades},t})}{dt} + \frac{\text{Var}(dq_{\text{news},t})}{dt}. \quad (A41)$$

<sup>34</sup> In the proof of Proposition 2 we have shown that  $bg < \frac{1-g}{1+g}$ , and hence  $\pi_\beta^F > 0$ .

In the slow model,  $\sigma_{q,\text{trades}}^2 = (\lambda^S)^2 \sigma_u^2 = \Sigma_0 + \frac{\sigma_v^2 \sigma_e^2}{\sigma_v^2 + \sigma_e^2}$ . Also,  $\sigma_{q,\text{news}}^2 = (\mu^S)^2 (\sigma_v^2 + \sigma_e^2) = \frac{\sigma_v^4}{\sigma_v^2 + \sigma_e^2}$ . Thus, in the slow model we have the following volatility decomposition:

$$\sigma_q^2 = \frac{\text{Var}(dq_t)}{dt} = \left( \Sigma_0 + \frac{\sigma_v^2 \sigma_e^2}{\sigma_v^2 + \sigma_e^2} \right) + \frac{\sigma_v^4}{\sigma_v^2 + \sigma_e^2} = \Sigma_0 + \sigma_v^2. \quad (\text{A42})$$

In the fast model,  $\sigma_{q,\text{trades}}^2 = (\lambda^F)^2 ((\gamma^F)^2 \sigma_v^2 + \sigma_u^2)$ , and using equation (A34) we compute  $\sigma_{q,\text{trades}}^2 = \frac{1+g}{g(2+b+bg)^2} \sigma_v^2$ . Also,  $\sigma_{q,\text{news}}^2 = (\mu^F)^2 ((1 - \rho^F \gamma^F)^2 \sigma_v^2 + \sigma_e^2 + (\rho^F)^2 \sigma_u^2) = \frac{(1+g)(1+b+bg)}{(2+b+bg)^2} \sigma_v^2$ . From (A33),  $\Sigma_0 + \sigma_v^2 = (c + 1) \sigma_v^2 = \frac{(1+g)^2(1+bg)}{g(2+b+bg)^2} \sigma_v^2$ , and thus

$$\sigma_q^2 = \frac{\text{Var}(dq_t)}{dt} = \frac{1+g}{g(2+b+bg)^2} \sigma_v^2 + \frac{(1+g)(1+b+bg)}{(2+b+bg)^2} \sigma_v^2 = \Sigma_0 + \sigma_v^2. \quad (\text{A43})$$

We now prove that the volatility component coming from quote updates is larger in the slow model:  $\frac{\sigma_v^2}{\sigma_v^2 + \sigma_e^2} = \frac{1}{1+b} > \frac{(1+g)(1+b+bg)}{(2+b+bg)^2}$ . Indeed, some algebraic manipulation shows that the difference is proportional to  $3 - g + 2b + bg - bg^2 = 2(1+b) + (1-g)(1+bg) > 0$ . Since the total volatility is the same, it also implies that the volatility component coming from the trades is larger in the fast model.

**PROOF OF COROLLARY 2:** In Theorems 1 and 2, we have proved that  $\Sigma_t = \Sigma_0(1-t)$ , regardless of whether the speculator is fast or slow. This proves the last part of the corollary. Furthermore, in both the slow model and the fast model,  $d\Sigma_t = dE((v_t - q_t)^2) = 2\text{Cov}(dv_t - dq_t, v_t - q_t) + \text{Var}(dv_t - dq_t)$ . Since the news  $dv_t$  is orthogonal to  $v_t - q_t$  in both models,  $d\Sigma_t = -2\text{Cov}(dq_t, v_t - q_t) - 2\text{Cov}(dq_t, dv_t) + \text{Var}(dv_t) + \text{Var}(dq_t)$ . But  $\frac{1}{dt} \text{Var}(dv_t) = \sigma_v^2$ , and, from Corollary 1,  $\sigma_q^2 = \frac{1}{dt} \text{Var}(dq_t) = \sigma_v^2 + \Sigma_0$ . We have just proved (25).

Equation (10) implies that in the slow model  $dq_t = \mu^S dz_t + \lambda^S dy_t$ . Since  $dy_t = dx_t + du_t$ , and  $dx_t$  has no volatility component ( $\gamma^S = 0$ ), we get  $\text{Cov}(dq_t, dv_t) = \mu^S \sigma_v^2 dt$ .

Equation (17) implies that in the fast model  $dq_t = \lambda^F dy_t + \mu^F (dz_t - \rho^F dy_t)$ . Since the volatility component of  $dx_t$  is equal to  $\gamma^F dv_t$ , we get  $\text{Cov}(dq_t, dv_t) = (\gamma^F (\lambda^F - \mu^F \rho^F) + \mu^F) \sigma_v^2 dt$ .

Next, we prove that  $\gamma^F (\lambda^F - \mu^F \rho^F) + \mu^F > \mu^S$ . Using (A34) and (A40), we need to show that  $\frac{2}{2+b+bg} > \frac{1}{1+b}$ , which is equivalent to  $1 > g$ . But this is true since  $g \in (0, 1)$ . Thus,  $\text{Cov}(dq_t, dv_t)$  is higher in the fast model. As  $d\Sigma_t = -\Sigma_0$  in both the fast model and the slow model, it follows from (25) that  $\text{Cov}(dq_t, v_t - q_t)$  is smaller when the speculator is fast.

**PROOF OF COROLLARY 3:** Expressions for the speculator's expected profit and its two components are given in (A38). Remember that  $a = \frac{\sigma_u^2}{\sigma_v^2} > 0$ ,  $b = \frac{\sigma_e^2}{\sigma_v^2} \geq 0$ , and  $c = \frac{\Sigma_0}{\sigma_v^2} > 0$ . When the speculator is slow, we deduce from (A38) that

$$\lim_{\Sigma_0 \rightarrow 0} \pi^S = \lim_{\Sigma_0 \rightarrow 0} \pi_\beta^S = \sigma_u \sigma_v \left( \frac{b}{1+b} \right)^{1/2} \geq 0, \quad (\text{A44})$$

where the last inequality is strict if  $b > 0$ , that is, if  $\sigma_e > 0$ . This proves the first part of Corollary 3.

Now consider the case in which the speculator is fast. In this case, remember from Theorem 2 that  $g$  solves  $F_b(g) = 1 + c$ , where  $F_b(x) = \frac{(1+bx)(1+x)^2}{x(2+b+bx)^2}$ . Hence, when  $\Sigma_0$  goes to zero,  $g$  solves  $F_b(g) = 1$ . Let  $g_0$  be this solution. It is unique and belongs to  $g_0 \in (0, 1]$  since (i)  $F_b(x)$  decreases with  $x$ , for  $x \in [0, 1]$ , (ii)  $F_b(0) = +\infty$ , and (iii)  $F_b(1) = \frac{1}{1+b}$ . Furthermore,  $g_0 = 1$  only when  $b = 0$ , that is, when  $\sigma_e = 0$ . Using these observations, we deduce from (A38) that

$$\lim_{\Sigma_0 \rightarrow 0} \pi^F = \sigma_u \sigma_v \frac{1}{g_0^{1/2}(2+b+bg_0)}, \quad (\text{A45})$$

$$\lim_{\Sigma_0 \rightarrow 0} \pi_\gamma^F = \sigma_u \sigma_v \frac{g_0^{1/2}(1+b+bg_0)}{2+b+bg_0} > 0, \quad (\text{A46})$$

and therefore

$$\lim_{\Sigma_0 \rightarrow 0} \pi_\beta^F = \sigma_u \sigma_v \frac{(1+g_0) \left( \frac{1-g_0}{1+g_0} - bg_0 \right)}{g_0^{1/2}(2+b+bg_0)} \geq 0, \quad (\text{A47})$$

where the last inequality is strict unless  $b = 0$ , that is, if  $\sigma_e = 0$ . Thus, we have shown that (i)  $\lim_{\Sigma_0 \rightarrow 0} \pi_\beta^k > 0$  iff  $\sigma_e > 0$  and (ii)  $\lim_{\Sigma_0 \rightarrow 0} \pi_\gamma^F > 0$ .  $\square$

**PROOF OF COROLLARY 4:** In the slow model, equation (8) and  $\gamma_t^S = 0$  imply that  $\text{Var}(dx_t) = (\beta_t^S)^2 \Sigma_t (dt)^2 = 0$ , since  $(dt)^2 = 0$ . Also,  $\text{Var}(du_t) = \sigma_u^2 dt$ . Thus,  $SPR_t^S = \frac{\text{Var}(dx_t)}{\text{Var}(dx_t) + \text{Var}(du_t)} = 0$ .

In the fast model, equation (15) implies  $\text{Var}(dx_t) = (\gamma^F)^2 \sigma_v^2 dt$ , and equation (19) implies  $(\gamma^F)^2 \sigma_v^2 = \sigma_u^2 g$ . Thus,  $SPR^F = \frac{\sigma_u^2 g dt}{\sigma_u^2 g dt + \sigma_u^2 dt} = \frac{g}{g+1}$ . From Theorem 2,  $g \in (0, 1)$ , and hence  $SPR^F > 0$ .

Before we proceed with the proof of Corollary 5, we derive some useful formulas.

**LEMMA A1:** Let  $\Lambda^k = \lambda^k - \mu^k \rho^k$  for  $k \in \{S, F\}$ . Then, for all  $s < t \in (0, 1)$ ,

$$\begin{aligned} \text{Cov}(v_s - q_s, v_t - q_t) &= \Sigma_s \left( \frac{1-t}{1-s} \right)^{\Lambda^k \beta_0^k}, \\ \frac{1}{ds} \text{Cov}(dv_s, v_t - q_t) &= (1 - \Lambda^k \gamma^k - \mu^k) \sigma_v^2 \left( \frac{1-t}{1-s} \right)^{\Lambda^k \beta_0^k}. \end{aligned} \quad (\text{A48})$$

**PROOF:** Fix  $s \in (0, 1)$ . Let  $X_t = \text{Cov}(v_s - q_s, v_t - q_t)$ . For  $t \geq s$ ,  $dX_t = \text{Cov}(v_s - q_s, dv_t - dq_t) = -\text{Cov}(v_s - q_s, dq_t) = -\Lambda^k \beta_t^k X_t dt = -\Lambda^k \frac{\beta_0^k}{1-t} X_t dt$ . Then,  $d \ln(X_t) = \Lambda^k \beta_0^k d \ln(1-t)$ . Also, at  $t = s$ , we have  $X_s = \Sigma_s$ . Thus, we have a first-order differential equation, with the solution given by the first equation in (A48).

Let  $Y_t = \frac{1}{ds} \text{Cov}(dv_s, v_t - q_t)$ . For  $t > s$ ,  $dY_t = \frac{1}{ds} \text{Cov}(dv_s, dv_t - dq_t) = -\frac{1}{ds} \text{Cov}(dv_s, dq_t) = -\Lambda^k \beta_t^k Y_t dt = -\Lambda^k \frac{\beta_0^k}{1-t} Y_t dt$ . Then,  $d \ln(Y_t) = \Lambda^k \beta_0^k d \ln(1-t)$ . At  $t =$

$s + ds$ , we have  $Y_{s+ds} = \frac{1}{ds} \text{Cov}(dv_s, v_s - q_s + dv_s - dq_s) = \frac{1}{ds} \text{Cov}(dv_s, dv_s) - \frac{1}{ds} s \text{Cov}(dv_s, \Lambda^k dy_t + \mu^k dz_t) = (1 - \Lambda^k \gamma^k - \mu^k) \sigma_v^2$ . Thus, we have a first-order differential equation, with the solution given by the second equation in (A48).

PROOF OF COROLLARY 5: In both the slow model and the fast model ( $k \in \{S, F\}$ ), the speculator's trading strategy is of the form  $dx_t = \beta_t^k (v_t - q_t) dt + \gamma^k dv_t$ , and the dealer's quote evolves according to  $dq_t = \mu^k (dz_t - \rho^k dy_t) + \lambda^k dy_t = \mu^k dz_t + \Lambda^k dy_t$ , where  $\Lambda^k = \lambda^k - \mu^k \rho^k$ .

For the slow model,  $\rho^S = 0$ , and hence  $\Lambda^S = \lambda^S$ . Using Lemma A1, we get

$$\text{Corr}(dx_t^S, dx_{t+\tau}^S) = \frac{\text{Cov}(v_t - q_t, v_{t+\tau} - q_{t+\tau})}{\text{Var}(v_t - q_t)^{1/2} \text{Var}(v_{t+\tau} - q_{t+\tau})^{1/2}} = \frac{\Sigma_t \left(\frac{1-t-\tau}{1-t}\right)^{\lambda^S \beta_0^S}}{\Sigma_t^{1/2} \Sigma_{t+\tau}^{1/2}}. \quad (\text{A49})$$

Since  $\Sigma_s = \Sigma_0(1 - s)$ , we obtain  $\text{Corr}(dx_t^S, dx_{t+\tau}^S) = \left(\frac{1-t-\tau}{1-t}\right)^{\lambda^S \beta_0^S - \frac{1}{2}}$ .

In the fast model, equation (A48) implies that the autocovariance of the speculator's order flow,  $\text{Cov}(dx_t^F, dx_{t+\tau}^F)$ , is of order  $(dt)^2$ . But the variance of the speculator's order flow is of order  $dt$ , and therefore the autocorrelation is of order  $dt$ , which as a number equals zero in continuous time.

PROOF OF COROLLARY 6: As in the proof of Corollary 5 and Lemma A1, for  $k \in \{S, F\}$ , we consider the following notation:  $X_{t,t+\tau} = \text{Cov}(v_t - q_t, v_{t+\tau} - q_{t+\tau})$  and  $Y_{t,t+\tau} = \frac{1}{dt} \text{Cov}(dv_t, v_{t+\tau} - q_{t+\tau})$ . Let  $\Lambda^k = \lambda^k - \mu^k \rho^k$ , and  $\alpha^k = \left(\frac{1-(t+\tau)}{1-t}\right)^{\Lambda^k \beta_0^k}$ . We first show that, as claimed in (30) and (31),  $\text{CPI}_t^k(\tau) = C_0^k + C_1^k(1 - \alpha^k)$ , where

$$C_0^S = 0, \quad C_1^S = \beta_0^S \Sigma_0, \\ C_0^F = (\mu^F + \Lambda^F \gamma^F) \gamma^F \sigma_v^2, \quad C_1^F = \beta_0^F \Sigma_0 + (1 - \mu^F - \Lambda^F \gamma^F) \gamma^F \sigma_v^2. \quad (\text{A50})$$

Let  $n^k = 1 - \mu^k - \Lambda^k \gamma^k$ . We write  $\text{CPI}_t^k(\tau) = \text{Cov}\left(\frac{dx_t}{dt}, q_{t+\tau} - q_t\right)$ . Since  $dx_t = \beta_t^k (v_t - q_t) dt + \gamma^k dv_t$ , we obtain

$$\begin{aligned} \text{CPI}_t^k(\tau) &= \beta_t^k \text{Cov}(v_t - q_t, q_{t+\tau} - q_t) + \frac{\gamma^k}{dt} \text{Cov}(dv_t, q_{t+\tau} - q_t) \\ &= \beta_t^k \text{Cov}(v_t - q_t, v_{t+\tau} - q_t) + \frac{\gamma^k}{dt} \text{Cov}(dv_t, v_{t+\tau} - q_t) - \beta_t^k X_{t,t+\tau} - \frac{\gamma^k}{dt} Y_{t,t+\tau}. \end{aligned} \quad (\text{A51})$$

But  $\text{Cov}(v_t - q_t, v_{t+\tau} - q_t) = \Sigma_t$ ,  $\text{Cov}(dv_t, v_{t+\tau} - q_t) = \sigma_v^2 dt$ , and Lemma A1 in the proof of Corollary 5 implies  $X_{t,t+\tau} = \Sigma_t \alpha^k$  and  $Y_{t,t+\tau} = n^k \sigma_v^2 \alpha^k$ . Thus,  $\text{CPI}_t^k(\tau) = \beta_0^k \Sigma_0 (1 - \alpha^k) + \gamma^k \sigma_v^2 (1 - n^k \alpha^k)$  (we know that  $\beta_t^k \Sigma_t = \beta_0^k \Sigma_0$  is constant). We get  $\text{CPI}_t^k(\tau) = C_0^k + C_1^k(1 - \alpha)$ , where  $C_0^k = (1 - n^k) \gamma^k \sigma_v^2$  and  $C_1^k = \beta_0^k \Sigma_0 + n^k \gamma^k \sigma_v^2$ . It is straightforward to check, for both  $k \in \{S, F\}$ , that  $C_0^k$  and  $C_1^k$  are indeed as in equation (A50). Moreover,  $n^F > 0$  as equation (A34) implies that  $n^F = 1 - \mu^F - \Lambda^F \gamma^F = \frac{b+bg}{2+b+bg} > 0$ .

<sup>35</sup> Note that  $\lambda^S \beta_0^S = 1 + \frac{\sigma_v^2 \sigma_d^2}{\Sigma_0(\sigma_v^2 + \sigma_d^2)} > 1$ .

PROOF OF PROPOSITION 6: We use the notation from the proof of Theorems 1 and 2, in particular the notational shortcut ( $a$ ,  $b$ , and  $c$ ) defined in (A27). In the slow model, we have

$$\beta_0^S = \frac{\sigma_u}{\Sigma_0^{1/2}} \left( 1 + \frac{\sigma_v^2 \sigma_e^2}{\Sigma_0 (\sigma_v^2 + \sigma_e^2)} \right)^{1/2} = \frac{a^{1/2}}{c} \left( c + \frac{b}{1+b} \right)^{1/2}. \tag{A52}$$

Thus,  $\beta_0^S$  is increasing in  $b = \frac{\sigma_e^2}{\sigma_v^2}$ , and decreasing in  $\nu = \frac{1}{\sigma_e}$ . Since  $\beta_t^S$  is proportional to  $\beta_0^S$ , it follows that  $\beta_t^S$  is decreasing in  $\nu$ .

In the fast model, let  $F(b, x) = F_b(x) = \frac{(1+bx)(1+x)^2}{x(2+b+bx)^2}$ . We compute its partial derivatives,  $\frac{\partial F}{\partial b} = -\frac{(1+x)^2(2+bx+bx^2)}{x(2+b+bx)^3}$  and  $\frac{\partial F}{\partial x} = -\frac{(1-x)(1+x)(2+b+3bx)}{x^2(2+b+bx)^3}$ . Remember that  $g$  is the solution of  $F(b, g) = 1 + c$  and this solution is in  $(0, 1]$ . Denote this solution by  $g(b, c)$ . We have  $\frac{\partial F}{\partial b} + \frac{\partial F}{\partial x} \frac{\partial g}{\partial b} = 0$  and  $\frac{\partial F}{\partial x} \frac{\partial g}{\partial c} = 1$ . We deduce that

$$\begin{aligned} \frac{\partial g(b, c)}{\partial b} &= -\frac{g(1+g)(2+bg+bg^2)}{(1-g)(2+b+3bg)}, \\ \frac{\partial g(b, c)}{\partial c} &= -\frac{g^2(2+b+bg)^3}{(1-g)(1+g)(2+b+3bg)}. \end{aligned} \tag{A53}$$

Thus,  $g(b, c)$  is decreasing in  $b$ , and hence is increasing in  $\nu$ . As  $\gamma^F = a^{1/2}g^{1/2}$ , it follows that  $\gamma^F$  is increasing in  $\nu$  as well.

In the proof of Theorem 2, we have also proved that

$$\beta_0^F = \frac{a^{1/2}}{cg^{1/2}} \left( \frac{1+g}{2+b+bg} - g \right). \tag{A54}$$

Using (A53), we compute  $\frac{\partial \beta_0^F}{\partial b} = \frac{a^{1/2}g^{1/2}(1+g)^2(2+3b+3bg+b^2g+b^2g^2)}{2c(1-g)(2+b+bg)(2+b+3bg)}$ . Thus,  $\beta_0^F$  is increasing in  $b$ , and hence is decreasing in  $\nu$ .

Finally, when  $\nu \rightarrow 0$ , we have  $\sigma_e \rightarrow +\infty$  and therefore  $b \rightarrow +\infty$ . Consider equation (23):  $g = \frac{(1+bg)(1+g)^2}{(2+b+bg)^2} \frac{1}{1+c}$ . Since  $g \in (0, 1)$ , the right-hand side of the equation is of the order  $\frac{1}{b}$ , and thus it converges to zero when  $b \rightarrow +\infty$ . This implies that  $\lim_{\nu \rightarrow 0} g = 0$ , which implies that  $\lim_{\nu \rightarrow 0} \gamma^F = 0$ . Moreover, we show that  $\lim_{\nu \rightarrow 0} \beta_0^S = \lim_{\nu \rightarrow 0} \beta_0^F$ . In the slow model, equation (A52) implies that  $\lim_{\nu \rightarrow 0} \beta_0^S = \frac{a^{1/2}}{c}(c+1)^{1/2}$ . In the fast model, equation (A54) implies that  $\lim_{\nu \rightarrow 0} \beta_0^F = \lim_{\nu \rightarrow 0} \frac{a^{1/2}}{c} \frac{1+g}{g^{1/2}b(1+\frac{2}{b}+g)} = \lim_{\nu \rightarrow 0} \frac{a^{1/2}}{c} \frac{1}{g^{1/2}b}$ . Thus, the proof of  $\lim_{\nu \rightarrow 0} \beta_0^S = \lim_{\nu \rightarrow 0} \beta_0^F$  is finished if we can show that  $\lim_{\nu \rightarrow 0} g^{1/2}b = \frac{1}{(1+c)^{1/2}}$ . To do so, note that equation (A33),  $(1+c)^{1/2} = \frac{(1+bg)^{1/2}(1+g)}{g^{1/2}(2+b+bg)}$ , implies  $g^{1/2}b = \frac{(1+bg)^{1/2}(1+g)}{(1+c)^{1/2}(1+\frac{2}{b}+g)}$ . Hence,  $\lim_{\nu \rightarrow 0} g^{1/2}b = \frac{1}{(1+c)^{1/2}}$ , which finishes the proof.  $\square$

PROOF OF COROLLARY 7: Equation (A53) shows that  $g$  is decreasing in  $b = \frac{\sigma_e^2}{\sigma_v^2}$ . Hence, if a variable  $X$  is an increasing function of  $g$ , then  $X$  is also increasing in  $\nu = \frac{1}{\sigma_e}$ . In the fast model, let  $TV^F = \frac{\text{Var}(dy_t)}{dt}$  be the trading volume, and

$SPR^F = \frac{\text{Var}(dx_t)}{\text{Var}(dy_t)}$  be the speculator's participation rate. Then  $TV^F = \sigma_u^2(1+g)$ , and  $SPR^F = \frac{g}{1+g}$  (see the proof of Corollary 4). Since both variables are increasing in  $g$ , they are also increasing in  $\nu$ .

Now, equation (A39) implies  $(\lambda^F)^2 = \frac{1+c}{a} \frac{1}{(1+bg)(1+g)^2}$ , where  $c = \frac{\sum_0}{\sigma_v^2} > 0$ . Using the formula for  $\frac{\partial g}{\partial b}$  in (A53), we compute  $\frac{\partial((1+bg)(1+g)^2)}{\partial b} = -\frac{g(1+g)^3(1+bg)}{1-g} < 0$ , and thus  $\frac{\partial \lambda^F}{\partial b} > 0$ . Since  $\nu = \frac{1}{\sigma_c} = \frac{1}{\sigma_v} \frac{1}{b^{1/2}}$ , this implies  $\frac{\partial \lambda^F}{\partial \nu} < 0$ , and  $\frac{\partial(1/\lambda^F)}{\partial \nu} > 0$ .  $\square$

PROOF OF COROLLARY 8: We need to show how the speculator's expected profit ( $\pi_0^F$ ) and its various components ( $\pi_\beta^F$  and  $\pi_\gamma^F$ ) depend on  $\nu = \frac{1}{\sigma_c}$ . In general, if a variable  $X$  is increasing in  $b = \frac{\sigma_c^2}{\sigma_v^2}$ , then it is decreasing in  $\nu$ , and vice versa. We now use the results from the proof of Proposition 3, collected in equation (A38). We normalize the expected profit,  $\tilde{\pi}_0^F = \frac{\pi_0^F}{\sigma_u \sigma_v}$ , and similarly all the other profits. For a variable  $X = f(b, g)$ , we study its dependence on  $b$  by computing  $\frac{dX}{db} = \frac{\partial f}{\partial b} + \frac{\partial f}{\partial g} \frac{\partial g}{\partial b}$ , where equation (A53) gives  $\frac{\partial g}{\partial b} = -\frac{g(1+g)(2+bg+bg^2)}{(1-g)(2+b+3bg)}$ .

If  $X = (\tilde{\pi}_\gamma^F)^2 = \frac{g(1+b+bg)^2}{(2+b+bg)^2}$ , then  $\frac{dX}{db} = -\frac{g(1+g)(1+b+bg)(2b+(2+3b+b^2)g+b(3+2b)g^2+b^2g^3)}{(1-g)(2+b+bg)^2(2+b+3bg)} < 0$ . Hence,  $\pi_\gamma^F$  is increasing in  $\nu$ .

In the proof of Proposition 3, we have seen that  $\pi_\beta^F = \beta_0^F \Sigma_0$ . Hence,  $\pi_\beta^F$  depends on  $\nu$  in the same way as  $\beta_0^F$ . But in Proposition 6, we have already shown that  $\beta_0^F$  is decreasing in  $\nu$ .

If  $X = \frac{1}{(\tilde{\pi}_0^F)^2} = g(2+b+bg)^2$ , then  $\frac{dX}{db} = -\frac{g^2(1+g)(2+b+bg)^2}{1-g} < 0$ . Thus,  $X$  is increasing in  $\nu$ , and  $\pi_0^F$  is decreasing in  $\nu$ .

If  $X = \frac{\pi_\gamma^F}{\pi_0^F} = g(1+b+bg)$ , then  $\frac{dX}{db} = -\frac{g(1+g)(2+b+bg)(1+(1+b)g+2bg^2)}{(1-g)(2+b+3bg)} < 0$ . Thus,  $\frac{\pi_\gamma^F}{\pi_0^F}$  is increasing in  $\nu$ .

Finally,  $\lim_{\nu \rightarrow 0} \frac{\pi_\gamma^F}{\pi_0^F} = \lim_{\nu \rightarrow 0} g(1+b+bg) = \lim_{\nu \rightarrow 0} g^{1/2} \cdot \lim_{\nu \rightarrow 0} g^{1/2} b \cdot \lim_{\nu \rightarrow 0} (1 + \frac{1}{b} + g) = 0 \cdot \frac{1}{(1+c)^{1/2}} \cdot 1 = 0$ , where the last equality follows from the proof of Proposition 6.  $\square$

PROOF OF COROLLARY 9: We need to show that  $\pi_0^F - \pi_0^S$  is increasing in  $\nu = \frac{1}{\sigma_c}$ , or equivalently, that it is decreasing in  $b = \frac{\sigma_c^2}{\sigma_v^2}$ . Let  $X_1 = (\frac{\pi_0^F}{\sigma_u \sigma_v})^2$  and  $X_2 = (\frac{\pi_0^S}{\sigma_u \sigma_v})^2$ . We now prove that  $(X_1)^{1/2} - X_2^{1/2}$  is decreasing in  $b$ . This is equivalent to  $\frac{dX_1}{db} \frac{1}{X_1^{1/2}} < \frac{dX_2}{db} \frac{1}{X_2^{1/2}}$ . Since we prove below that  $\frac{dX_1}{db} > 0$  and  $\frac{dX_2}{db} > 0$ , it is enough to prove that  $(\frac{dX_1}{db})^2 \frac{1}{X_1} < (\frac{dX_2}{db})^2 \frac{1}{X_2}$ .  $\square$

Using (A38), we get  $X_1 = \frac{1}{g(2+b+bg)^2}$  and  $X_2 = c + \frac{b}{1+b}$ , where  $c = \frac{\sum_0}{\sigma_v^2}$ . Using the same method as in the proof of Corollary 8, we compute  $\frac{dX_1}{db} = \frac{1+g}{(1-g)(2+b+bg)^2}$ . Since  $c$  does not depend on  $b$ ,  $\frac{dX_2}{db} = \frac{1}{(1+b)^2}$ . Now we eliminate  $c$  from the formula for  $X_2$ : from equation (A33),  $c + 1 = \frac{(1+bg)(1+g)^2}{g(2+b+bg)^2}$ . Then,  $X_2 = c + 1 - \frac{1}{1+b} = \frac{(1-g)^2(1+b+bg)}{g(2+b+bg)^2(1+b)}$ . After some algebra,  $(\frac{dX_2}{db})^2 \frac{1}{X_2} - (\frac{dX_1}{db})^2 \frac{1}{X_1}$  is proportional to  $(2+b +$

$bg)^4 - (1 + g)^2(1 + b + bg)(1 + b)^3$ . (The constant is  $\frac{g(1-g^2)}{(1+b+bg)(2+b+bg)^2(1+b)^3}$ .) Hence, the proof is finished if we show that  $(2 + b + bg)^4 > (1 + g)^2(1 + b + bg)(1 + b)^3$ . This follows by multiplying the inequalities (i)  $2 + b + bg > 1 + b + bg$  and (ii)  $(2 + b + bg)^3 > (1 + g)^3(1 + b)^3 > (1 + g)^2(1 + b)^3$ , and hence it is enough to prove that  $2 + b + bg > (1 + g)(1 + b)$ . But this inequality is equivalent to  $1 > g$ , which finishes the proof.  $\square$

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### Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

**Appendix S1:** Internet Appendix.

