

Equilibrium Pricing and Trading Volume under Preference Uncertainty

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Information collection and processing in financial institutions is challenging. This can delay the observation by traders of the exact capital charges and constraints of their institution. During this delay, traders face preference uncertainty. In this context, we study optimal trading strategies and equilibrium prices in a continuous centralized market. We focus on liquidity shocks, during which preference uncertainty is likely to matter most. Preference uncertainty generates allocative inefficiency, but need not reduce prices. Progressively learning about preferences generate round-trip trades, which increase volume relative to the frictionless market. In a cross section of liquidity shocks, the initial price drop is positively correlated with total trading volume. Across traders, the number of round-trips is negatively correlated with trading profits and average inventory.

Keywords: Information processing, Trading volume, Liquidity shock, Preference uncertainty, Equilibrium pricing

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1. INTRODUCTION

Financial firms' investment choices are constrained by capital requirements and investment guidelines, as well as risk-exposure and position limits. To assess the bindingness and the cost of these constraints, so as to determine the corresponding constrained optimal investment position, each financial firm must collect and aggregate data from several trading desks and divisions. This is a difficult task, studied theoretically by Vayanos (2003), who analyses the challenges raised by the aggregation of risky positions within a financial firm subject to communication constraints. These challenges have also been emphasized by several regulators and consultants.¹

1. See Basel Committee on Banking Supervision (2009) and Ernst & Young (2012, p. 9): "Many firms face challenges extracting and aggregating appropriate data from multiple siloed systems, which translate into fragmented

Because data collection and aggregation is challenging, it takes time. For example, Ernst & Young (2012, p. 58) finds that “53% of [respondents in its study] aggregate counterparty exposure across business lines by end of day, 27% report it takes two days, and 20% report much longer processes”.² This delays the incorporation of relevant information into investment decisions, particularly in times of market stress.³

From a theoretical perspective, these stylized facts imply that, during the time it takes to reassess financial and regulatory constraints, financial firms’ traders make decisions under *preference uncertainty*.⁴ The goal of this article is to examine the consequences of such preference uncertainty for trading strategies, equilibrium pricing, and aggregate trading volume.

To do so, we focus on situations where the market is hit by an aggregate liquidity shock, reducing firms’ willingness and ability to hold assets (see Berndt *et al.*, 2005; Greenwood, 2005; Coval and Stafford, 2007). As mentioned above, it is in such times of stress that preference uncertainty is likely to be most severe. To cope with the shock, financial firms establish hedges, raise new capital, and adjust positions in several assets and contracts. This process is complex and lengthy, as it involves transactions conducted by different desks in different markets and also because it takes time to check it has been completed. During the corresponding data collection and processing period, there is uncertainty about the actual preferences of the firm.

To model this situation, we consider an infinite-horizon, continuous-time market for one asset. There is a continuum of infinitely lived, risk-neutral and competitive financial firms who derive a non-linear utility flow, denoted by $v(\theta, q)$, from holding q divisible shares of this asset, as in Gârleanu (2009) and Lagos and Rocheteau (2009). At the time of the liquidity shock, the utility flow parameter drops to θ_ℓ for some of the firms, as in Weill (2004, 2007) and Duffie *et al.* (2007). This drop reflects the increase in capital charges and the additional regulatory costs of holding the asset induced by the liquidity shock. Then, as time goes by, the firms hit by the shock progressively switch back to a high valuation, $\theta_h > \theta_\ell$. This switch occurs when a firm has successfully established the hedges and adjustments in capital and position necessary to absorb the liquidity shock and correspondingly recover a high valuation for the asset. To model this we assume each firm is associated with a Poisson process and switches back to high valuation at the first jump in this process. Furthermore, to model preference uncertainty we assume each firm is represented in the market by a trader who observes her firm’s current valuation for the asset, θ , at Poisson distributed “updating times”. Each firm is thus exposed to two Poisson processes: one jumps with its valuation for the asset, and the other jumps when its trader observes updated information about that valuation. For tractability, we assume that these processes are independent and independent across firms.

In this context, a trader does not continuously observe the utility flow generated for her firm by the position she takes. She, however, designs and implements the trading strategy that is optimal

management information on the degree of risk facing the organization”. Ernst & Young (2012, p. 20), however, also notes that financial firms use these data to assess their risk appetite and that “close to half [the respondents] (49%) report that stress testing results are significantly incorporated into risk management decision making”.

2. See also Ernst & Young (2012, p. 76): “The most prominent challenge is the sheer amount of time it takes to conduct stress testing [via] what is often a manual process of conducting test and gathering results across portfolios and businesses”. Similarly, the Institute for International Finance (2011, p. 50) mentions, some of the respondents to its study “say that their process lacks the capability to produce near-real-time and real-time reports on exposure and limit usage”.

3. As noted by Mehta *et al.* (2012, p. 7): “Most banks calculate economic capital on a daily (30%) or weekly (40%) basis, actively using it for risk steering and definition of limits in accordance with the risk appetite”. See also Mehta *et al.* (2012, p. 5): “Across all banks, the survey found that average Value-at-Risk run time ranges between 2 and 15 hours; in stressed environments, it can take much longer”.

4. By “preference uncertainty” we mean that traders view the utility function of their institution as a random variable, but we don’t use the word “uncertainty” in the Knightian sense.

for the firm, given her information. Thus, when a trader observes updated information about the preferences of her firm, she designs a new trading plan, specifying the process of her asset holdings until the next information update, based on rational expectations about future variables and decisions. At each point time, the corresponding demand from a trader is increasing in the probability that her firm has high valuation. Substituting demands in the market clearing condition gives rise to equilibrium prices. We show equilibrium existence and uniqueness. By the law of large numbers, the cross-sectional aggregate distribution of preferences, information sets and demands is deterministic, and so is the equilibrium price.⁵

Unconstrained efficiency would require that low-valuation traders sell to high-valuation traders. Such reallocation, however, is delayed by preference uncertainty. Some traders hold more shares than they would if they knew the exact current status of their firm, while others hold less shares. This does not necessarily translate into lower prices, however. Preference uncertainty has two effects on asset demand, going in opposite directions. On the one hand, demand increases because traders who currently have low valuation believe they may have a high valuation with positive probability. On the other hand, demand decreases because traders who currently have high valuation fear they may have low valuation. If the utility function is such that demand is concave in the probability that the firm has high valuation, the former effect dominates the latter, so that preference uncertainty actually increases prices. The opposite holds if asset demand is convex. We also analyse in closed form a specification where demand is neither globally concave nor convex and show that preference uncertainty may increase prices when the liquidity shock hits, but subsequently lowers them as the shock subsides.

In a static model, preference uncertainty would lead to lower trading volume, because it reduces the dispersion of valuations across traders. Indeed, $\mathbb{V}[\mathbb{E}(\theta|\mathcal{F})] < \mathbb{V}[\theta]$, where \mathcal{F} is the information set of a trader under preference uncertainty. The opposite can occur in our dynamic model, because trades arise due to *changes* in expected valuations, which happens more often with preference uncertainty than with known preferences. More precisely, when traders observe their firm still has low valuation, they sell a block of shares. Then, until the next updating time, they remain uncertain about the exact valuation of their firm. They anticipate, however, that it is more and more likely that their firm has emerged from the shock. Correspondingly, under natural conditions, they gradually buy back shares, which they may well sell back at their next updating time if they learn their firm still has low valuation. This generates round trips, and larger trading volume than when preferences are known. In some sense, preference uncertainty implies a *tâtonnement* process, by which the allocation of the asset progressively converges towards the efficient allocation. Successive corrections in this *tâtonnement* process generate excess volume relative to the known preference case. When the frequency of information updates increases, the size of the round-trip trades decreases, but their number becomes larger. We show that, as the frequency of information updates goes to infinity and preference uncertainty vanishes, the two effects balance out exactly, so that the excess volume converges to some non-zero limit.

The condition under which preference uncertainty raises prices is related to the condition for excess volume (more precisely the latter is necessary for the former.) Increased prices reflect the demand coming from traders who think their firm may have switched to high valuation, while in fact it still has low valuation. This demand also gives rise to the build-up of inventories, that are then unwound via a block sale when the trader observes her firm still has low valuation. And it is these round-trip trades that are at the origin of excess volume.

5. In Biais *et al.* (2012a), we analysed an extension of our framework where the market is subject to recurring aggregate liquidity shocks, occurring at Poisson arrival times. In Appendix B.5, we consider the case when the number of traders is finite and the law of large numbers no longer applies. While, in both extensions, the price becomes stochastic, the qualitative features of our equilibrium are upheld.

A natural measure of the magnitude of the liquidity shock is the fraction of traders initially hit. As this fraction increases, both the initial price drop and total trading volume increase. Thus, one empirical implication of our analysis is that, in a cross section of liquidity shocks, the magnitude of the initial price drop should be positively correlated with the total trading volume following the shock. Our theoretical analysis also generates implications for the cross section of traders in a given liquidity shock episode. A trader whose institution recovers rapidly holds large inventories, and makes only a few round trips. In contrast, a trader whose institution recovers late engages in many successive round trips. Correspondingly she holds inventory during short periods of time. Furthermore, the traders whose institutions recover late earn low trading profits since they buy late, at high prices. Thus, our analysis implies negative correlation, across traders, between the number of round trips and trading profits, and positive correlation between average inventories and trading profits. The latter correlation can be interpreted in terms of reward to liquidity supply: Traders with low-valuation demand liquidity. Traders accommodating this demand hold inventories at a profit.

Our assumption that institutions are unable to collect and process all information instantaneously is in the spirit of the rational inattention literature, which emphasizes limits to information processing (see, e.g. Lynch, 1996; Reis, 2006a; 2006b, Mankiw and Reis; 2002; Gabaix and Laibson, 2002; Alvarez *et al.*, 2011; Alvarez *et al.*, 2010.) Our analysis complements theirs by focusing on a different object: financial institutions and traders during liquidity shocks.

Much of our formalism builds on search models of over-the-counter (OTC) markets, such as those of Duffie *et al.* (2005), Weill (2007), Gârleanu (2009), Lagos and Rocheteau (2009), Lagos *et al.* (2011), and Pagnotta and Philippon (2011). In particular, we follow these models in assuming that investors' valuations change randomly. That being said, the centralized, continuous, limit order market we consider is very different from the fragmented dealer market they consider. Correspondingly, what happens between times at which our traders observe their firm's valuation differs from what happens in search models of OTC markets between times at which traders contact the market. During this time interval, in our approach, traders engage in active trading strategies, while they stay put in OTC markets. Hence in our analysis, the friction can lead to excess volume, while in the above models of OTC markets, the friction reduces trading volume.⁶

One of the major implications of our theoretical analysis is that each trader will generally engage in several consecutive round trips. The round trips arising in our centralized market are different, however, from those arising in dealer markets.⁷ Dealers aim, after a sequence of round-trip trades, to hold zero net inventory position and, in turning over their position, earn the realized bid-ask spread. This logic, underlying Grossman and Miller (1988), stands in contrast with that in our model where there are no designated dealers and round trips are not motivated by the desire to move back to an ideal zero net position.

Gromb and Vayanos (2002, 2010) and Brunnermeier and Pedersen (2009) also study liquidity shocks in markets with frictions. They consider traders' funding constraints, while we consider information processing constraints. In contrast with these papers, in our framework frictions do not necessarily amplify the initial price drop and can increase trading volume.

6. The excess volume induced by preference uncertainty also contrasts with the reduction in volume generated by Knightian uncertainty in Easley and O'Hara (2010).

7. They also differ from those arising in fragmented OTC markets such as those analysed by Afonso and Lagos (2011), Atkeson *et al.* (2012), and Babus and Kondor (2012). In these models, excess volume arises because all trades are bilateral, which give investors incentives to provide immediacy to each other, buying from those with lower valuation than them, and then selling to those with higher valuation. In our model, excess volume arises even though all trades occur in a centralized market.

The consequences of informational frictions in our analysis vastly differ from those of asymmetric information on common values. The latter create a “speculative” motive for trade. With rational traders, however, this does not increase trading volume, but reduces it, due to adverse selection (see Akerlof, 1970).⁸

The next section presents our model. Section 3 presents the equilibrium. The implications of our analysis are outlined in Section 4. Section 5 briefly concludes. The main proofs are in the appendix. A Supplementary Appendix collects the proofs omitted in the article. It also discusses a model in which institutions choose their information collection effort as well as what happens with a finite number of traders.

2. MODEL

2.1. *Assets and agents*

Time is continuous and runs forever. A probability space (Ω, \mathcal{F}, P) is fixed, as well as an information filtration satisfying the usual conditions (Protter, 1990).⁹ There is an asset in positive supply $s > 0$ exchanged in a centralized continuous market. The economy is populated by a $[0, 1]$ -continuum of infinitely-lived financial firms (banks, funds, insurers, etc...) discounting the future at the same rate $r > 0$.

Financial firms can either be in a high valuation state, θ_h , or in a low-valuation state, θ_ℓ . The firm’s utility flow from holding q units of the asset in state $\theta \in \{\theta_\ell, \theta_h\}$ is denoted by $v(\theta, q)$, and satisfies the following conditions. First, utilities are strictly increasing and strictly concave in q , and they satisfy

$$v_q(\theta_\ell, q) < v_q(\theta_h, q),$$

for all $q > 0$. That is, low-valuation firms have lower marginal utility than high-valuation firms and, correspondingly, demand less assets.¹⁰ Second, in order to apply differential arguments, we assume that, for both $\theta \in \{\theta_\ell, \theta_h\}$, $v(\theta, q)$ is three times continuously differentiable in $q > 0$ and satisfies the Inada conditions $v_q(\theta, 0) = +\infty$ and $v_q(\theta, \infty) = 0$. Finally, firms can produce (or consume) a non-storable numéraire good at constant marginal cost (utility) normalized to 1.

2.2. *Liquidity shock*

To model liquidity shocks we follow Weill (2004, 2007) and Duffie *et al.* (2007). All financial firms are *ex ante* identical: before the shock, each firm is in the high-valuation state, $\theta = \theta_h$, and holds s shares of the asset. At time zero, the liquidity shock hits a fraction $1 - \mu_{h,0}$ of financial firms, who make a switch to low valuation, $\theta = \theta_\ell$. The switch from $\theta = \theta_h$ to $\theta = \theta_\ell$ induces a drop in utility flow, reflecting the increase in capital charges and additional regulatory costs of holding the asset induced by the liquidity shock. The shock, however, is transient. In practice firms can respond to liquidity shocks by hedging their positions, adjusting them, and raising capital. Once

8. See Appendix B.3 for a formal argument. Of course the effect of adverse selection disappears if uninformed traders are noise traders. Noise traders do not optimize so, by assumption, never worry about adverse selection.

9. To simplify the exposition, for most stated equalities or inequalities between stochastic processes, we suppress the “almost surely” qualifier as well as the corresponding product measure over times and events.

10. In order for the asset demand of low-valuation firms to be lower than that of high valuation, we only need to rank marginal utilities, not utilities. If utilities are bounded below, however, we can without loss of generality assume that $v(\theta, 0) = 0$, so that the ranking of marginal utilities implies that of utilities, that is, $v(\theta_\ell, q) < v(\theta_h, q)$ for all $q > 0$.

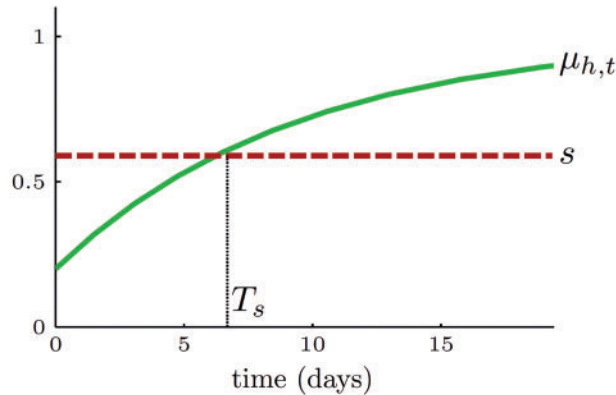


FIGURE 1

The measure of high-valuation institutions (plain green) and the asset supply (dashed red).

they have completed this process successfully, they recover from the shock and switch back to a high valuation for the asset. To model this, we assume that, for each firm, there is a random time at which it reverts to the high-valuation state, $\theta = \theta_h$, and then remains there forever. For simplicity, we assume that recovery times are exponentially distributed, with parameter γ , and independent across firms. Denote by $\mu_{h,t}$ the fraction of financial firms with high valuation at time t . By the law of large numbers, the flow of firms who become high valuation at time t is equal to¹¹ $\mu'_{h,t} = \gamma(1 - \mu_{h,t})$, implying that:

$$1 - \mu_{h,t} = (1 - \mu_{h,0}) e^{-\gamma t}, \quad (1)$$

as illustrated in Figure 1. Because there is no aggregate uncertainty, in all what follow we will focus on equilibria in which aggregate outcomes (price, allocation, etc...) are deterministic functions of time.

2.3. Preference uncertainty

Each firm is represented in the market by one trader. As discussed above, the process by which the firm recovers from the liquidity shock is complex. As a result, it takes time to collect and analyse the data about this process. To model the corresponding delay, we assume a trader does not observe the preference parameter θ of her firm in continuous time. Instead, for each trader, there is a counting process, N_t , such that she updates her information about θ at each jump of N_t . Updating occurs according to a Poisson process with intensity ρ . Thus times between updates are

11. For simplicity and brevity, we do not formally prove how the law of large numbers applies to our context. To establish the result precisely, one would have to follow Sun (2006), who relies on constructing an appropriate measure for the product of the agent space and the event space. Another foundation for this law of motion is provided by Bélanger and Giroux (2012) who study aggregate population dynamics with a large but finite number of agents.

exponentially distributed.¹² For simplicity, the different traders' processes are independent from each other and from everything else.¹³

2.4. Holding plans and intertemporal utilities

When a trader observes the current value of her θ at some time $t > 0$, she designs a new asset holding plan, $q_{t,u}$, for all subsequent times $u \geq t$ until her next updating time, at which point she designs a new holding plan, and so on. Each holding plan is implemented, in our centralized market, by the placement and updating of sequences of limit orders. At each point in time, the collection of the current limit orders of a trader determines her demand function.

Formally, letting $\mathcal{T} = \{(t, u) \in \mathbb{R}_+^2 : t \leq u\}$, the collection of *asset holding plans* is a stochastic process

$$q : \mathcal{T} \times \Omega \rightarrow \mathbb{R}_+$$

$$(t, u, \omega) \mapsto q_{t,u}(\omega),$$

which is adapted with respect to the filtration generated by θ_t and N_t . That is, a trader's asset holdings at time u can only depend on the information she received until time t , her last updating time: the history of her updating and valuation processes up to time t . Note that, given that there is no aggregate uncertainty and given our focus on equilibria with deterministic aggregate outcomes, we do not need to make the holding plan contingent on any aggregate information such as, *e.g.* the market price, since the later is a deterministic function of u . We impose, in addition, mild technical conditions ensuring that intertemporal values and costs of the holding plan are well defined: we assume that a trader must choose holding plans which are bounded, have bounded variation with respect to u for any t , and which generate absolutely integrable discounted utility flows. In all what follow, we will say that a holding plan is admissible if it satisfies these measurability and regularity conditions.

Now consider a trader's intertemporal utility. For any time $u \geq 0$, let τ_u denote the last updating time before u , with the convention that $\tau_u = 0$ if there has been no updating on θ since time zero. Note that τ_u has an atom of mass $e^{-\rho u}$ at $\tau_u = 0$, and a density $\rho e^{-\rho(u-t)}$ for $t \in (0, u]$.¹⁴ At time u , the trader follows the plan she designed at time τ_u , so she holds a quantity $q_{\tau_u,u}$ of assets. Thus, the trader's *ex ante* intertemporal utility can be written:

$$V(q) = \mathbb{E} \left[\int_0^\infty e^{-ru} v(\theta_u, q_{\tau_u,u}) du \right], \tag{2}$$

where the expectation is taken over θ_u and τ_u . Next, consider the intertemporal cost of buying and selling assets. During $[u, u + du]$, the trader follows the plan chosen at time τ_u , which prescribes

12. A possible foundation for this assumption is the following. The risk-management unit must evaluate the firm's position in N dimensions and sends the aggregate result when all N assessments have been conducted. Denote by T_n the time it takes to conduct the evaluation of position n . The delay after which the risk-management unit will inform the trader of the aggregate result is equal to $\max\{T_1, \dots, T_N\}$. Assuming the T_n are i.i.d. with cdf $F(t) = (1 - e^{-\rho t})^{1/N}$, we have that $\max\{T_1, \dots, T_N\}$ is exponentially distributed with parameter ρ .

13. To simplify notations, we do not index the different processes by trader-specific subscripts. Rather we use the same generic notation, " N_t ", for all traders.

14. To see this, note that for any $t \in [0, u]$, the probability that $\tau_u \leq t$ is equal to the probability that there has been no updating time during $(t, u]$, *i.e.* to the probability of the event $N_u - N_t = 0$. Since the counting process for updating times follows a Poisson distribution, this probability is equal to $e^{-\rho(u-t)}$.

that her holdings must change by $dq_{\tau_u, u}$. Denoting the price at time u by p_u , the cost of buying or selling asset during $[u, u + du]$ is, then, $p_u dq_{\tau_u, u}$. Therefore, the *ex ante* intertemporal cost of buying and selling assets writes

$$C(q) = \mathbb{E} \left[\int_0^\infty e^{-ru} p_u dq_{\tau_u, u} \right], \quad (3)$$

which is well defined under natural regularity conditions about p_u .¹⁵

2.5. Market clearing

The market clearing condition requires that, at each date $u \geq 0$, aggregate asset holdings be equal to s , the asset supply. In our mass-one continuum setting, aggregate asset holdings are equal to the cross-sectional average asset holding. Moreover, by the law of large numbers, and given *ex ante* identical traders, the cross-sectional average asset holding is equal to the expected asset holding of a representative trader. Hence, the market clearing condition at time u can be written:

$$\mathbb{E}[q_{\tau_u, u}] = s \quad (4)$$

for all $u \geq 0$, where the expectation is taken with respect to τ_u and to θ_{τ_u} , reflecting the aggregation of asset demands over a population of traders with heterogeneous updating times and uncertain preferences.

3. EQUILIBRIUM

An *equilibrium* is made up of an admissible holding plan q and of a price path p such that: (i) given the price path p the holding plan q maximizes the intertemporal net utility $V(q) - C(q)$, where $V(q)$ and $C(q)$ are given by (2) and (3) and (ii) the optimal holding plan is such that the market clearing condition (4) holds at all times. In this subsection we characterize the demands of traders for any given price path and then, substituting demands in the market-clearing condition, we show existence and uniqueness of equilibrium. We conclude the section by establishing that this equilibrium is socially optimal.

3.1. Asset demands

Focusing on equilibria in which the price path is deterministic, bounded, and continuously differentiable,¹⁶ we define the holding cost of the asset at time u :

$$\xi_u = rp_u - \dot{p}_u, \quad (5)$$

which is equal to the cost of buying a share of the asset at time u and reselling it at $u + du$, i.e., the time value of money, rp_u , minus the capital gain, \dot{p}_u .

15. For example, it will be well defined in the equilibrium we study, where p_u is deterministic and continuous.

16. As argued above, deterministic price paths are natural given the absence of aggregate uncertainty. Further, in the environment that we consider, one can show that the equilibrium price must be continuous (see Biais *et al.*, 2012b). The economic intuition is as follows. If the price jumps at time t , all traders who receive an updating opportunity shortly before t would want to “arbitrage” the jump: they would find it optimal to buy an infinite quantity of assets and re-sell these assets just after the jump. This would contradict market clearing.

Lemma 1. *A trader’s intertemporal net utility can be written:*

$$V(q) - C(q) = p_0 s + \mathbb{E} \left[\int_0^\infty e^{-ru} \left\{ \mathbb{E}[v(\theta_u, q_{\tau_u, u}) | \mathcal{F}_{\tau_u}] - \xi_u q_{\tau_u, u} \right\} du \right]. \tag{6}$$

At time τ_u , her most recent updating time before time u , the trader received information θ_{τ_u} about her valuation, and she chose the holding plan $q_{\tau_u, u}$. Thus, she expects to derive utility $\mathbb{E}[v(\theta_u, q_{\tau_u, u}) | \mathcal{F}_{\tau_u}]$ at time u , and to incur the opportunity cost $\xi_u q_{\tau_u, u}$.

Lemma 1 implies that an optimal holding plan can be found via optimization at each information set. Formally, a trader’s optimal asset holding at time u solves:

$$q_{\tau_u, u} = \arg \max_q \mathbb{E}[v(\theta_u, q) | \mathcal{F}_{\tau_u}] - \xi_u q.$$

Let $\pi_{\tau_u, u}$ denote the probability that $\theta_u = \theta_h$ given the value of θ_{τ_u} observed at t . The trader’s problem can be rewritten as

$$q_{\tau_u, u} = \arg \max_q \pi_{\tau_u, u} v(\theta_h, q) + (1 - \pi_{\tau_u, u}) v(\theta_\ell, q) - \xi_u q,$$

so the first-order necessary and sufficient condition is:

$$\pi_{\tau_u, u} v_q(\theta_h, q_{\tau_u, u}) + (1 - \pi_{\tau_u, u}) v_q(\theta_\ell, q_{\tau_u, u}) = \xi_u. \tag{7}$$

This equation means that each trader’s expected marginal utility of holding the asset during $[u, u + du]$ is equal to the opportunity cost of holding the asset during that infinitesimal time interval. It implies the standard equilibrium condition that marginal utilities are equalized across traders. Analysing the first-order condition (7), we obtain the following lemma:

Lemma 2. *There exists a unique solution to (7), which is a function of ξ_u and $\pi_{\tau_u, u}$ only, and which we correspondingly denote by $D(\pi, \xi)$. The function $D(\pi, \xi)$ is strictly increasing in π and strictly decreasing in ξ , is twice continuously differentiable in (π, ξ) , and it satisfies $\lim_{\xi \rightarrow 0} D(0, \xi) = \infty$ and $\lim_{\xi \rightarrow \infty} D(1, \xi) = 0$.*

Note that a trader’s demand is increasing in the probability of being a high valuation, π . This follows directly from the assumption that high-valuation traders have higher marginal utility than low valuation.

3.2. Existence and uniqueness

Consider some time $u \geq 0$ and a trader whose most recent updating time is τ_u . If $\theta_{\tau_u} = \theta_h$, then the trader knows for sure that $\theta_u = \theta_h$ and so she demands $D(1, \xi_u)$ units of the asset at time u . If $\theta_{\tau_u} = \theta_\ell$, then

$$\pi_{\tau_u, u} \equiv \frac{\mu_{h, u} - \mu_{h, \tau_u}}{1 - \mu_{h, \tau_u}} = 1 - e^{-\gamma(u - \tau_u)}, \tag{8}$$

and her demand is $D(\pi_{\tau_u, u}, \xi_u)$. Therefore, the market-clearing condition (4) writes:

$$\mathbb{E} \left[\mu_{h, \tau_u} D(1, \xi_u) + (1 - \mu_{h, \tau_u}) D(\pi_{\tau_u, u}, \xi_u) \right] = s. \tag{9}$$

The first term in the expectation represents the aggregate demand of high-valuation traders, i.e., traders who discovered at their last updating time, τ_u , that their firm had a high valuation, $\theta_{\tau_u} = \theta_h$. Likewise, the second term represents the demand of low-valuation traders.

Note that aggregate demand, on the left-hand side of (9), inherits the properties of $D(\pi, \xi)$: it is continuous in (π, ξ) , strictly decreasing in ξ , goes to infinity when $\xi \rightarrow 0$, and to zero when $\xi \rightarrow \infty$. Thus, this equation has a unique solution, ξ_u , which is easily shown to be a bounded and continuous function of time, u . The equilibrium price is obtained as the present discounted value of future holding costs:

$$p_t = \int_t^\infty e^{-r(u-t)} \xi_u du.$$

While the holding cost ξ_u measures the cost of buying the asset at u net of the benefit of reselling at $u + du$, the price p_t measures the cost of buying at t and holding until the end of time. Taking stock:

Proposition 1. *There exists a unique equilibrium. The holding cost at time u is the unique solution of (9), and is bounded and continuous. The asset holding of a time- τ_u high-valuation trader is $q_{h,u} \equiv D(1, \xi_u)$, and the asset holding of a time- τ_u low-valuation trader is $q_{\ell, \tau_u, u} \equiv D(\pi_{\tau_u, u}, \xi_u)$.*

3.3. Constrained efficiency

To study constrained efficiency we define a collection of holding plans to be *feasible* if it is admissible and if it satisfies the resource constraint, which is equivalent to the market-clearing condition (4). Furthermore, we say that a collection of holding plans, q , *Pareto dominates* some other collection of holding plans, q' , if it is possible to generate a Pareto improvement by switching from q' to q while making time zero transfers among investors. Because utilities are quasi linear, q Pareto dominates q' if and only if $V(q) > V(q')$.

Proposition 2. *The holding plan arising in the equilibrium characterized in Proposition 1 is the unique maximizer of $V(q)$ among all feasible holding plans.*

The proposition reflects that, in our setup, there are no externalities. To assess the robustness of this welfare theorem, we consider (in Supplementary Appendix B.4) a simple variant of the model, with three stages: *ex ante* banks choose how much effort to exert, to increase the precision of the information signal about their own type, *interim* banks receive their signal and trade in a centralized market, *ex post* banks discover their types and payoffs realize. In this context, again, we find that the equilibrium is constrained Pareto efficient: both the choice of effort and the allocation coincide with the one that a social planner would choose.

4. IMPLICATIONS

4.1. Known preferences

To understand the implications of preference uncertainty, we first need to consider the benchmark in which traders continuously observe their valuation, θ_u . In that case, the last updating time is equal to the current time, $\tau_u = u$, and the market clearing condition (9) becomes:

$$\mu_{h,u} D(1, \xi_u^*) + (1 - \mu_{h,u}) D(0, \xi_u^*) = s. \quad (10)$$

Clearly, since $\mu_{h,u}$ is increasing and since $D(1, \xi) > D(0, \xi)$ for all ξ , it must be the case that both ξ_u^* and the price

$$p_t^* = \int_t^\infty e^{-r(u-t)} \xi_u^* du,$$

increase over time. Correspondingly, demand of high- and low-valuation traders, given by $D(1, \xi_u^*)$ and $D(0, \xi_u^*)$, must be decreasing over time.

The intuition is the following. High-valuation traders are more willing to hold the asset than low-valuation traders. In a sense, the high-valuation traders absorb the asset that low-valuation traders are not willing to hold, which can be interpreted as liquidity supply. As time goes by, the mass of traders with high valuation goes up. Thus, the amount of asset each individual trader of a given valuation needs to hold goes down. Correspondingly, traders' marginal valuation for the asset increases, and so does the price.

While this increase in price is predictable, it does not generate arbitrage opportunities. It just reflects the dynamics of the optimal allocation of the asset in a context where traders' willingness to hold the asset is finite. Such finite willingness to hold the asset arises from the concavity of the utility function, which can be interpreted as a proxy for risk aversion, or for various "limit to arbitrage" frictions. While they expect the price to be higher in the future, investors are not willing to buy more of it now, because that would entail an opportunity cost of holding the asset (ξ_u) greater than their marginal valuation for the asset.

4.2. *The impact of preference uncertainty on price*

Let ξ_u^* be the opportunity cost of holding the asset when preferences are known. Starting from this benchmark, what would be the effect of preference uncertainty? Would it raise asset demand and asset prices?

A global condition for higher demand and higher prices: Given any holding cost ξ , preference uncertainty increases asset demand at time u , relative to the case of known preferences, if and only if:

$$\mathbb{E} \left[\mu_{h,\tau_u} D(1, \xi) + (1 - \mu_{h,\tau_u}) D(\pi_{\tau_u,u}, \xi) \right] > \mu_{h,u} D(1, \xi) + (1 - \mu_{h,u}) D(0, \xi). \quad (11)$$

The left-hand side is the aggregate demand at time u under preference uncertainty, and the right-hand side is the aggregate demand under known preferences. Using the definition of $\pi_{\tau_u,u}$, this inequality can be rearranged into:

$$\mathbb{E} \left[(1 - \mu_{h,\tau_u}) D(\pi_{\tau_u}, \xi) \right] > \mathbb{E} \left[(1 - \mu_{h,\tau_u}) \left\{ \pi_{\tau_u,u} D(1, \xi) + (1 - \pi_{\tau_u,u}) D(0, \xi) \right\} \right]. \quad (12)$$

To interpret this inequality, note first that preference uncertainty has no impact on the demand of high-valuation time- τ_u traders, because they know for sure that they will keep a high valuation forever after τ_u . Thus, preference uncertainty only has an impact on the demand of the measure $1 - \mu_{h,\tau_u}$ of time- τ_u low-valuation traders. The left-hand side of (12) is the time- u demand of these time- τ_u low-valuation traders, under preference uncertainty. The right-hand side is the demand of these same traders, but under known preferences.

Equation (12) reveals that preference uncertainty has two effects on asset demand, going in opposite directions. With known preferences, a fraction $\pi_{\tau_u,u}$ of time- τ_u low-valuation traders

would have known for sure that they had a high valuation at time u : preference uncertainty *decreases* their demand, from $D(1, \xi)$ to $D(\pi_{\tau_u, u}, \xi)$. But the complementary fraction, $1 - \pi_{\tau_u, u}$, would have known for sure that they had a low valuation at time u : preference uncertainty *increases* their demand, from $D(0, \xi)$ to $D(\pi_{\tau_u, u}, \xi)$.

Clearly one sees that inequality (12) holds for all u and τ_u if demand $D(\pi, \xi)$ is strictly concave in π .

Proposition 3. *If $D(\pi, \xi)$ is strictly concave in π for all ξ , then the holding cost and the price are strictly larger with preference uncertainty than with known preferences.*

Concavity implies that, under preference uncertainty, the increase in demand of low-valuation traders dominates the decrease for high-valuation traders.¹⁷ To illustrate the proposition, consider the iso-elastic utility:

$$v(\theta, q) = \theta \frac{q^{1-\sigma} - 1}{1-\sigma}, \quad (13)$$

as in Lagos and Rocheteau (2009, Propositions 5 and 6). Then, after calculating demand, one sees that preference uncertainty increases prices if $\sigma > 1$, and decreases prices when $\sigma < 1$.

Note that the global concavity condition of Proposition 3 does not hold in other cases of interest. In particular, preferences in the spirit of Duffie *et al.* (2005) tend to generate demands that are locally convex for low π and locally concave for high π .¹⁸ To address such cases, we now develop a less demanding local condition which applies when $u \simeq 0$, *i.e.* just after the liquidity shock, or for all u when $\rho \rightarrow \infty$, *i.e.* when traders face small preference uncertainty.

A local condition for higher prices when $u \simeq 0$: Heuristically, just after the liquidity shock, $\tau_u = 0$ for most of the population, so we only need to study (12) for $\tau_u = 0$. Moreover, low-valuation traders only had a short time to switch to a high type, and so the probability $\pi_{0, u}$ is close to zero. Because $\pi_{0, u} \simeq \frac{\mu'_{h, 0}}{1 - \mu_{h, 0}} \times u \simeq 0$ and demand is differentiable, we can make a first-order Taylor expansion of (12) in $\pi_{0, u}$, and the condition for higher holding cost becomes:

Proposition 4. *When $u > 0$ is small, time- u demand is larger under preference uncertainty, and so is the equilibrium holding cost ξ_u if:*

$$D_\pi(0, \xi_0^*) > D(1, \xi_0^*) - D(0, \xi_0^*). \quad (14)$$

where ξ_0^* is the time-zero holding cost with known preferences.

17. This result is in line with those previously derived by Gârleanu (2009) and Lagos and Rocheteau (2009) in the context of OTC markets. In these papers, traders face a form of preference uncertainty, because they are uncertain about their stochastic utility flows in between two contact times with dealers. When OTC market frictions increase, inter-contact times are larger, and so is preference uncertainty. In line with Proposition 3, in Gârleanu (2009) the friction does not affect asset prices when the asset demand is linear in agent's type. Also in line with Proposition 3, in Lagos and Rocheteau (2009) making the friction more severe increases the price if the utility function is sufficiently concave.

18. In Duffie *et al.* (2005), preferences are of the form $\theta \min\{1, q\}$. Then, when $\xi \in (\theta_\ell, \theta_h)$, demand is a step function of π : it is zero when $\pi\theta_h + (1-\pi)\theta_\ell < \xi$, equal to $[0, 1]$ when $\pi\theta_h + (1-\pi)\theta_\ell = \xi$, and equal to one when $\pi\theta_h + (1-\pi)\theta_\ell > \xi$. With a smooth approximation of $\min\{1, q\}$, demand becomes a smooth approximation of this step function. It tends to be convex for small π , rises rapidly when $\pi\theta_h + (1-\pi)\theta_\ell \simeq \xi$, and becomes concave for large π . In Section 4.4, we offer a detailed study of equilibrium in a specification where asset demands are neither globally concave nor convex in π .

The left-hand side of (14) represents the per-capita increase in demand for low types, and the right-hand side the per-capita decrease in demand for high types. The proposition shows that the holding cost is higher under preference uncertainty if: (i) $D_\pi(0, \xi)$ is large, i.e., low types react very strongly to changes in their expected valuation and (ii) $D(1, \xi) - D(0, \xi)$ is bounded, i.e. high types do not demand too much relative to low types.

Note that the demand shift of low and high valuations are driven by different considerations. For the large population of low types, what matters is a change at the intensive margin: by how much each low type changes its demand in response to a small change in expected valuation, which is approximately equal to $\pi_{0,u} D_\pi(0, \xi_0^*)$. For the small population of high types, on the other hand, what matters is a change at the extensive margin: by how much demand changes because a small fraction of high types turn into low types, which is approximately equal to $\pi_{0,u} [D(1, \xi) - D(0, \xi)]$. This distinction will be especially important in our discussion of trading volume, because we can have very large intensive margin changes even if holdings are bounded.

A local condition for higher prices when $\rho \rightarrow \infty$: When $\rho \rightarrow \infty$, most traders had their last updating time shortly before u , approximately at $\tau_u = u - \frac{1}{\rho}$. This is intuitively similar to the situation analysed in the previous paragraph: when $u \simeq 0$, all traders had their last updating time shortly before u as well, at $\tau_u = 0$. Going through the same analysis, which can be thought of heuristically as using $\pi_{u - \frac{1}{\rho}, u}$ instead of $\pi_{0,u}$, we arrive at:

Proposition 5. *For all $u > 0$ and for ρ large enough, time- u demand is larger under preference uncertainty, and so is the equilibrium holding cost if:*

$$D_\pi(0, \xi_u^*) > D(1, \xi_u^*) - D(0, \xi_u^*), \quad (15)$$

where ξ_u^* is the time- u holding cost with known preferences. Moreover, the holding cost admits the first-order approximation:

$$\xi_u(\rho) = \xi_u^* - \frac{\mu'_{h,u}}{\rho} \frac{D_\pi(0, \xi_u^*) - [D(1, \xi_u^*) - D(0, \xi_u^*)]}{\mu_{h,u} D_\xi(1, \xi_u^*) + (1 - \mu_{h,u}) D_\xi(0, \xi_u^*)} + o_\alpha\left(\frac{1}{\rho}\right),$$

where $o_\alpha(1/\rho)$ is a function such that $\sup_{u \geq \alpha} |\rho o_\alpha(1/\rho)| \rightarrow 0$ as $\rho \rightarrow \infty$, for any $\alpha > 0$.

Condition (15) follows heuristically by replacing ξ_0^* by ξ_u^* in condition (14). To interpret the approximation formula, we first observe that, to a first-order approximation, the extra demand at time u induced by preference uncertainty can be written:

$$\frac{\mu'_{h,u}}{\rho} \left\{ D_\pi(0, \xi_u^*) - [D(1, \xi_u^*) - D(0, \xi_u^*)] \right\} \quad (16)$$

by taking a first-order Taylor approximation of the difference between the left-hand side and the right-hand side of (12), for $\tau_u \simeq u - \frac{1}{\rho}$. Thus, the holding cost has to move by an amount equal to this extra demand, in equation (16), divided by the negative of the slope of the demand curve, $\mu_{h,u} D_\xi(1, \xi_u^*) + (1 - \mu_{h,u}) D_\xi(0, \xi_u^*)$.

4.3 The impact of preference uncertainty on volume

We now turn from the effects of preference uncertainty on prices to its consequences for trading volume. Does preference uncertainty increase or reduce volume? To study this question, we focus

on the case where preference uncertainty is least likely to affect volume, as the friction is very small, *i.e.* it takes only a very less amount of time for traders to find out exactly what the objective of the financial firm is. To do so, we study the limit of the trading volume as ρ goes to infinity. One could expect that, as the friction vanishes, trading volume goes to its frictionless counterpart. We will show, however, that it is not the case, and we will offer an economic interpretation for that wedge.

If the opportunity holding cost were equal to ξ_u^* (which is the price prevailing in the benchmark case in which preferences are known) then traders' holding plans would be:

$$q_{\ell,t,u}^* \equiv D(\pi_{t,u}, \xi_u^*) \quad \text{and} \quad q_{h,u}^* \equiv D(1, \xi_u^*).$$

Thus, when low-valuation traders know their preferences with certainty, they hold $q_{\ell,u,u}^* = D(0, \xi_u^*)$ at all times. With these notations, (twice) the instantaneous trading volume is:¹⁹

$$2V^* = \mu_{h,u} \left| \frac{dq_{h,u}^*}{du} \right| + (1 - \mu_{h,u}) \left| \frac{dq_{\ell,u,u}^*}{du} \right| + \mu'_{h,u} |q_{h,u}^* - q_{\ell,u,u}^*|. \quad (17)$$

The first and second terms account for the flow sale of high- and low-valuation traders. The last term accounts for the lumpy purchases of the flow $\mu'_{h,u}$ of traders who switch from low to high valuation.

With preference uncertainty, (twice) the instantaneous trading volume is

$$2V = \mathbb{E} \left[\mu_{h,\tau_u} \left| \frac{dq_{h,u}}{du} \right| + (1 - \mu_{h,\tau_u}) \left| \frac{\partial q_{\ell,\tau_u,u}}{\partial u} \right| \right. \\ \left. + \rho(1 - \mu_{h,\tau_u}) \left\{ \pi_{\tau_u,u} |q_{h,u} - q_{\ell,\tau_u,u}| + (1 - \pi_{\tau_u,u}) |q_{\ell,u,u} - q_{\ell,\tau_u,u}| \right\} \right], \quad (18)$$

where the expectation, taken over τ_u , reflects the aggregation of trades over a population of agents with heterogeneous updating times.

The terms of the first line of equation (18) represent the flow trades of the traders who do not update their holding plans. The *partial* derivative with respect to u , $\frac{\partial q_{\ell,\tau_u,u}}{\partial u}$, reflects the fact that these traders follow a plan chosen at some earlier time, τ_u . In contrast, with known preferences, traders update their holding plans continuously so the corresponding term in equation (17) involves the *total* derivative, $\frac{dq_{\ell,u,u}^*}{du} = \frac{\partial q_{\ell,u,u}^*}{\partial t} + \frac{\partial q_{\ell,u,u}^*}{\partial u}$.

The terms on the second line of equation (18) represent the lumpy trades of the traders who update their holding plans. There is a flow $\rho(1 - \mu_{h,\tau_u})$ of time- τ_u low-valuation traders who update their holding plans. Out of this flow, a fraction $\pi_{\tau_u,u}$ find out that they have a high valuation, and make a lumpy adjustment to their holdings equal to $|q_{h,u} - q_{\ell,\tau_u,u}|$. The complementary fraction $1 - \pi_{\tau_u,u}$ find out they have a low valuation and make the adjustment $|q_{\ell,u,u} - q_{\ell,\tau_u,u}|$.

To compare the volume with known versus uncertain preferences, we consider the $\rho \rightarrow \infty$ limit. As shown formally in the Appendix, in order to evaluate this limit, we can replace $q_{h,u}$ and $q_{\ell,t,u}$ by their limits $q_{h,u}^*$ and $q_{\ell,t,u}^*$, and use the approximation $\tau_u \simeq u - \frac{1}{\rho}$. After a little algebra, we obtain that:

$$\lim_{\rho \rightarrow \infty} 2V = 2V^\infty = \mu_{h,u} \left| \frac{dq_{h,u}^*}{du} \right| + (1 - \mu_{h,u}) \left| \frac{\partial q_{\ell,u,u}^*}{\partial u} \right| + \mu'_{h,u} |q_{h,u}^* - q_{\ell,u,u}^*| + (1 - \mu_{h,u}) \left| \frac{\partial q_{\ell,u,u}^*}{\partial t} \right|.$$

19. Equation (17) gives *twice* the volume because it double counts each trade as a sale and a purchase.

Subtracting the volume with known preferences, $2V^*$, we obtain:

$$2V^\infty - 2V^* = (1 - \mu_{h,u}) \left\{ \left| \frac{\partial q_{\ell,u,u}^*}{\partial t} \right| + \left| \frac{\partial q_{\ell,u,u}^*}{\partial u} \right| - \left| \frac{dq_{\ell,u,u}^*}{du} \right| \right\}, \quad (19)$$

which is positive by the triangle inequality, and strictly so if the partial derivatives are of opposite sign.

To interpret these derivatives, note that under preference uncertainty, at time u some low-valuation traders receive the bad news that they still have a low valuation while others receive no news.

The traders who receive bad news at time u switch from the time- τ_u to the time- u holding plan. When $\tau_u \simeq u - \frac{1}{\rho}$, they change their holdings by an amount proportional to the partial derivatives with respect to t , $\frac{\partial q_{\ell,u,u}^*}{\partial t} < 0$. This derivative is negative, implying that these traders they sell upon receiving bad news.

The traders who receive no news trade the amount prescribed by their time- τ_u holding plan. When $\tau_u \simeq u - \frac{1}{\rho}$, this changes their holding by an amount proportional to the partial derivative with respect to u , $\frac{\partial q_{\ell,u,u}^*}{\partial u}$. Thus, when $\frac{\partial q_{\ell,u,u}^*}{\partial u} > 0$, traders with no news build up their inventories.

Under preference uncertainty, the changes in holdings due to $\frac{\partial q_{\ell,u,u}^*}{\partial t}$ and $\frac{\partial q_{\ell,u,u}^*}{\partial u}$ contribute separately to the trading volume, explaining the first two terms of (19). With known preferences, in contrast, all low-valuation traders are continuously aware that they have a low valuation, and so their holdings change by an amount equal to the total derivative. This explains the last term of (19).

In the time series, the above analysis implies that, when $\frac{\partial q_{\ell,u,u}^*}{\partial u} > 0$, low-valuation traders engage in round-trip trades. Consider a trader who finds out at two consecutive updating times, u and $u + \varepsilon$, that she has a low valuation. In between the two updating times, when ε is small, she builds up inventories since $\frac{\partial q_{\ell,u,u}^*}{\partial u} > 0$. At the updating time $u + \varepsilon$, she receives bad news, switches holding plan, and thus sells, since $\frac{\partial q_{\ell,u,u}^*}{\partial t} < 0$. Thus round trips arise only if $\frac{\partial q_{\ell,u,u}^*}{\partial u} > 0$. Correspondingly, the next proposition states that preference uncertainty generates excess volume when $\rho \rightarrow \infty$ if and only if $\frac{\partial q_{\ell,u,u}^*}{\partial u} > 0$.

Proposition 6. *As $\rho \rightarrow \infty$, the excess volume is equal to:*

$$V^\infty - V^* = (1 - \mu_{h,u}) \max \left\{ \frac{\partial q_{\ell,u,u}^*}{\partial u}, 0 \right\},$$

where $\frac{\partial q_{\ell,u,u}^*}{\partial u} > 0$ if and only if:

$$D_\pi(0, \xi_u^*) > [D(1, \xi_u^*) - D(0, \xi_u^*)] \frac{(1 - \mu_{h,u}) D_\xi(0, \xi_u^*)}{\mu_{h,u} D_\xi(1, \xi_u^*) + (1 - \mu_{h,u}) D_\xi(0, \xi_u^*)}. \quad (20)$$

To understand why a trader may increase her holding shortly after an updating time, *i.e.* why $\frac{\partial q_{\ell,u,u}^*}{\partial u} > 0$, note that:

$$(1 - \mu_{h,u}) \frac{\partial q_{\ell,u,u}^*}{\partial u} = (1 - \mu_{h,u}) D_\pi(0, \xi_u^*) \frac{\partial \pi_{t,u}}{\partial u} + (1 - \mu_{h,u}) D_\xi(0, \xi_u^*) \frac{d\xi_u^*}{du}. \quad (21)$$

The equation reveals two effects going in opposite directions. On the one hand, the first term is positive, reflecting the fact that a low-valuation trader expects that she may switch to a high valuation, which increases her demand over time. On the other hand, the second term is negative because the price increases over time and, correspondingly, decreases demand. If low-valuation traders' demands are very sensitive to changes in expected valuation, then $D_\pi(0, \xi_u^*)$ is large and preference uncertainty creates extra volume. A sufficient condition for this to be the case is that demand is weakly concave with respect to π . In the case of iso-elastic preferences, (13), this arises if $\sigma \geq 1$.

One sees that the condition (15) for preference uncertainty to increase demand is closely related to condition (20) for it to create excess volume. This is natural given that both phenomena can be traced back to low-valuation traders' willingness to increase their holdings as their probability of being high valuation increases. But the former condition turns out to be stronger than the later: excess volume is necessary but not sufficient for higher demand.

Note that the result is robust to changing our assumption about the information of time zero investors and assuming they learn their initial preference shock at their first updating time, instead of time 0. The reason is that, at any time $u > 0$, time-zero investors have measure $e^{-\rho u} = o(1/\rho)$, which in our Taylor approximation for large ρ implies that they have a negligible contribution to trading volume. This is reflected in the heuristic calculations developed above, according to which the excess volume result is entirely driven by the group of investors who *recently* updated their information.

Finally, the excess volume of Proposition 6 depends on having a *large* frequency of information updates, as measured by ρ . Indeed, when the frequency is small, $\rho \rightarrow 0$, then the opposite can occur: the instantaneous volume can be *smaller* with preference uncertainty instead of being larger. Intuitively, when ρ is small, investors trade less because they do not receive any news about their preference switch. A stark example arises with the parametric case studied in Section 4.4, where we will find that the instantaneous volume goes to zero when $\rho \rightarrow 0$.

4.4 An analytical example

To illustrate our results and derive further implications, we now consider the following analytical example. We let $s \in (0, 1)$, $\mu_{h,0} < s$, $\sigma > 0$ and we assume that preferences are given by:

$$v(\theta, q) = m(q) - \delta \mathbb{I}_{\{\theta = \theta_\ell\}} \frac{m(q)^{1+\sigma}}{1+\sigma}, \quad (22)$$

for some $\delta \in (0, 1]$ and where

$$m(q) \equiv \begin{cases} 1 - \frac{\ln(1 + e^{1/\varepsilon[1 - q^{1-\varepsilon}/(1-\varepsilon)]})}{\ln(1 + e^{1/\varepsilon})} & \text{if } \varepsilon > 0 \\ \min\{q, 1\} & \text{if } \varepsilon = 0. \end{cases}$$

When $\varepsilon > 0$ and is small, the function $m(q)$ is approximately equal to $\min\{q, 1\}$,²⁰ and it satisfies the smoothness and Inada conditions of Section 2.1, so all the results derived so far can be applied.

When $\varepsilon = 0$ the function $m(q)$ is exactly equal to $\min\{q, 1\}$ and so it no longer satisfies these regularity conditions. Nevertheless, existence and uniqueness can be established up to small

20. We follow Eeckhout and Kircher (2010) who use a closely related function to approximate a smooth but almost frictionless matching process.

adjustments in the proof. Moreover, equilibrium objects are continuous in ε , in the following sense:

Proposition 7. *As $\varepsilon \rightarrow 0$ the holding cost, holding plans and the asymptotic excess volume converge pointwise to their $\varepsilon = 0$ counterparts.*

This continuity result allows us to concentrate, for the remainder of this section, on the $\varepsilon = 0$ equilibrium, which can be solved in closed form.

When $\varepsilon = 0$, the marginal valuation of a high-valuation trader is equal to one as long as q is lower than 1, and equal to 0 for larger values of q . Hence, her demand is a step function of the holding cost, ξ :

$$D(1, \xi) = \begin{cases} 1 & \text{if } \xi < 1 \\ \in [0, 1] & \text{if } \xi = 1 \\ 0 & \text{if } \xi > 1. \end{cases}$$

Also for $\varepsilon = 0$, the demand of a trader who expects to be of high valuation with probability $\pi < 1$ is:

$$D(\pi, \xi) = \min \left\{ \left(\frac{1 - \xi}{\delta(1 - \pi)} \right)^{\frac{1}{\sigma}}, 1 \right\}. \tag{23}$$

When, $\varepsilon = 0$ and $\sigma \rightarrow 0$, our specification nests the case analysed in Duffie *et al.* (2005) and the demand of low-valuation traders is a step function of both the holding cost and the probability π of having a high valuation.²¹ When $\sigma > 0$ our specification generates smoother demands for low-valuation traders, as with the iso-elastic specification (13) of Lagos and Rocheteau (2009). In particular, for $q < 1$ preferences are iso-elastic, and $v_q > 0$ while $v_{qq} < 0$. Note however that when preferences are as in (13), demand is either globally concave or convex in π , so that preference uncertainty either always increases or decreases prices. In contrast, for the specification we consider, in line with Duffie *et al.* (2005), demands are neither globally concave nor convex in π . Correspondingly, we will show that preference uncertainty can increase prices in certain market conditions and decrease prices in others.

The equilibrium holding cost is easily characterized. First, $\xi_u \leq 1$ for otherwise aggregate demand would be zero. Second, $\xi_u = 1$ if and only if $u \geq T_f$, where T_f solves $\mathbb{E}[\mu_{h, \tau_{T_f}}] = s$. In other words, $\xi_u = 1$ if and only if the measure of traders who know that they have a high valuation, $\mathbb{E}[\mu_{h, \tau_u}]$, is greater than the asset supply, s . In that case, high-valuation traders absorb all the supply while holding $q_{h,u} \leq 1$, and therefore have a marginal utility $v_q(\theta_h, q_{h,u}) = 1$. Low-valuation traders, on the other hand, hold no asset. In this context $p = 1/r$.

When $u < T_f$, then $\xi_u < 1$. All high-valuation traders hold one unit, and low-valuation traders hold positive amounts. The holding cost, ξ_u , is the unique solution of:

$$\mathbb{E}[\mu_{h, \tau_u} + (1 - \mu_{h, \tau_u})D(\pi_{\tau_u, u}, \xi)] = s. \tag{24}$$

21. See Addendum III in Biais *et al.* (2012b) for a proof that the equilibrium is indeed continuous at $\sigma = 0$: precisely, we show that, as $\sigma \rightarrow 0$, equilibrium objects converges pointwise, almost everywhere, to their $\sigma = 0$ counterparts.

4.4.1 Known preferences. With known preferences, the above characterization can be applied by setting $\tau_u = u$ for all u . In this case T_f is the time T_s solving $\mu_{h,T_s} = s$. When $u \geq T_s$:

$$q_{h,u}^* \in [0, 1], \quad q_{\ell,u,u}^* = 0, \quad \text{and} \quad \xi_u^* = 1,$$

that is, all assets are held by high-valuation traders, the holding cost is 1 and the price is $1/r$. When $u < T_s$:

$$q_{h,u}^* = 1, \quad q_{\ell,u,u}^* = \frac{s - \mu_{h,u}}{1 - \mu_{h,u}}, \quad \text{and} \quad \xi_u^* = 1 - \delta \left(q_{\ell,u,u}^* \right)^\sigma < 1.$$

In this case, there are $\mu_{h,u}$ high-valuation traders who each hold one share, and $1 - \mu_{h,u}$ low-valuation traders who hold the residual supply $s - \mu_{h,u}$. The holding cost, ξ_u^* , is equal to the marginal utility of a low-valuation trader and is less than one. Notice that the per-capita holding of low-valuation traders, $q_{\ell,u,u}^*$, decreases over time. This reflects that, as time goes by, more and more firms recover from the shock, switch to $\theta = \theta_h$ and increase their holdings. As a result, the remaining low-valuation traders are left with less shares to hold.

4.4.2 Holding plans. With preference uncertainty, we already know that $q_{h,u} = 1$ for all $u < T_f$, $q_{h,u} \in [0, 1]$ and $q_{\ell,\tau_u,u} = 0$ for all $u \geq T_f$. The only thing left to derive are the holdings of low-valuation traders when $u < T_f$.

Proposition 8. *Suppose preferences are given by (22) and that $\varepsilon = 0$. When $u < T_f$, low valuation traders hold $q_{\ell,\tau_u,u} = \min \left\{ (1 - \mu_{h,\tau_u})^{1/\sigma} Q_u, 1 \right\}$, where Q_u is a continuous function such that $Q_0 = \frac{s - \mu_{h,0}}{(1 - \mu_{h,0})^{1+1/\sigma}}$ and $Q_{T_f} = 0$. Moreover, Q_u is a hump-shaped function of u if condition (20) holds evaluated at $u = 0$, which, for the preferences given in (22) is equivalent to:*

$$q_{\ell,0,0} = q_{\ell,0,0}^* = \frac{s - \mu_{h,0}}{1 - \mu_{h,0}} > \frac{\sigma}{1 + \sigma}, \quad (25)$$

Otherwise Q_u is strictly decreasing in u .

At time 0, all traders know their valuation for sure, so the allocation must be the same as with known preferences. In particular, $q_{\ell,0,0} = q_{\ell,0,0}^* = \frac{s - \mu_{h,0}}{1 - \mu_{h,0}}$, which determines Q_0 . At time T_f high-valuation traders absorb the entire supply. Hence, $Q_{T_f} = 0$. For times $u \in (0, T_f)$, asset holdings are obtained by scaling down Q_u by $(1 - \mu_{h,\tau_u})^{1/\sigma}$, where τ_u is the last updating time of the trader. This follows because low-valuation traders have iso-elastic holding costs, so their asset demands are homogenous. Correspondingly, if a low-valuation trader holds less than one unit, $q_{\tau_u,u} < 1$, then, substituting (8) in (23) we have:

$$q_{\ell,\tau_u,u} = D(\pi_{\tau_u,u}, \xi_u) = (1 - \mu_{h,\tau_u})^{1/\sigma} Q_u, \quad \text{where} \quad Q_u \equiv \left(\frac{1 - \xi_u}{\delta(1 - \mu_{h,u})} \right)^{1/\sigma}.$$

Otherwise $q_{\ell,\tau_u,u} = 1 < (1 - \mu_{h,\tau_u})^{1/\sigma} Q_u$. If Q_u is hump shaped and achieves its maximum at some time T_ψ , then the holding plan of a trader with updating time $\tau_u \leq T_\psi$ will be hump-shaped, and the holding plan of a trader with updating time $\tau_u > T_\psi$ will be decreasing.

As shown in the previous section, (20) is the condition under which the holding plans of low valuation traders are increasing with time, near time zero. If this condition holds at time 0, it

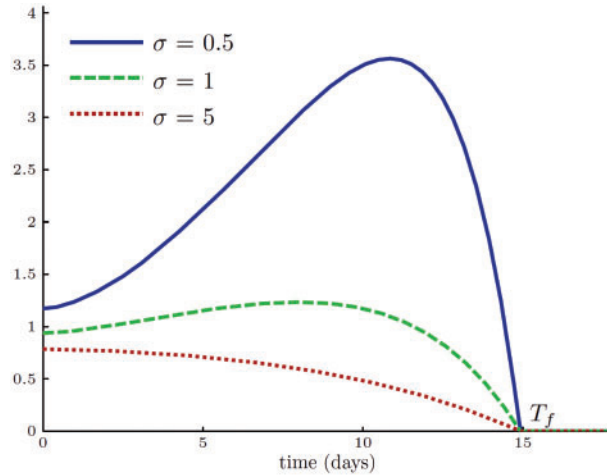


FIGURE 2
The function Q_u for various values of σ .

implies that holding plans defined at time 0 are hump shaped. Because holdings plans at later times are obtained by scaling down time=0 holding plans, they also are hump shaped.

Finally, note that, with the preference specification (22), the demand of high-valuation traders is inelastic when $\xi_0^* < 1$, *i.e.* $D_\xi(1, \xi_0^*) = 0$. This implies that the condition under which preference uncertainty increases trading volume, (20), is equivalent to the simpler condition under which it increases demand, (15).

The closed form expression for equation (25) reveals some natural comparative static. When σ is small, (25) is more likely to hold. Indeed, utility is close to linear, $D_\pi(0, \xi_0^*)$ is large, and traders' demands are very sensitive to changes in the probability of being high valuation. When s is large or when $\mu_{h,0}$ is small, (25) is also more likely to hold. In that case the liquidity shock is more severe. Hence, shortly after the initial aggregate shock, the inflow of traders who receive good news is not large enough to absorb the sales of the traders who currently receive bad news. In equilibrium, some of these sales are absorbed by traders who received the bad news at earlier updating times, $\tau_u < u$. Indeed, these "early" low-valuation traders anticipate that, as time has gone by since their last updating time, $\tau_u < u$, their valuation is more and more likely to have reverted upwards $\pi_{\tau_u, u} > 0$. These traders find it optimal to buy if their utility is not too concave, *i.e.* if σ is not too high. Correspondingly, their holding plan can be increasing, and hence the function Q_u is hump shaped, as depicted in Figure 2 for $\sigma = 0.5$ and 1. But for the larger value of $\sigma = 5$, the function Q_u is decreasing (the parameter values used for this figure are discussed in Section 4.4.5).

4.4.3 Trading volume. Proposition 8 provides a full characterization of the equilibrium holdings process, which can be compared to its counterpart without preference uncertainty. Holdings with known preferences are illustrated by the dash-dotted red curve in Figure 3: as long as a trader has not recovered from the shock, her holdings decline smoothly, and, as soon as she recovers, her holdings jump to 1. Holdings under preference uncertainty are quite different, as illustrated by the solid green curve in Figure 3. Consider a trader who is hit by a liquidity shock at time zero. After time zero, if (25) holds, the trader's holding plan, illustrated by the dotted blue curve, progressively buys back. If, at the next updating time, t_2 , the trader learns that her valuation is still low, then she sells again. These round-trip trades continue until updating

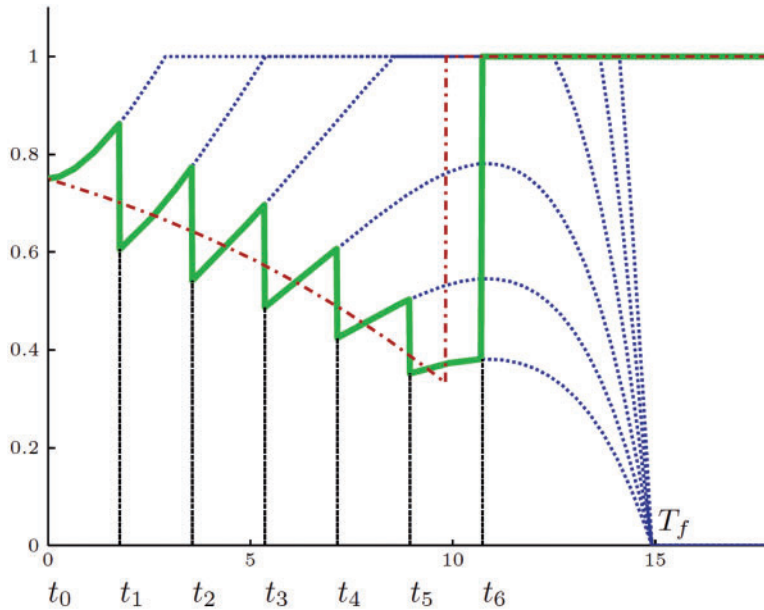


FIGURE 3

The realized holdings with continuous-updating are represented by the downward sloping curve (dash-dotted red). The realized holdings with infrequent updating are represented by the sawtooth curve (solid green). The consecutive holding plans with infrequent updating are represented by hump-shaped curves (dotted blue).

time t_6 when the trader finds out her valuation has recovered, at which point her holdings jump to 1. Thus, as we argued before, while the friction we consider implies less frequent observations of preferences, it does not induce less frequent *trading*, quite to the contrary. The hump-shaped asset holding plans shown in Figure 3 create round-trip trades and generate extra trading volume relative to the known preference case.²²

As we know, this extra volume persists even in the $\rho \rightarrow \infty$ limit: although the round trip trades of a low-valuation trader become smaller and smaller, they occur more and more frequently. Note that, while the above analytical results on excess volume were obtained for the asymptotic case where ρ goes to infinity, Figure 3 illustrates that, even for finite ρ , preference uncertainty generates excess volume relative to the case where preferences are known.

Proposition 9. *Suppose preferences are given by (22) and that $\varepsilon = 0$. Then the asymptotic excess volume is equal to:*

$$\begin{aligned} V^\infty - V^* &= \gamma(1 - \mu_{h,u}) \max \left\{ D_\pi(0, \xi_u^*) - [D(1, \xi_u^*) - D(0, \xi_u^*)], 0 \right\} \\ &= \gamma \max \left\{ \frac{s - \mu_{h,u}}{\sigma} - (1 - s), 0 \right\}. \end{aligned}$$

22. As illustrated in Figure 3, both with known preferences and with preference uncertainty, the agent is continuously trading. Transactions costs, as analysed by Constantinides (1986), Dumas and Luciano (1991), and Vayanos (1998), would reduce trading volume, as agents would wait until their positions get significantly unbalanced before engaging in trades. We conjecture that equilibrium dynamics would remain similar to that in Figure 3, except that holdings would be step functions. This would reduce the number of round trips but not altogether eliminate them.

The first equality involves, once again, the same terms as in condition (15): there is excess volume if demand is sufficiently sensitive to changes in the probability of being high valuation.

The second equality yields comparative statics of excess volume with respect to exogenous parameters. When σ decreases, demand becomes more sensitive to changes in the probability of being high valuation, and volume increases. When γ increases, low-valuation traders expect to change valuation faster, increase their demand by more, which increases excess volume.

4.4.4 Price. In the context of this analytical example we can go beyond the analysis of holding costs offered in the general case, and discuss detailed properties of the equilibrium price:

Proposition 10. *Suppose preferences are given by (22) and that $\varepsilon = 0$. Then, the price is continuously differentiable, strictly increasing for $u \in [0, T_f)$, and constant equal to $1/r$ for $u \geq T_f$. Moreover:*

- For $u \in [T_s, T_f)$, the price is strictly lower than with known preferences.
- For $u \in [0, T_s]$, if (25) does not hold, then the price is strictly lower than with known preferences. But if s is close to 1 and σ is close to 0, then at time 0 the price is strictly higher than with known preferences.

This proposition complements our earlier asymptotic results in various ways. First it characterizes the impact of preference uncertainty on price as opposed to holding cost; second, it offers results about the price path when ρ is finite; and third, it links price impact to fundamental parameters, such as s and σ .

The first bullet point follows because, from time T_s to time T_f , $\xi_u < \xi_u^* = 1$. But it is not necessarily true for all $u \in [0, T_s)$. When (25) does not hold, then low-valuation traders do not create extra demand in between their updating times: to the contrary, they continue to sell their assets, and in equilibrium the price is smaller than its counterpart with known preferences. When (25) holds, then low-valuation traders increase their holdings in between updating times and the price at time zero can be larger than its counterpart with known preferences. This effect is stronger when low-valuation traders are marginal for a longer period, that is, when the shock is more severe (s close to one) and when their utility flow is not too concave (σ close to zero).

4.4.5 Empirical Implications. To illustrate numerically some key implications of the model, we select in Table 1 parameter values to generate effects comparable to empirical observations about liquidity shocks in large equity markets. Hendershott and Seasholes (2007) find liquidity price pressure effects of the order of 10 to 20 basis points, with duration ranging

TABLE 1
Parameter values

Parameter		Value
Discount rate	r	0.05
Updating intensity	ρ	250
Asset supply	s	0.8
Initial mass	$\mu_{h,0}$	0.2
Recovery intensity	γ	25
Utility cost	δ	1
Curvature of utility flow	σ	{0.5, 1, 5}

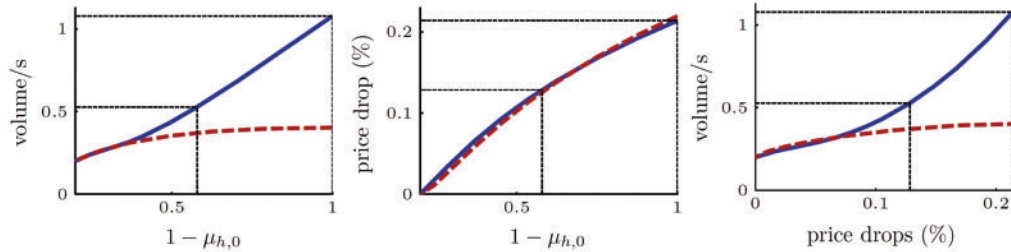


FIGURE 4

The relationship between size of the shock, $1 - \mu_{h,0}$, and volume per unit of asset supply (left panel), between the size of the shock and price drop in percent (middle panel), and between price drop and volume (right panel), for known preferences (dashed red curves) vs. uncertain preferences (plain blue curves).

from 5 to 20 days. During the liquidity event described in Khandani and Lo (2008), the price pressure subsided in about 4 trading days. Adopting the convention that there are 250 trading days per year, setting γ to 25 means that an investor takes on average 10 days to switch back to high valuation. Setting the asset supply to $s=0.8$ and the initial mass of high-valuation traders to $\mu_{h,0}=0.2$ then implies that with continuous updating the time it takes the market to recover from the liquidity shock (as proxied by T_f) is approximately 15 days. For these parameter values, setting the discount rate to $r=0.05$ and the holding cost parameter to $\delta=1$ implies that the initial price pressure generated by the liquidity shock is between 10 and 20 basis points.²³ Finally, in line with the survey evidence cited in the introduction, we set the updating intensity to $\rho=250$, *i.e.* we assume that each trader receives updated information about the utility flow she generates once every day, on average.

Excess volume and liquidity shock: One of the main insights of our analysis is that preference uncertainty generates round trip trades, which in turn lead to excess trading volume after a liquidity shock. One natural measure of the size of the liquidity shock is the fraction of traders initially hit, $1 - \mu_{h,0}$. The larger this fraction, the smaller $\mu_{h,0}$ and hence $\mu_{h,u}$ at any time u , and, by (A.1) in Proposition 9, the larger the excess volume. The left panel of Figure 4 illustrates this point by plotting total volume against initial price drop, for shocks of various sizes ($1 - \mu_{h,0}$). As can be seen in the figure, the larger is $1 - \mu_{h,0}$, the larger the total trading volume.

The middle panel of Figure 4 shows that the initial price drop, at time 0, is also increasing in the size of the liquidity shock. The right panel of Figure 4 illustrates these two points together by plotting total volume against initial price drop, a relationship that is perhaps easier to measure empirically. It shows that preference uncertainty generates a large elasticity of volume to price drop. Consider for instance an increase of $1 - \mu_{h,0}$ from 0.6 to one. The left panel of the figure indicates that the volume increases from 0.53 to 1.08, by about 105%. The price impact, on the other hand, increases from 0.13 to 0.21 basis points, by about 65%. Taken together, the elasticity

23. Duffie *et al.* (2007) provide a numerical analysis of liquidity shocks in OTC markets. They choose parameters to match stylized facts from illiquid corporate bond markets. Because we focus on more liquid electronic exchanges, we chose very different parameter values. For example in their analysis the price takes one year to recover while in ours it takes less than two weeks. While the price impact of the shock in our numerical example is relatively low, it would be larger for lower values of γ and r . For example if r were 10% and the recovery time 20 days, then the initial price impact of the shock would go up from 13 to 60 basis points. Note however that, with non-negative utility flow, the initial price impact of the shock is bounded above by $1 - e^{-rT}$.

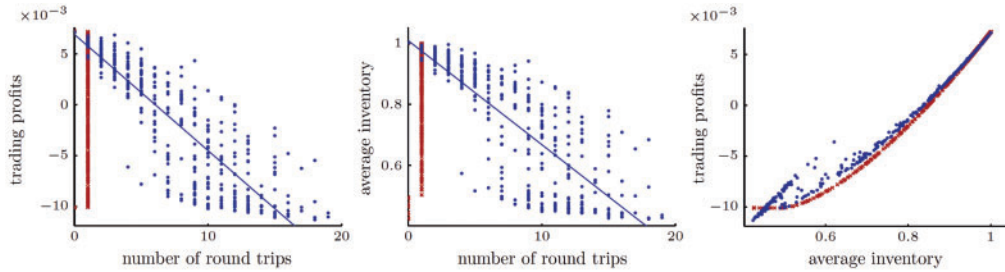


FIGURE 5

The relationship between individual trading volume and trading profits (left panel), between individual average inventory and trading profits (middle panel), and between individual average inventory and trading profits (right panel), for known preferences (red x-marks) vs. uncertain preferences (blue dots).

of volume to price impact under preference uncertainty is around 1.6. As also shown in the figure, the elasticity with known preferences is an order of magnitude smaller, about 0.11.

Trading patterns in a cross section of traders: While the above discussion bears on the empirical implications of our model for a cross section of liquidity shocks, our analysis also delivers implications for the cross section of traders within one liquidity shock.

Ex ante, all traders are identical, but ex post they differ, because they had different sample paths of valuations and information updates. Traders whose valuation recover early and who observe this rapidly, buy the asset early in the liquidity cycle, at a low price, and then hold it. In contrast, traders whose valuations remain low throughout the major part of the liquidity cycle, buy the asset later, at a high price and so they make lower trading profits.²⁴ These same traders are the ones who have many information updates leading them to engage in many round trips. Therefore, as shown in Appendix B.2, in the cross section of traders, there is a negative *correlation* between trading profits and the number of round trips.

The left panel of Figure 5 illustrates, in our numerical example, the model-generated cross-sectional relationship between trading profits and the number of round trips (measured by the number of times two consecutive trades by the same trader were of opposite signs, *e.g.* a purchase followed by a sale.) Each blue dot represents the trading profits and number of round trips of one trader in a cross-section of 500 traders, under preference uncertainty. The cross-section is representative in the sense that traders' characteristics (initial type, recovery time, and updating times) are drawn independently according to their "true", model-implied, probability distribution. The figure reveals that, with preference uncertainty, there is strong negative relationship between trading profits and the number of round trips. In contrast, with known preferences (as illustrated by the red x-marks in the figure), there is no cross-sectional variation in the number of round trips, and therefore no such relation. Hence, for the cross section of traders, our model generates qualitatively different predictions for the known preferences and uncertain preferences cases. The middle panel of Figure 5 illustrates the negative relation between the number of round trips and average inventories. Note that, once again, the pattern arising under preference uncertainty (blue dots) is significantly different from that arising with known preferences (red x-marks).

Since traders whose valuations recover early buy early and keep large holdings throughout the cycle, their behaviour can be interpreted as liquidity supply. Put together, the negative relations

24. Trading profits, here, are understood as reflecting only the proceeds from sales minus the cost of purchases, in the same spirit as in (3). That is, they do not factor in the utility flow $v(\theta, q)$ earned by the financial firm.

(i) between number of round trips and trading profits, and (ii) number of round trips and average inventory, imply a positive relation between average inventory and trading profits. It is illustrated in the right panel of Figure 5. The figure illustrates that, in our model, liquidity supply is profitable in equilibrium.

5 CONCLUSION

Information collection, processing, and dissemination in financial institutions is challenging, as emphasized in practitioners' surveys and consultants' reports Ernst & Young (2012), Institute for International Finance (2011), Mehta *et al.* (2012). Completing these tasks is necessary for financial institutions to assess the bite of the regulatory and financial constraints they face, and their corresponding constrained optimal positions. As long as traders are not perfectly informed about the optimal position for their institution, they face preference uncertainty. We analyse optimal trading and equilibrium pricing in this context.

We focus on liquidity shocks, during which preference uncertainty is likely to matter most. Preference uncertainty generates allocative inefficiency, but need not reduce prices. As traders progressively learn about the preferences of their institution they conduct round-trip trades. This generates excess trading volume relative to the frictionless case. In a cross section of liquidity shocks, the initial price drop is positively correlated with total trading volume. Across traders, the number of round trips of a trader is negatively correlated with her trading profits.

While information collection, processing, and dissemination frictions within financial institutions are very important in practice, to the best of our knowledge, Vayanos (2003) offers the only previous theoretical analysis of this issue. This seminal paper studies the optimal way to organize the firm to aggregate information. It therefore characterizes the endogenous structure of the information factored into the decisions of the financial institution, but it does not study the consequences of this informational friction for market equilibrium prices. Thus, the present paper complements Vayanos (2003), since we take as given the informational friction, but study its consequences for market pricing and trading. It would be interesting, in further research, to combine the two approaches: endogenize the organization of the firm and the aggregation of information, as in Vayanos (2003), *and* study the consequences of the resulting informational structure for market equilibrium, as in the present study.

Another important, but challenging, avenue of further research would be to take into account interconnections and externalities among institutions. In the present model, individual valuations (θ_h or θ_ℓ) are exogenous. In practice, however, these valuations could be affected by others' actions. To study this, one would need a microfoundation for the endogenous determination of the valuations θ_h and θ_ℓ . For example, in an agency theoretic context, valuations could be affected by the pledgeable income of an institution (see Tirole, 2006; Biais *et al.*, 2013). Thus, price changes, reducing the value of the asset held by an institution (or increasing its liabilities), would reduce its pledgeable income. In turn, this would reduce its ability to invest in the asset, which could push its valuation down to θ_ℓ . The analysis of the dynamics equilibrium prices and trades in this context is left for further research.

A PROOFS

A.1 Proof of Lemma 1

Let us begin by deriving a convenient expression for the intertemporal cost of buying and selling assets. For this we let $\tau_0 \equiv 0 < \tau_1 < \tau_2 < \dots$ denote the sequence of updating times. For accounting purposes, we can always assume that, at her n -th updating time, the investor sells all of her assets, q_{τ_{n-1}, τ_n} , and purchases a new initial holding q_{τ_n, τ_n} . Thus, the

expected inter-temporal *cost* of following the successive holding plans can be written:

$$C(q) = \mathbb{E} \left[-p_0 s + \sum_{n=0}^{\infty} \left\{ e^{-r\tau_n} p_{\tau_n} q_{\tau_n, \tau_n} + \int_{\tau_n}^{\tau_{n+1}} p_u dq_{\tau_n, u} e^{-ru} - e^{-r\tau_{n+1}} p_{\tau_{n+1}} q_{\tau_n, \tau_{n+1}} \right\} \right].$$

Given that p_u is continuous and piecewise continuously differentiable, and that $u \mapsto q_{\tau_n, u}$ has bounded variations, we can integrate by part (see Theorem 6.2.2 in Carter and Van Brunt, 2000), keeping in mind that $d/du(e^{-ru} p_u) = -e^{-ru} \xi_u$. This leads to:

$$C(q) = \mathbb{E} \left[-p_0 s + \sum_{n=0}^{\infty} \int_{\tau_n}^{\tau_{n+1}} e^{-ru} \xi_u q_{\tau_n, u} du \right] = -p_0 s + \mathbb{E} \left[\int_0^{\infty} e^{-ru} \xi_u q_{\tau_u, u} du \right].$$

In the above, the first equality follows by adding and subtracting $q_{0, u} = s$, and by noting that $q_{0, u}$ is constant; the second equality follows by using our “ τ_u ” notation for the last updating time before u . With the above result in mind, we find that we can rewrite the intertemporal payoff net of cost as:

$$\begin{aligned} V(q) - C(q) &= p_0 s + \mathbb{E} \left[\int_0^{\infty} e^{-ru} \left(v(\theta_u, q_{\tau_u, u}) - \xi_u q_{\tau_u, u} \right) du \right] \\ &= p_0 s + \mathbb{E} \left[\int_0^{\infty} e^{-ru} \mathbb{E} [v(\theta_u, q_{\tau_u, u}) - \xi_u q_{\tau_u, u} | \mathcal{F}_{\tau_u}] du \right], \end{aligned}$$

after switching the order of summation and applying the law of iterated expectations. \parallel

A.2 Proof of Lemma 2

Because of the Inada conditions, the left-hand side of (7) goes to infinity when $q_{\tau_u, u}$ goes to 0, and to 0 when $q_{\tau_u, u}$ goes to infinity. Because of the strict concavity of $q \mapsto v(\theta, q)$, the left-hand side of (7) is strictly decreasing for all $q_{\tau_u, u} \in (0, \infty)$. Hence there exists a unique solution to (7). Since this solution only depends on ξ_u and $\pi_{\tau_u, u}$, we denote it by: $D(\pi, \xi)$. Since $v_q(\theta_h, q) > v_q(\theta_\ell, q)$, when $\pi_{\tau_u, u}$ is raised the left-hand side of (7) is shifted upwards, while the left-hand side is shifted upward when ξ_u is raised. Hence, $D(\pi, \xi)$ is strictly increasing in π and decreasing in ξ . Finally, since $v(\theta, q)$ is three times continuously differentiable, $D(\pi, \xi)$ is twice continuously differentiable. When $\pi = 0$ and ξ goes to 0, (7) implies that $v_q(\theta_\ell, D(0, \xi))$ goes to 0, the refore $D(0, \xi)$ goes to infinity by the Inada conditions. Similarly, When $\pi = 1$ and ξ goes to infinity, (7) and the Inada conditions imply that $D(1, \xi)$ goes to zero. \parallel

A.3 Proof of Proposition 1

Aggregate demand is

$$e^{-\rho u} [\mu_{h,0} D(1, \xi) + (1 - \mu_{h,0}) D(\pi_{0,u}, \xi)] + \int_0^u \rho e^{-\rho(u-t)} [\mu_{h,t} D(1, \xi) + (1 - \mu_{h,t}) D(\pi_{t,u}, \xi)] dt.$$

Clearly, this is a strictly decreasing and continuous function of ξ , which is greater than s if $\xi = v_q(\theta_\ell, s)$, and smaller than s if $\xi = v_q(\theta_h, s)$. Thus, we can apply the intermediate value theorem to establish that a unique equilibrium holding cost exists. Because aggregate demand is continuous in (ξ, u) and because ξ_u is bounded, ξ_u must be continuous in u . \parallel

A.4 Proof of Proposition 2

Let us begin with a preliminary remark. By definition, any feasible allocation q' satisfies the market-clearing condition $\mathbb{E}[q'_{\tau_u, u}] = s$. Taken together with the fact that $\xi_u = rp_u - \dot{p}_u$ is deterministic, this implies:

$$\begin{aligned} C(q') &= -p_0 s + \mathbb{E} \left[\int_0^{\infty} e^{-ru} \xi_u q'_{\tau_u, u} du \right] = -p_0 s + \int_0^{\infty} e^{-ru} \mathbb{E}[q'_{\tau_u, u}] \xi_u du \\ &= -p_0 s + \int_0^{\infty} e^{-ru} \xi_u s du = 0. \end{aligned} \tag{A.1}$$

With this in mind, consider the equilibrium asset holding plan of Proposition 1, q , and suppose it does not solve the planning problem. Then there is a feasible asset holding plan q' that achieves a strictly higher value, *i.e.*

$$V(q') > V(q).$$

But, we just showed above that $C(q') = 0$. Subtracting the zero inter-temporal cost from both sides, we obtain that $V(q') - C(q') > V(q) - C(q)$, which contradicts individual optimality. Uniqueness of the planning solution follows because the planner's objective is strictly concave and the constraint set is convex. \parallel

A.5 Proof of Proposition 3

Note that:

$$\pi_{\tau_u, u} = \frac{\mu_{hu} - \mu_{h\tau_u}}{1 - \mu_{h\tau_u}} = \frac{1 - \mu_{hu}}{1 - \mu_{h\tau_u}} \times 0 + \frac{\mu_{hu} - \mu_{h\tau_u}}{1 - \mu_{h\tau_u}} \times 1.$$

Hence, if $D(\pi, \xi)$ is strictly concave in π , we have that:

$$\begin{aligned} & \mu_{h\tau_u} D(1, \xi) + (1 - \mu_{h\tau_u}) D(\pi_{\tau_u, u}, \xi) \\ & > \mu_{h\tau_u} D(1, \xi) + (1 - \mu_{h\tau_u}) \left[\frac{1 - \mu_{hu}}{1 - \mu_{h\tau_u}} D(0, \xi) + \frac{\mu_{hu} - \mu_{h\tau_u}}{1 - \mu_{h\tau_u}} D(1, \xi) \right] \\ & = \mu_{hu} D(1, \xi) + (1 - \mu_{hu}) D(0, \xi). \end{aligned}$$

Taking expectations with respect to τ_u on the left-hand side, we find that, for all ξ , demand is strictly higher with preference uncertainty. The result follows. \parallel

A.6 Proof of Proposition 4

Consider a first-order Taylor expansion of aggregate asset demand under preference uncertainty when $u \simeq 0$, evaluated at ξ_0^* :

$$\begin{aligned} & e^{-\rho u} [\mu_{h,0} D(1, \xi_0^*) + (1 - \mu_{h,0}) D(\pi_{0,u}, \xi_0^*)] \tag{A.2} \\ & \quad + \int_0^u \rho e^{-\rho(u-t)} [\mu_{h,t} D(1, \xi_0^*) + (1 - \mu_{h,t}) D(\pi_{0,t}, \xi_0^*)] dt \\ & = [1 - \rho u] [\mu_{h,0} D(1, \xi_0^*) + (1 - \mu_{h,0}) D(0, \xi_0^*) + D_\pi(0, \xi_0^*) \mu'_{h0} u] \\ & \quad + \rho u [\mu_{h,0} D(1, \xi_0^*) + (1 - \mu_{h,0}) D(0, \xi_0^*)] + o(u) \\ & = s + D_\pi(0, \xi_0^*) \mu'_{h,0} u + o(u). \tag{A.3} \end{aligned}$$

The second equality follows from the fact that, by definition, $\mu_{h,0} D(1, \xi_0^*) + (1 - \mu_{h,0}) D(0, \xi_0^*) = s$. Now consider a first-order Taylor expansion of aggregate demand under known preferences:

$$\mu_{h,u} D(1, \xi_0^*) + (1 - \mu_{h,u}) D(0, \xi_0^*) = s + [D(1, \xi_0^*) - D(0, \xi_0^*)] \mu'_{h,0} u + o(u). \tag{A.4}$$

Clearly, for small u , the aggregate asset demand at ξ_0^* is larger under preference uncertainty if:

$$D_\pi(0, \xi_0^*) > D(1, \xi_0^*) - D(0, \xi_0^*).$$

Since ξ_u^* is continuous at $u=0$, this condition also ensures that, as long as u is small enough, aggregate demand at ξ_u^* is larger under preference uncertainty. The result follows. \parallel

A.7 Asymptotics: Propositions 5 and 6

A.7.1 Preliminary results Our maintained assumption on $v(\theta, q)$ implies that equilibrium holding costs will remain in the compact $[\underline{\xi}, \bar{\xi}]$, where $\underline{\xi}$ and $\bar{\xi}$ solve:

$$D(0, \underline{\xi}) = s \quad \text{and} \quad D(1, \bar{\xi}) = s.$$

The net demand at time u of traders who had their last information update at time $t < u$ is:

$$\mathcal{D}(t, u, \xi) \equiv \mu_{h,t} D(1, \xi) + (1 - \mu_{h,t}) D(\pi_{t,u}, \xi) - s.$$

It will be enough to study net demand over the domain $\Delta \times [\underline{\xi}, \bar{\xi}]$, where $\Delta \equiv \{(t, u) \in \mathbb{R}_+^2 : t \leq u\}$.

Lemma 3 (Properties of net demand). *The net demand $\mathcal{D}(t, u, \xi)$ is twice continuously differentiable over $\Delta \times [\underline{\xi}, \bar{\xi}]$, with bounded first and second derivatives. Moreover $\mathcal{D}_\xi(t, u, \xi) < 0$ and is bounded away from zero.*

All results follow from direct calculations of first and second derivatives. The details can be found in Appendix B.1.1. Next, we introduce the following notation. For any $\alpha > 0$, we let $\Delta_\alpha \equiv \{(t, u) \in \mathbb{R}_+^2 : t \leq u \text{ and } u \geq \alpha\}$. Fix some function $g(\rho)$ such that $\lim_{\rho \rightarrow \infty} g(\rho) = 0$. We say that a function $h(t, u, \rho)$ is a $o_\alpha[g(\rho)]$ if it is bounded over $\Delta \times \mathbb{R}_+$, and if:

$$\lim_{\rho \rightarrow \infty} \frac{h(t, u, \rho)}{g(\rho)} = 0, \quad \text{uniformly over } (t, u) \in \Delta_\alpha.$$

To establish our asymptotic results we repeatedly apply the following Lemma.

Lemma 4. Suppose $f(t, u, \rho)$ is twice continually differentiable with respect to t , and that $f(t, u, \rho)$, $f_t(t, u, \rho)$ and $f_{t,t}(t, u, \rho)$ are all bounded over $\Delta \times \mathbb{R}_+$. Then, for all $\alpha > 0$,

$$\mathbb{E}[f(\tau_u, u, \rho)] = e^{-\rho u} f(0, u, \rho) + \int_0^u e^{-\rho(u-t)} \rho f(t, u, \rho) dt = f(u, u, \rho) - \frac{1}{\rho} f_t(u, u, \rho) + o_\alpha\left(\frac{1}{\rho}\right).$$

This follows directly after two integration by parts:

$$\begin{aligned} \int_0^u \rho e^{-\rho(u-t)} f(t, u, \rho) dt &= f(u, u, \rho) - \frac{1}{\rho} f_t(u, u, \rho) \\ &+ e^{-\rho u} \left[-f(0, u, \rho) + \frac{1}{\rho} f_t(0, u, \rho) \right] + \frac{1}{\rho^2} \int_0^u \rho e^{-\rho(u-t)} f_{t,t}(t, u, \rho) dt. \quad \parallel \end{aligned}$$

We sometimes also use a related convergence result that apply under weaker conditions:

Lemma 5. Suppose that $f(t, u)$ is bounded, and continuous $t = u$. Then $\lim_{\rho \rightarrow \infty} \mathbb{E}[f(\tau_u, u)] = f(u, u)$.

Let M be an upper bound of $f(t, u)$ and, fixing $\varepsilon > 0$, let η be such that $|f(t, u) - f(u, u)| < \varepsilon$ for all $t \in [u - \eta, u]$. We have:

$$\begin{aligned} |\mathbb{E}[f(\tau_u, u)] - f(u, u)| &\leq \mathbb{E}[|f(\tau_u, u) - f(u, u)| \mathbb{I}_{\{\tau_u \in [0, u - \eta]\}}] + \mathbb{E}[|f(\tau_u, u) - f(u, u)| \mathbb{I}_{\{\tau_u \in [u - \eta, u]\}}] \\ &\leq 2Me^{-\rho\eta} + \varepsilon, \end{aligned}$$

since the probability that $\tau_u \in [0, u - \eta]$ is equal to $e^{-\rho\eta}$. The result follows by letting $\rho \rightarrow \infty$. \parallel

A.7.2 Proof of Proposition 5 Let $\xi_u(\rho)$ denote the market clearing holding cost at time u when the preference uncertainty parameter is ρ , i.e. the unique solution of:

$$e^{-\rho u} \mathcal{D}(0, u, \xi) + \int_0^u \rho e^{-\rho(u-t)} \mathcal{D}(t, u, \xi) dt = 0.$$

Note that $\xi_u(\rho) \in [\underline{\xi}, \bar{\xi}]$. Similarly, let ξ_u^* denote the frictionless market clearing holding cost, solving $\mathcal{D}(u, u, \xi) = 0$, which also belongs to $[\underline{\xi}, \bar{\xi}]$.

The first step is to show that $\xi_u(\rho)$ converges point wise towards ξ_u^* . For this we note that $\xi_u(\rho)$ belongs to the compact $[\underline{\xi}, \bar{\xi}]$ so it admits at least one convergence subsequence, with a limit that we denote by $\hat{\xi}_u$. Now use Lemma 4 with $f(t, u, \rho) = \mathcal{D}(t, u, \xi_u(\rho))$, and recall from Lemma 3 that $\mathcal{D}(t, u, \xi)$ has bounded first and second derivatives. This implies that:

$$0 = e^{-\rho u} \mathcal{D}(0, u, \xi_u(\rho)) + \int_0^u \rho e^{-\rho(u-t)} \mathcal{D}(t, u, \xi_u(\rho)) dt = \mathcal{D}(u, u, \xi_u(\rho)) + o_\alpha(1). \quad (\text{A.5})$$

Letting ρ go to infinity we obtain, by continuity, that $\mathcal{D}(u, u, \hat{\xi}_u) = 0$, implying that $\hat{\xi}_u = \xi_u^*$. Therefore, ξ_u^* is the unique accumulation point of $\xi_u(\rho)$, and so must be its limit.

The second step is to show that $\xi_u(\rho) = \xi_u^* + o_\alpha(1)$. For this we use again (A.5), but with a first-order Taylor expansion of $\mathcal{D}(u, u, \xi_u(\rho))$. This gives:

$$0 = \mathcal{D}(u, u, \xi_u^*) + \mathcal{D}_\xi(u, u, \hat{\xi}_u(\rho)) [\xi_u(\rho) - \xi_u^*] + o_\alpha(1),$$

where $\hat{\xi}_u$ lies in between ξ_u^* and $\xi_u(\rho)$. Given that $\mathcal{D}(u, u, \xi_u^*) = 0$ by definition, and that \mathcal{D}_ξ is bounded away from zero, the result follows.

Now, for the last step, we use again to Lemma 4, with $f(t, u, \rho) = \mathcal{D}(t, u, \xi_u(\rho))$:

$$\begin{aligned} 0 &= \mathcal{D}(u, u, \xi_u(\rho)) - \frac{1}{\rho} \mathcal{D}_t(u, u, \xi_u(\rho)) \\ &+ e^{-\rho u} \frac{1}{\rho} \mathcal{D}_t(0, u, \xi_u(\rho)) + \frac{1}{\rho^2} \int_0^u \rho e^{-\rho t} \mathcal{D}_{t,t}(t, u, \xi(\rho)) dt. \end{aligned}$$

Clearly, by our maintained assumptions on $\mathcal{D}(t, u, \xi)$, the terms on the second line add up to a $o_\alpha(1/\rho)$. Also, recall that $\xi_u(\rho) = \xi_u^* + o_\alpha(1)$ and note that $\mathcal{D}_t(t, u, \xi)$ has bounded derivatives and thus is uniformly continuous over $\Delta \times [\underline{\xi}, \bar{\xi}]$, implying that $\mathcal{D}_t(u, u, \xi_u(\rho)) = \mathcal{D}_t(u, u, \xi_u^*) + o_\alpha(1)$. Taken together, we obtain:

$$\begin{aligned} 0 &= \mathcal{D}(u, u, \xi_u(\rho)) - \frac{1}{\rho} \mathcal{D}_t(u, u, \xi_u^*) + o_\alpha\left(\frac{1}{\rho}\right) \\ &= \mathcal{D}(u, u, \xi_u^*) + \mathcal{D}_\xi(u, u, \xi_u^*) [\xi_u(\rho) - \xi_u^*] \\ &+ \frac{1}{2} \mathcal{D}_{\xi,\xi}(u, u, \hat{\xi}_u(\rho)) [\xi_u(\rho) - \xi_u^*]^2 - \frac{1}{\rho} \mathcal{D}_t(u, u, \xi_u^*) + o_\alpha\left(\frac{1}{\rho}\right), \end{aligned}$$

for some $\hat{\xi}_u(\rho)$ in between ξ_u^* and $\xi_u(\rho)$. Keeping in mind that $\mathcal{D}(u, u, \xi_u^*) = 0$, we obtain:

$$\left[\xi_u(\rho) - \xi_u^*(\rho) \right] \left\{ 1 + \frac{\mathcal{D}_{\xi, \xi}(u, u, \hat{\xi}_u(\rho))}{2\mathcal{D}_{\xi}(u, u, \xi_u^*)} [\xi_u(\rho) - \xi_u^*] \right\} = \frac{1}{\rho} \frac{\mathcal{D}_t(u, u, \xi_u^*)}{\mathcal{D}_{\xi}(u, u, \xi_u^*)} + o_{\alpha} \left(\frac{1}{\rho} \right).$$

The term in the curly bracket on the left-hand side is $1 + o_{\alpha}(1)$, and the result follows after substituting the explicit expression of $\mathcal{D}_t(u, u, \xi_u^*)$ and $\mathcal{D}_{\xi}(u, u, \xi_u^*)$. \parallel

A.7.3 Proof of Proposition 6 We first need to establish further asymptotic convergence results. First:

Lemma 6. *For large ρ , the time derivative of the holding cost admits the approximation:*

$$\frac{d\xi_u(\rho)}{du} = \frac{d\xi_u^*}{du} + o_{\alpha}(1).$$

The proof is in Appendix B.1.2. Now, using Propositions 5 and 6, it follows that:

Lemma 7. *The holding plans and their derivatives converge α -uniformly to their frictionless counterparts:*

$$\begin{aligned} q_{\ell, t, u} &= q_{\ell, t, u}^* + o_{\alpha}(1), & \frac{\partial q_{\ell, t, u}}{\partial t} &= \frac{\partial q_{\ell, t, u}^*}{\partial t} + o_{\alpha}(1), & \frac{\partial q_{\ell, t, u}}{\partial u} &= \frac{\partial q_{\ell, t, u}^*}{\partial u} + o_{\alpha}(1) \\ q_{h, u} &= q_{h, u}^* + o_{\alpha}(1), & \frac{dq_{h, u}}{du} &= \frac{dq_{h, u}^*}{du} + o_{\alpha}(1). \end{aligned}$$

The proof is in Appendix B.1.3. With these results in mind, let us turn to the various components of the volume, in equation (18). The first term of equation (18) is:

$$\mathbb{E} \left[\mu_{h, \tau_u} \left| \frac{dq_{h, u}}{du} \right| \right] = \mathbb{E} \left[\mu_{h, \tau_u} \left| \frac{dq_{h, u}^*}{du} \right| + o_{\alpha}(1) \right] = \mu_{h, u} \left| \frac{dq_{h, u}^*}{du} \right| + o(1),$$

where the first equality follows from Lemma 7. The second equality follows from Lemma 4 and from the observation that, by dominated convergence, $\mathbb{E}[o_{\alpha}(1)] \rightarrow 0$ as $\rho \rightarrow \infty$.²⁵ The second term of equation (18) is:

$$\mathbb{E} \left[(1 - \mu_{h, \tau_u}) \left| \frac{\partial q_{\ell, t, u}}{\partial u} \right| \right] = \mathbb{E} \left[(1 - \mu_{h, \tau_u}) \left| \frac{\partial q_{\ell, \tau_u, u}^*}{\partial u} \right| + o_{\alpha}(1) \right] \rightarrow (1 - \mu_{h, u}) \left| \frac{\partial q_{\ell, t, u}^*}{\partial u} \right|,$$

by an application of Lemma 5. For the first term on the second line of equation (18) is:

$$\begin{aligned} \rho \mathbb{E} \left[(1 - \mu_{h, \tau_u}) \pi_{\tau_u, u} |q_{h, u} - q_{\ell, \tau_u, u}| \right] &= \rho \mathbb{E} \left[(\mu_{h, u} - \mu_{h, \tau_u}) (q_{h, u} - q_{\ell, \tau_u, u}) \right] \\ &= \mu'_{h, u} (q_{h, u} - q_{\ell, u, u}) + o_{\alpha}(1) \rightarrow \mu'_{h, u} (q_{h, u}^* - q_{\ell, u, u}^*), \end{aligned}$$

where the first equality follows from the definition of $\pi_{\tau_u, u}$ and from the observation that $q_{h, u} \geq q_{\ell, \tau_u, u}$, and where the second equality follows from an application of Lemma 4. The limit follows from Lemma 7. Finally, using the same logic, the last term on the second line of equation (18) is:

$$\begin{aligned} \rho \mathbb{E} \left[\mu_{h, \tau_u} (1 - \pi_{\tau_u, u}) |q_{\ell, u, u} - q_{\ell, \tau_u, u}| \right] &= \rho \mathbb{E} \left[(1 - \mu_{h, u}) (q_{\ell, \tau_u, u} - q_{\ell, u, u}) \right] \\ &= (1 - \mu_{h, u}) \frac{\partial q_{\ell, u, u}}{\partial t} + o_{\alpha}(1) \rightarrow (1 - \mu_{h, u}) \frac{\partial q_{\ell, u, u}^*}{\partial t}. \end{aligned}$$

Collecting terms, we obtain the desired formula for $V^{\infty} - V^*$. Next, consider the necessary and sufficient condition for $\frac{\partial q_{\ell, u, u}^*}{\partial u} > 0$. We first note that:

$$\frac{\partial q_{\ell, u, u}^*}{\partial u} = D_{\pi}(0, \xi_u^*) \frac{\partial \pi_{u, u}}{\partial u} + D_{\xi}(0, \xi_u^*) \frac{d\xi_u^*}{du}.$$

The holding cost ξ_u^* solves:

$$\mu_{h, u} D(1, \xi) + (1 - \mu_{h, u}) D(0, \xi) = s \Rightarrow \frac{d\xi_u^*}{du} = \frac{\mu'_{h, u} [D(1, \xi_u^*) - D(0, \xi_u^*)]}{\mu_{h, u} D_{\xi}(1, \xi_u^*) + (1 - \mu_{h, u}) D_{\xi}(0, \xi_u^*)},$$

using the Implicit Function Theorem. The result follows by noting that $\mu'_{h, u} = \gamma(1 - \mu_{h, u})$ and $\frac{\partial \pi_{u, u}}{\partial u} = \gamma$. \parallel

25. Indeed for $g(t, u, \rho) = o_{\alpha}(1)$, $|\mathbb{E}[g(\tau_u, u, \rho)]| \leq \sup_{t \in [0, \alpha]} |g(t, u, \rho)| e^{-\rho(u-\alpha)} + \int_{\alpha}^u \rho |g(t, u, \rho)| e^{-\rho(u-t)} dt$. The first term goes to zero because $g(t, u, \rho)$ is bounded, and the second one goes to zero because $g(t, u, \rho)$ converges uniformly to 0 over $[\alpha, u]$.

A.8 Proof of Proposition 7

To clarify the exposition, our notations in this section are explicit about the fact that utility functions and equilibrium objects depend on ε : e.g. we write $m(q, \varepsilon)$ instead of $m(q)$, $\xi_u(\varepsilon)$ instead of ξ_u , etc. We begin with preliminary properties of the function $m(q, \varepsilon)$.

Lemma 8. *The function $m(q, \varepsilon)$ is continuous in $(q, \varepsilon) \in [0, \infty)^2$, and satisfies $m(0, \varepsilon) = 0$ and $\lim_{q \rightarrow \infty} m(q, \varepsilon) = 1$. For $\varepsilon > 0$, it is strictly increasing, strictly concave, three times continuously differentiable over $(q, \varepsilon) \in [0, \infty) \times (0, \infty)$, and satisfies Inada conditions $\lim_{q \rightarrow 0} m_q(q, \varepsilon) = \infty$, and $\lim_{q \rightarrow \infty} m_q(q, \varepsilon) \rightarrow 0$. Lastly, its first and second derivatives satisfy, for all $q > 0$:*

$$\lim_{\varepsilon \rightarrow 0} m_q(q, \varepsilon) = \begin{cases} 1 & \text{if } q < 1 \\ 1/(1+e) & \text{if } q = 1 \\ 0 & \text{if } q > 1 \end{cases} \quad \lim_{\varepsilon \rightarrow 0} m_{qq}(q, \varepsilon) = \begin{cases} 0 & \text{if } q < 1 \\ -\infty & \text{if } q = 1 \\ 0 & \text{if } q > 1 \end{cases}$$

The proof is in Appendix B.1.4. Having shown that $m(q, \varepsilon)$ is continuous even at points such that $\varepsilon = 0$, we can on to apply the Maximum Theorem (see, e.g. Stokey and Lucas, 1989, Theorem 3.6) to show that demands are continuous in all parameters. We let:

$$D(\pi, \xi, \varepsilon) = \operatorname{argmax}_{\xi q \leq 2} m(q, \varepsilon) - \delta(1 - \pi) \frac{m(q, \varepsilon)^{1+\sigma}}{1+\sigma} - \xi q.$$

Note that the constraint $q \leq 2/\xi$ is not binding: when $q > 2/\xi$, the objective is strictly negative since $m(q, \varepsilon) \leq 1$, and so it can be improved by choosing $q = 0$. Thus, the maximization problem satisfies the condition of the Theorem of the Maximum: the objective is continuous in all variables, and the constraint set is a compact valued continuous correspondence of ξ . This implies that $D(\pi, \xi, \varepsilon)$ is non-empty, compact valued, and upper hemi continuous. Moreover, the demand is single-valued in all cases except when $\pi = 1$, $\xi = 1$, and $\varepsilon = 0$, in which case it is equal to $[0, 1]$.

Existence and uniqueness of equilibrium: When $\varepsilon > 0$, this follows directly from Proposition 1. When $\varepsilon = 0$, an equilibrium holding cost solves:

$$\mathbb{E}[\mu_{h, \tau_u} q_{h, \tau_u, u} + (1 - \mu_{h, \tau_u}) D(\pi_{\tau_u, u}, \xi, 0)] = s.$$

for some $q_{h, \tau_u, u} \in D(1, \xi, 0)$. One verifies easily that, for $\pi = 1$, $D(1, \xi, 0) = 1$ for all $\xi \in (0, 1)$, $D(1, \xi, 0) = [0, 1]$ for $\xi = 1$, and $D(1, \xi, 0) = 0$ for all $\xi > 1$. For $\pi < 1$, $D(\pi, \xi, 0) = 0$ for all $\xi \geq 1$. One sees that $\xi \leq 1$ or otherwise the market cannot clear. Then, there are two cases:

- If $\mathbb{E}[\mu_{h, \tau_u}] \geq s$, and $\xi < 1$, then $D(1, \xi, 0) = 1$ and the market cannot clear. Thus, the market clearing holding cost is $\xi_u = 1$, high-valuation holdings are indeterminate but must add up to s , and low-valuation holdings are equal to zero.
- If $\mathbb{E}[\mu_{h, \tau_u}] < s$, then $\xi < 1$. Otherwise, if $\xi = 1$, $D(\pi, \xi, 0) = 0$ for all $\pi < 1$ and the market cannot clear. Because $D(1, \xi, 0) = 1$ and $D(\pi, \xi, 0)$ is strictly decreasing, in this case as well there a unique market clearing holding cost, $\xi_u(0)$. High-valuation traders hold $q_{h, u}(0) = 1$, and low-valuation traders hold $q_{\ell, \tau_u, u} = D(\pi_{\tau_u, u}, \xi, 0)$. ||

Convergence of holding costs: For all $\varepsilon \geq 0$, let $\xi_u(\varepsilon)$ be the market clearing holding cost at time u . For $\varepsilon > 0$, let $\underline{\xi}(\varepsilon) \equiv m_q(s, \varepsilon)(1 - \delta m(s, \varepsilon)^\sigma)$ and $\bar{\xi}(\varepsilon) \equiv m_q(s, \varepsilon)$, so that $D(1, \bar{\xi}(\varepsilon), \varepsilon) = D(1, \underline{\xi}(\varepsilon), \varepsilon) = s$. Clearly, for all $\varepsilon > 0$, $\xi_u(\varepsilon) \in [\underline{\xi}(\varepsilon), \bar{\xi}(\varepsilon)]$. Moreover, from Lemma 8, it follows that $\lim_{\varepsilon \rightarrow 0} \underline{\xi}(\varepsilon) = 1 - \delta \lim_{\varepsilon \rightarrow 0} \bar{\xi}(\varepsilon) = 1$, so that $\underline{\xi}(\varepsilon)$ and $\bar{\xi}(\varepsilon)$ remain in some compact $[\underline{\xi}, \bar{\xi}]$, s.t. $\underline{\xi} > 0$.

Thus, $\xi_u(\varepsilon)$ has at least one convergence subsequence as $\varepsilon \rightarrow 0$, with some limit $\hat{\xi}_u$. When $\pi < 1$, we have by upper hemi continuity that a subsequence of $D(\pi, \xi_u(\varepsilon), \varepsilon)$ converges to some $\hat{q}_{h, u} \in D(1, \hat{\xi}_u, 0)$, and we have by continuity that $D(\pi, \xi_u(\varepsilon), \varepsilon) \rightarrow D(\pi, \hat{\xi}_u, 0)$, and when $\pi = 1$. Moreover, by dominated convergence, $\mathbb{E}[(1 - \mu_{h, \tau_u}) D(\pi_{\tau_u, u}, \xi_u(\varepsilon), \varepsilon)] \rightarrow \mathbb{E}[(1 - \mu_{h, \tau_u}) D(\pi_{\tau_u, u}, \hat{\xi}_u, 0)]$. Taken together, we obtain that:

$$\mathbb{E}[\mu_{h, \tau_u} \hat{q}_{h, u} + (1 - \mu_{h, \tau_u}) D(\pi_{\tau_u, u}, \hat{\xi}_u, 0)] = s,$$

where $\hat{q}_{h, u} \in D(1, \hat{\xi}_u, 0)$. Therefore, $\hat{\xi}_u$ is an equilibrium holding cost of the $\varepsilon = 0$ economy, which we know must be equal to $\xi_u(0)$. Thus, the unique accumulation point of $\xi_u(\varepsilon)$ is $\xi_u(0)$, and so it must be its limit. ||

Convergence of holding plans: Let $q_{\ell,t,u}(\varepsilon) \equiv D(\pi_{t,u}, \xi_u(\varepsilon), \varepsilon)$. Let $q_{h,u}(\varepsilon) \equiv D(1, \xi_u, \varepsilon)$ whenever the correspondence is single-valued. When it is multi-valued, which only arises when $\varepsilon = 0$ and $\xi_u(0) = 1$, we let $q_{h,u} \equiv s/\mathbb{E}[\mu_{h,\tau_u}]$. We have two cases to consider:

- If $\pi \in (0, 1)$, or if $\pi = 1$ and $\xi_u(0) < 1$, then $D(\pi, \xi, \varepsilon)$ is single valued and thus continuous in a neighbourhood of $(\xi_u(0), 0)$. Therefore $D(\pi, \xi_u(\varepsilon), \varepsilon) \rightarrow D(\pi, \xi_u(0), 0)$, i.e. $\varepsilon > 0$ holding plan converge to their $\varepsilon = 0$ counterparts.
- If $\pi = 1$ and $\xi_u(0) = 1$, which occurs when $\mathbb{E}[\mu_{h,\tau_u}] \geq s$, we have:

$$\mathbb{E}[\mu_{h,\tau_u}]q_{h,u}(\varepsilon) + \mathbb{E}[(1 - \mu_{h,\tau_u})q_{\ell,\tau_u,u}(\varepsilon)] = s.$$

But we have just shown that $q_{\ell,\tau_u,u}(\varepsilon) \rightarrow 0$. Moreover, $q_{\ell,\tau_u,u}(\varepsilon) \leq q_{h,u}(\varepsilon) \leq s/\mathbb{E}[\mu_{h,\tau_u}]$ otherwise the market would not clear. Thus, by dominated convergence, $\mathbb{E}[(1 - \mu_{h,\tau_u})q_{\ell,\tau_u,u}(\varepsilon)] \rightarrow 0$. From the market clearing condition, this implies that $q_{h,u}(\varepsilon) \rightarrow q_{h,u}(0) = s/\mathbb{E}[\mu_{h,\tau_u}]$. \parallel

Convergence of the volume: Fixing some $\varepsilon > 0$ positive and small enough, the specification of preferences satisfy our basic regularity conditions. So, we have that the excess volume when $\rho \rightarrow \infty$ converges to:

$$\max \left\{ (1 - \mu_{h,u}) \frac{\partial q_{\ell,u,u}^*(\varepsilon)}{\partial u}, 0 \right\}$$

where, as before, we use the star “ \star ” to index holding plans and holding costs in the equilibrium with known preferences. Next, we will show that:

$$\lim_{\varepsilon \rightarrow 0} \max \left\{ (1 - \mu_{h,u}) \frac{\partial q_{\ell,u,u}^*(\varepsilon)}{\partial u}, 0 \right\} = \gamma \max \left\{ \frac{s - \mu_{h,u}}{\sigma} - (1 - s), 0 \right\},$$

which is equal to the excess volume in the $\varepsilon = 0$ equilibrium, as is shown formally later in Appendix A.9.3. For this we recall that:

$$q_{\ell,t,u}^*(\varepsilon) = D(\pi_{t,u}, \xi_u^*(\varepsilon)) \Rightarrow \frac{\partial q_{\ell,u,u}^*(\varepsilon)}{\partial u} = D_\pi(0, \xi_u^*(\varepsilon)) \frac{\partial \pi_{u,u}}{\partial u} + D_\xi(0, \xi_u^*(\varepsilon)) \frac{d\xi_u^*(\varepsilon)}{du}. \quad (\text{A.6})$$

The holding cost $\xi_u^*(\varepsilon)$ solves:

$$\mu_{h,u}D(1, \xi, \varepsilon) + (1 - \mu_{h,u})D(0, \xi, \varepsilon) = s.$$

Thus, by the Implicit Function Theorem, the time derivative of $\xi_u^*(\varepsilon)$ is:

$$\frac{d\xi_u^*(\varepsilon)}{du} = - \frac{\mu'_{h,u} [D(1, \xi_u^*(\varepsilon), \varepsilon) - D(0, \xi_u^*(\varepsilon), \varepsilon)]}{\mu_{h,u}D_\xi(1, \xi_u^*(\varepsilon), \varepsilon) + (1 - \mu_{h,u})D_\xi(0, \xi_u^*(\varepsilon), \varepsilon)}.$$

For any (π, ξ, ε) , with $\varepsilon > 0$, the demand $D(\pi, \xi, \varepsilon)$ is the unique solution of

$$0 = H(q, \pi, \xi, \varepsilon) - \xi, \quad \text{where} \quad H(q, \pi, \xi, \varepsilon) \equiv m_q(q, \varepsilon) [1 - \delta(1 - \pi)m(q, \varepsilon)^\sigma].$$

Using the Implicit Function Theorem, $D_\pi(\pi, \xi, \varepsilon) = -H_\pi/H_q$ and $D_\xi(\pi, \xi, \varepsilon) = -H_\xi/H_q$, all evaluated at $q = D(\pi, \xi, \varepsilon)$ and (π, ξ, ε) . To evaluate the limit of (A.6) as $\varepsilon \rightarrow 0$, we start with some preliminary asymptotic results.

Step 1: preliminary results. By continuity of $m(q, \varepsilon)$ we have:

$$\lim_{\varepsilon \rightarrow 0} m(q_{\ell,u,u}^*(\varepsilon), \varepsilon) = m(q_{\ell,u,u}^*(0), 0) = q_{\ell,u,u}^*(0) \quad (\text{A.7})$$

$$\lim_{\varepsilon \rightarrow 0} m(q_{h,u}^*(\varepsilon), \varepsilon) = m(q_{h,u}^*(0), 0) = q_{h,u}^*(0), \quad (\text{A.8})$$

where, on both lines, the second equality follows by noting that, since $\xi_u^*(0) > 0$ (otherwise the market would not clear), we have $q_{\ell,u,u}^*(\varepsilon) \leq 1$ and $q_{h,u}^*(0) \leq 1$. The first-order condition of low-valuation traders is:

$$\xi_u^*(\varepsilon) = m_q(q_{\ell,u,u}^*(\varepsilon), \varepsilon) (1 - \delta m(q_{\ell,u,u}^*(\varepsilon), \varepsilon)^\sigma).$$

Taking $\varepsilon \rightarrow 0$ limits on both sides we obtain that:

$$\xi_u^*(0) = \lim_{\varepsilon \rightarrow 0} m_q(q_{\ell,u,u}^*(\varepsilon), \varepsilon) (1 - \delta (q_{\ell,u,u}^*(0))^\sigma).$$

When $\varepsilon = 0$, since $q_{\ell,u,u}^*(0) \leq s < 1$, the first-order condition of a low-valuation trader is $\xi_u^*(0) = (1 - \delta (q_{\ell,u,u}^*(0))^\sigma)$. Hence:

$$\lim_{\varepsilon \rightarrow 0} m_q(q_{\ell,u,u}^*(\varepsilon), \varepsilon) = 1. \quad (\text{A.9})$$

Using (5) and the fact that $q_{\ell,u,u}^*(0) \leq s < 1$, this implies in turns that:

$$\lim_{\varepsilon \rightarrow 0} (q_{\ell,u,u}^*(\varepsilon))^{-\varepsilon} = 1. \quad (\text{A.10})$$

Turning to the first-order condition of a high-valuation trader, we obtain:

$$\lim_{\varepsilon \rightarrow 0} m_q(q_{h,u}^*(\varepsilon)) = \xi_u^*(0). \quad (\text{A.11})$$

If $\mu_{h,u} < s$, then $\xi_u^*(0) < 1$ and it follows from the analytical expression of $m_q(q, \varepsilon)$, in equation (5), that:

$$\lim_{\varepsilon \rightarrow 0} e^{\frac{1}{\varepsilon} \left(1 - \frac{q_{h,u}^*(\varepsilon)}{1-\varepsilon}\right)} < \infty.$$

Therefore the analytical expression of $m_{qq}(q, \varepsilon)$, in equation (6), implies that:

$$\lim_{\varepsilon \rightarrow 0} m_{qq}(q_{h,u}^*(\varepsilon), \varepsilon) = -\infty \text{ if } \mu_{h,u} < s. \quad (\text{A.12})$$

Lastly, when $\mu_{h,u} < s$, $\xi_u^*(0) < 1$ implies that $q_{\ell,u,u}^*(0) > 1$ and, using (6) together with (A.10), that

$$\lim_{\varepsilon \rightarrow 0} m_{qq}(q_{\ell,u}^*(\varepsilon), \varepsilon) = 0 \text{ if } \mu_{h,u} < s. \quad (\text{A.13})$$

Step 2: limit of volume when $\mu_{h,u} < s$. We have

$$D_\pi(0, \xi_u^*(\varepsilon)) = - \frac{\delta m_q m^\sigma}{m_{qq}(1 - \delta m^\sigma) - \delta \sigma m_q^2 m^{\sigma-1}}$$

where m , m_q and m_{qq} are all evaluated at $q_{\ell,u,u}^*(\varepsilon)$ and ε . Using (A.7), (A.9), and (A.13), we obtain that:

$$\lim_{\varepsilon \rightarrow 0} D_\pi(0, \xi_u^*(\varepsilon)) = \frac{q_{\ell,u,u}^*(0)}{\sigma}.$$

A similar argument shows that:

$$\lim_{\varepsilon \rightarrow 0} D_\xi(0, \xi_u^*(\varepsilon)) = \lim_{\varepsilon \rightarrow 0} \frac{1}{m_{qq}(1 - \delta m^\sigma) - \delta \sigma m_q^2 m^{\sigma-1}} = - \frac{1}{\delta \sigma (q_{\ell,u,u}^*)^{\sigma-1}}.$$

Lastly, using (A.8), (A.11), and (A.12), we have

$$\lim_{\varepsilon \rightarrow 0} D_\xi(1, \xi_u^*(\varepsilon)) = \lim_{\varepsilon \rightarrow 0} \frac{1}{m_{qq}(1 - \delta m^\sigma) - \delta \sigma m_q^2 m^{\sigma-1}} = 0,$$

where m , m_q , and m_{qq} are evaluated at $q_{h,u}^*(\varepsilon)$ and ε . Now using these limits in (A.6) we obtain that

$$\lim_{\varepsilon \rightarrow 0} \frac{\partial q_{\ell,u,u}^*(\varepsilon)}{\partial u} = \gamma \frac{q_{\ell,u,u}^*(0)}{\sigma} + \gamma (q_{h,u}^*(0) - q_{\ell,u,u}^*(0)).$$

When $\varepsilon = 0$ and $\mu_{h,u} < s$, $\xi_u^*(0) < 1$ implying that $q_{h,u}^*(0) = 1$ and, from market clearing, that $q_{\ell,u,u}^*(0) = (s - \mu_{h,u}) / (1 - \mu_{h,u})$. Plugging these expressions into the above, we obtain the desired result.

Step 3: limit when $\mu_{h,u} \geq s$. In this case we note that

$$\frac{\partial q_{\ell,u,u}^*(\varepsilon)}{\partial u} \leq \gamma D_\pi(0, \xi_u^*(\varepsilon)) \leq \gamma \frac{\delta m_q m^\sigma}{-m_{qq}(1 - \delta m^\sigma) + \delta \sigma m_q^2 m^{\sigma-1}} \leq \gamma \frac{\delta m_q m^\sigma}{\delta \sigma m_q^2 m^{\sigma-1}} = \frac{\gamma m}{\delta m_q} \rightarrow 0,$$

using (A.7) and (A.9), where m , m_q and m_{qq} are evaluated at $q_{\ell,u,u}^*(\varepsilon)$ and ε . Clearly, this implies that

$$\lim_{\varepsilon \rightarrow 0} \max \left\{ (1 - \mu_{h,u}) \frac{\partial q_{\ell,u,u}^*(\varepsilon)}{\partial u}, 0 \right\} = 0 = \gamma \max \left\{ \frac{s - \mu_{h,u}}{\sigma} - (1 - s), 0 \right\},$$

given that $\mu_{h,u} \geq s$. ||

A.9 Proof of Propositions 8, 9, and 10

A.9.1 A characterization of equilibrium object Let

$$S_u \equiv s - \mathbb{E}[\mu_{h,\tau_u}] = e^{-\rho u}(s - \mu_{h,0}) + \int_0^u \rho e^{-\rho(u-t)}(s - \mu_{h,t}) dt, \quad (\text{A.14})$$

the gross asset supply in the hand of investors, minus the maximum (unit) demand of high-valuation investors. Keeping in mind that T_s is the time such that $\mu_{h,T_s} = s$, one sees that S_u has a unique root $T_f > T_s$. Indeed, $S_u > 0$ for all $u \in [0, T_s)$ and S_u goes to minus infinity when u goes infinity, so S_u has a root $T_f > T_s$. It is unique because $\dot{S}_{T_f} = \rho(s - \mu_{h,T_f}) < 0$ since $T_f > T_s$.

We already know from the text that, when $u \geq T_f$, $\xi_u = 1$, $q_{h,u} \in [0, 1]$ and $q_{\ell,\tau_u,u} = 0$.²⁶ Now consider $u \in [0, T_f]$. In that case, $S_u > 0$ and we already know that $\xi_u < 1$, which implies that high-valuation investors hold one unit, $q_{h,u} = 1$. Replacing expression (8) for $\pi_{\tau_u,u}$ into the asset demand (23) we obtain that:

$$q_{\ell,\tau_u,u} = \min\{(1 - \mu_{h,\tau_u})^{1/\sigma} Q_u, 1\}, \quad \text{where } Q_u \equiv \left(\frac{1 - \xi_u}{\delta(1 - \mu_{h,u})} \right)^{1/\sigma}.$$

This is the formula for holding plans in Proposition 8 Plugging this back into the market clearing condition (24), we obtain that Q_u solves:

$$e^{-\rho u}(1 - \mu_{h,0}) \min\{(1 - \mu_{h,0})^{1/\sigma} Q_u, 1\} + \int_0^u \rho e^{-\rho(u-t)}(1 - \mu_{h,t}) \min\{(1 - \mu_{h,t})^{1/\sigma} Q_u, 1\} dt = S_u. \quad (\text{A.15})$$

The left-hand side of (A.15) is continuous, strictly increasing for $Q_u < (1 - \mu_{h,u})^{-1/\sigma}$ and constant for $Q_u \geq (1 - \mu_{h,u})^{-1/\sigma}$. It is zero when $Q_u = 0$, and strictly larger than S_u when $Q_u = (1 - \mu_{h,u})^{-1/\sigma}$ since $s < 1$. Therefore, equation (A.15) has a unique solution and the solution satisfies $0 < Q_u < (1 - \mu_{h,u})^{-1/\sigma}$.

Now, turning to the price, the definition of Q_u implies that the price solves $rp_u = 1 - \delta(1 - \mu_{h,u})Q_u^\sigma + \dot{p}_u$ for $u < T_f$. For $u \geq T_f$, the fact that high-valuation traders are indifferent between any asset holdings in $[0, 1]$ implies that $rp_u = 1 + \dot{p}_u$. But the price is bounded and positive, so it follows that $p_u = 1/r$. Since the price is continuous at T_f , this provides a unique candidate equilibrium price path. Clearly this candidate is C^1 over $(0, T_f)$ and (T_f, ∞) . To show that it is continuously differentiable at T_f note that, given $Q_{T_f} = 0$ and $p_{T_f} = 1/r$, the ODE $rp_u = 1 - \delta(1 - \mu_{h,u})Q_u^\sigma + \dot{p}_u$ implies that $\dot{p}_{T_f} = 0$. Obviously, since the price is constant after T_f , $\dot{p}_{T_f} = 0$ as well. We conclude that \dot{p}_u is continuous at $u = T_f$ as well.

Next, we show that the candidate equilibrium thus constructed is indeed an equilibrium. For this recall that $0 < Q_u < (1 - \mu_{h,u})^{-1/\sigma}$, which immediately implies that $0 < 1 - rp_u + \dot{p}_u < 1$ for $u < T_f$. It follows that high-valuation traders find it optimal to hold one unit. Now one can directly verify that, for $u < T_f$, the problem of low-valuation traders is solved by $q_{\ell,u} = \min\{(1 - \mu_{h,t})^{-1/\sigma} Q_u, 1\}$. For $u \geq T_f$, $1 - rp_u + \dot{p}_u = 0$ and so the problem of high-valuation traders is solved by any $q_{\ell,u} \in [0, 1]$, while the problem of low-valuation traders is clearly solved by $q_{\ell,u} = 0$. The asset market clears at all dates by construction.

A.9.2 Concluding the proof of Proposition 8: the shape of Q_u We begin with preliminary results that we use repeatedly in this appendix. For the first preliminary result, consider equation (A.15) after removing the min operator in the integral:

$$\begin{aligned} e^{-\rho u}(1 - \mu_{h,0})^{1+1/\sigma} \bar{Q}_u + \int_0^u \rho e^{-\rho(u-t)}(1 - \mu_{h,t})^{1+1/\sigma} \bar{Q}_u dt &= S_u \\ \iff \bar{Q}_u &= \frac{(s - \mu_{h,0}) + \int_0^u \rho e^{\rho t}(s - \mu_{h,t}) dt}{(1 - \mu_{h,0})^{1+1/\sigma} + \int_0^u \rho e^{\rho t}(1 - \mu_{h,t})^{1+1/\sigma} dt}. \end{aligned} \quad (\text{A.16})$$

Now, whenever $(1 - \mu_{h,0})^{1/\sigma} \bar{Q}_u \leq 1$, it is clear that \bar{Q}_u also solves equation (A.15). Given that the solution of (A.15) is unique it follows that $Q_u = \bar{Q}_u$. Conversely if $Q_u = \bar{Q}_u$, subtracting (A.15) from (A.16) shows that:

$$\begin{aligned} & \left((1 - \mu_{h,0})^{1/\sigma} \bar{Q}_u - \min\{(1 - \mu_{h,0})^{1/\sigma} \bar{Q}_u, 1\} \right) \\ & + \int_0^u \rho e^{-\rho(u-t)}(1 - \mu_{h,t}) \left((1 - \mu_{h,t})^{1/\sigma} \bar{Q}_u - \min\{(1 - \mu_{h,t})^{1/\sigma} \bar{Q}_u, 1\} \right) dt = 0. \end{aligned}$$

26. The holding of high-valuation trader is indeterminate. However, it is natural to assume that they have identical holdings, $q_{h,u} = s/\mathbb{E}[\mu_{h,\tau_u}]$. Indeed, we have seen that this is the limit of high-valuation investors' holdings as $\varepsilon \rightarrow 0$.

Since the first term and the integrand are positive, this can only be true if $(1 - \mu_{h,0})^{1/\sigma} \bar{Q}_u \leq 1$. Taken together, we obtain:

Lemma 9 (A useful equivalence) $\bar{Q}_u \leq (1 - \mu_{h,0})^{-1/\sigma}$ if and only if $Q_u = \bar{Q}_u$.

The next Lemma, proved in Section B.1.5, provides basic properties of \bar{Q}_u :

Lemma 10 (Preliminary results about \bar{Q}_u) *The function \bar{Q}_u is continuous, satisfies $\bar{Q}_0 = \frac{s - \mu_{h,0}}{(1 - \mu_{h,0})^{1+1/\sigma}}$ and $\bar{Q}_{T_f} = 0$. It is strictly decreasing over $(0, T_f]$ if $\frac{s - \mu_{h,0}}{1 - \mu_{h,0}} \leq \frac{\sigma}{1 + \sigma}$ and hump shaped otherwise.*

Taken together, Lemma 9 and Lemma 10 immediately imply that

Lemma 11. *The function Q_u satisfies $Q_0 = \frac{s - \mu_{h,0}}{(1 - \mu_{h,0})^{1+1/\sigma}}$, $Q_{T_f} = 0$. If $\frac{s - \mu_{h,0}}{1 - \mu_{h,0}} \leq \frac{\sigma}{1 + \sigma}$, then it is strictly decreasing over $(0, T_f]$. If $\frac{s - \mu_{h,0}}{1 - \mu_{h,0}} > \frac{\sigma}{1 + \sigma}$ and $\bar{Q}_u \leq (1 - \mu_{h,0})^{-1/\sigma}$ for all $u \in (0, T_f]$, Q_u is hump shaped over $(0, T_f]$.*

The only case that is not covered by the Lemma is when $\frac{s - \mu_{h,0}}{1 - \mu_{h,0}} > \frac{\sigma}{1 + \sigma}$ and $\bar{Q}_u > (1 - \mu_{h,0})^{-1/\sigma}$ for some $u \in (0, T_f]$. In this case, note that for u small and u close to T_f , we have that $\bar{Q}_u < (1 - \mu_{h,0})^{-1/\sigma}$. Given that \bar{Q}_u is hump shaped, it follows that the equation $\bar{Q}_u = (1 - \mu_{h,0})^{-1/\sigma}$ has two solutions, $0 < T_1 < T_2 < T_f$. For $u \in (0, T_1]$ (resp. $u \in [T_2, T_f]$), $\bar{Q}_u \leq (1 - \mu_{h,0})^{-1/\sigma}$ and is increasing (resp. decreasing), and thus Lemma 9 implies that $Q_u = \bar{Q}_u$ and increasing (resp. decreasing) as well. We first establish that Q_u is piecewise continuously differentiable. Let $\Psi(Q) \equiv \inf\{\psi \geq 0 : (1 - \mu_{h,\psi})^{1/\sigma} Q \leq 1\}$, and $\psi_u \equiv \Psi(Q_u)$. Thus, $\psi_u > 0$ if and only if $u \in (T_1, T_2)$. We have:

Lemma 12. Q_u is continuously differentiable except in T_1 and T_2 , and

$$Q'_u = \frac{\rho e^{\rho u} (s - \mu_{h,u} - (1 - \mu_{h,u})^{1+1/\sigma} Q_u)}{\mathbb{I}_{\{\psi_u=0\}} (1 - \mu_{h,0})^{1+1/\sigma} + \int_{\psi_u}^u \rho e^{\rho t} (1 - \mu_{h,t})^{1+1/\sigma} dt}. \quad (\text{A.17})$$

The proof is in Appendix B.1.6. In particular, (A.17) implies that $Q_{T_1^+}$ has the same sign as $Q_{T_1^-}$, which is positive, and that $Q_{T_2^-}$ has the same sign as $Q_{T_2^+}$, which is negative. Thus, Q_u changes sign at least once in (T_1, T_2) . To conclude, in Section B.1.7, we establish:

Lemma 13. *The derivative Q'_u changes sign only once in (T_1, T_2) .*

A.9.3 Proof of Proposition 9: asymptotic volume In Section B.1.8 of the Supplementary Appendix, we prove the following asymptotic results:

Lemma 14. *As ρ goes to infinity:*

$$T_f(\rho) \downarrow T_s \quad (\text{A.18})$$

$$Q_u(\rho) = \frac{s - \mu_{h,u}}{(1 - \mu_{h,u})^{1+1/\sigma}} - \frac{1}{\rho} \frac{\gamma}{(1 - \mu_{h,u})^{1/\sigma}} \left[\left(1 + \frac{1}{\sigma}\right) \frac{s - \mu_{h,u}}{1 - \mu_{h,u}} - 1 \right] + o\left(\frac{1}{\rho}\right), \quad \forall u \in [0, T_s] \quad (\text{A.19})$$

$$T_\psi(\rho) = \arg \max_{u \in [0, T_f(\rho)]} Q_u(\rho) \longrightarrow \arg \max_{u \in [0, T_s]} \frac{s - \mu_{h,u}}{(1 - \mu_{h,u})^{1+1/\sigma}}. \quad (\text{A.20})$$

With this in mind we can study the asymptotic behavior of volume.

Basic formulas: As above, let T_ψ denote the argmax of the function Q_u . For any time $u < T_\psi$ and some time interval $[u, u + du]$, the only traders who *sell* are those who have an updating time during this time interval, and who find out that they have a *low* valuation. Thus, trading volume during $[u, u + du]$ can be computed as the volume of assets sold by these investors, as follows. Just before their updating time, low-valuation investors hold on average:

$$\mathbb{E}[q_{\ell, \tau_u, u}] = e^{-\rho u} q_{\ell, 0, u} + \int_0^u \rho e^{-\rho(u-t)} q_{\ell, t, u} dt. \quad (\text{A.21})$$

Instantaneous trading volume is then:

$$V_u = \rho(1 - \mu_{h,u}) \left(\mathbb{E}[q_{\ell, \tau_u, u}] - q_{\ell, u, u} \right), \quad (\text{A.22})$$

where $\rho(1 - \mu_{h,u})$ is the measure of low-valuation investors having an updating time, the term in large parentheses is the average size of low-valuation investors' sell orders, and $q_{\ell,u,u}$ is their asset holding right after the updating time.

For any time $u \in (T_\psi, T_f)$ and some time interval $[u, u + du]$, the only traders who *buy* are those who have an updating time during this time interval, and who find out that they have switched from a low to a high valuation. Trading volume during $[u, u + du]$ can be computed as the volume of assets purchased by these traders:

$$V_u = \rho \mathbb{E}[(1 - \mu_{h,\tau_u})\pi_{\tau_u,u}(1 - q_{\ell,\tau_u,u})] = \rho \mathbb{E}[(\mu_{h,u} - \mu_{h,\tau_u})(1 - q_{\ell,\tau_u,u})], \tag{A.23}$$

where the second equality follows by definition of $\pi_{\tau_u,u}$.

Finally, for $u > T_f$, the trading volume is not zero since high-valuation traders continue to buy from the low valuation investors having an updating time:

$$V_u = \rho(1 - \mu_{hu})\mathbb{E}[q_{\ell,\tau_u,u}]. \tag{A.24}$$

Taking the $\rho \rightarrow \infty$ limit. We first note that $q_{\ell,u,u}(\rho) = \min\{(1 - \mu_{h,u})^{1/\sigma} Q_u, 1\} = (1 - \mu_{h,u})^{1/\sigma} Q_u(\rho)$. Next, we need to calculate an approximation for:

$$\mathbb{E}[q_{\ell,\tau_u,u}] = q_{\ell,0,u}e^{-\rho u} + \int_0^u \min\{(1 - \mu_{h,t})^{1/\sigma} Q_u(\rho), 1\} \rho e^{-\rho(u-t)} dt.$$

For this we follow the same calculations leading to equation (14) in the proof of Lemma 14, but with $f(t, \rho) = \min\{(1 - \mu_{h,t})^{1/\sigma} Q_u(\rho), 1\}$. This gives:

$$\begin{aligned} \mathbb{E}[q_{\ell,\tau_u,u}] &= f(0, u)e^{-\rho u} + \int_0^u \rho e^{-\rho(u-t)} f(t, \rho) dt = f(u, \rho) - \frac{1}{\rho} f_t(u, \rho) + o\left(\frac{1}{\rho}\right) \\ &= (1 - \mu_{h,u})^{1/\sigma} Q_u(\rho) + \frac{1}{\rho} \frac{\gamma}{\sigma} \frac{s - \mu_{h,u}}{1 - \mu_{h,u}} + o\left(\frac{1}{\rho}\right) = \frac{s - \mu_{h,u}}{1 - \mu_{h,u}} + \frac{\gamma}{\rho} \frac{1 - s}{1 - \mu_{h,u}} + o\left(\frac{1}{\rho}\right), \end{aligned}$$

where the second line follows from plugging equation (A.19) into the first line. Substituting this expression into equations (A.22) and (A.23), we find after some straightforward manipulation that, when ρ goes to infinity, $V_u \rightarrow \gamma(s - \mu_{h,u})/\sigma$ for $u < T_\psi(\infty)$, $V_u \rightarrow \gamma(1 - s)$ for $u \in (\lim T_\psi(\infty), T_s)$, and $V_u \rightarrow 0$ for $u > T_s$.

The trading volume in the Walrasian equilibrium is equal to the measure of low-valuation investors who become high-valuation investors: $\gamma(1 - \mu_{h,u})$, times the amount of asset they buy at that time: $1 - (s - \mu_{h,u})/(1 - \mu_{h,u})$. Thus the trading volume is $\gamma(1 - s)$. To conclude the proof, note that after taking derivatives of $Q_u(\infty)$ with respect to u , it follows that

$$Q'_{T_\psi(\infty)}(\infty) = 0 \Leftrightarrow \frac{s - \mu_{hT_\psi(\infty)}}{\sigma} = 1 - s$$

which implies in turn that $\gamma(s - \mu_{h,u})/\sigma > \gamma(1 - s)$ for $u < T_\psi(\infty)$.

A.9.4 Proof of Proposition 10 We have already argued that the price is continuously differentiable. To prove that it is strictly increasing for $u \in [0, T_f)$, we let $\Delta_u \equiv (1 - \mu_{h,u})^{1/\sigma} Q_u$ for $u \leq T_f$, and $\Delta_u = 0$ for $u \geq T_f$. In Section B.1.9 of the Supplementary Appendix, we show that:

Lemma 15. *The function Δ_u is strictly decreasing over $(0, T_f]$.*

Now, in terms of Δ_u , the price writes:

$$p_u = \int_u^\infty e^{-r(y-u)} (1 - \delta \Delta_y^\sigma) dy = \int_0^\infty e^{-rz} (1 - \delta \Delta_{z+u}^\sigma) dz,$$

after the change of variable $y - u = z$. Since Δ_u is strictly decreasing over $u \in (0, T_f)$, and constant over $[T_f, \infty)$, it clearly follows from the above formula that p_u is strictly increasing over $u \in (0, T_f)$. Next:

First bullet point: when (25) does not hold. Given the ODEs satisfied by the price path, it suffices to show that, for all $u \in (0, T_s)$,

$$(1 - \mu_{h,u})Q_u^\sigma > \left(\frac{s - \mu_{h,u}}{1 - \mu_{h,u}}\right)^\sigma.$$

Besides, when condition (25) holds, it follows from Lemma 9 and Lemma 10 that:

$$Q_u = \bar{Q}_u = \frac{s - \mu_{h,0} + \int_0^u e^{\rho t} (s - \mu_{h,t}) dt}{(1 - \mu_{h,0})^{1+1/\sigma} + \int_0^u e^{\rho t} (1 - \mu_{h,t})^{1+1/\sigma} dt}.$$

Plugging the above and rearranging, we are left with showing that:

$$F_u = (s - \mu_{h,u}) \left[(1 - \mu_{h,0})^{1+1/\sigma} + \int_0^u e^{\rho t} (1 - \mu_{h,t})^{1+1/\sigma} dt \right] - (1 - \mu_{h,u})^{1+1/\sigma} \left[s - \mu_{h,0} + \int_0^u e^{\rho t} (s - \mu_{h,t}) dt \right] < 0.$$

But we know from the proof of Lemma 10, equation (7), that F_u has the same sign as Q'_u , which we know is negative at all $u > 0$ since $\frac{s - \mu_{h,0}}{1 - \mu_{h,0}} \leq \frac{\sigma}{\sigma + 1}$.

Second bullet point: when condition (25) holds and when s is close to 1 and σ is close to 0. The price at time 0 is equal to:

$$p_0 = \int_0^{+\infty} e^{-ru} \xi_u du.$$

With known preferences, $\xi_u^* = 1 - \delta \left(\frac{s - \mu_{h,u}}{1 - \mu_{h,u}} \right)^\sigma = 1 - \delta \left(1 - \frac{1-s}{1 - \mu_{h,0}} e^{\gamma u} \right)^\sigma$ for $u < T_s$, and $\xi_u^* = 1$ for $u > T_s$. Therefore $p_0^* = 1/r - \delta J^*(s)$, where:

$$J^*(s) \equiv \int_0^{T_s} e^{-ru} \left(1 - \frac{1-s}{1 - \mu_{h,0}} e^{\gamma u} \right)^\sigma du,$$

where we make the dependence of $J^*(s)$ on s explicit. Similarly, with preference uncertainty, the price at time 0 is equal to $p_0 = 1/r - \delta J(s)$, where:

$$J(s) \equiv \int_0^{T_f} e^{-ru} (1 - \mu_{h,u}) Q_u^\sigma du.$$

We begin with a Lemma proved in Section B.1.10:

Lemma 16. *When s goes to 1, both $J^*(s)$ and $J(s)$ go to $1/r$.*

Therefore, p_0 goes to $(1 - \delta)/r$ both with continuous and infrequent updating. Besides, with continuous updating:

$$p_0(s_0) = (1 - \delta)/r + \delta \int_{s_0}^1 J^{*\prime}(s) ds,$$

and with infrequent updating:

$$p_0(s_0) = (1 - \delta)/r + \delta \int_{s_0}^1 J'(s) ds.$$

The next two lemmas compare $J^{*\prime}(s)$ and $J'(s)$ for s in the neighbourhood of 1 when σ is not too large. The first Lemma is proved in Section B.1.11.

Lemma 17. *When s goes to 1:*

$$\begin{aligned} J^{*\prime}(s) &\sim \sigma \times \text{constant} && \text{if } r > \gamma, \\ J^{*\prime}(s) &\sim \Gamma_1(\sigma) \log((1-s)^{-1}) && \text{if } r = \gamma, \\ J^{*\prime}(s) &\sim \Gamma_2(\sigma) (1-s)^{-1+r/\gamma} && \text{if } r < \gamma, \end{aligned}$$

where the constant terms $\Gamma_1(\sigma)$ and $\Gamma_2(\sigma)$ go to 0 when $\sigma \rightarrow 0$.

In this Lemma and all what follows $f(s) \sim g(s)$ means that $f(s)/g(s) \rightarrow 1$ when $s \rightarrow 1$. The second Lemma is proved in Section B.1.12 of the supplementary appendix:

Lemma 18. *Assume $\gamma + \gamma/\sigma - \rho > 0$. There exists a function $\tilde{J}'(s) \leq J'(s)$ such that, when s goes to 1:*

$$\begin{aligned} \tilde{J}'(s) &\rightarrow +\infty && \text{if } r > \gamma, \\ \tilde{J}'(s) &\sim \Gamma_3(\sigma) \log((1-s)^{-1}) && \text{if } r = \gamma, \\ \tilde{J}'(s) &\sim \Gamma_4(\sigma) (1-s)^{-1+r/\gamma} && \text{if } r < \gamma, \end{aligned}$$

where the constant terms $\Gamma_3(\sigma)$ and $\Gamma_4(\sigma)$ go to strictly positive limits when $\sigma \rightarrow 0$.

Lemmas 17 and 18 imply that, if σ is close to 0, then $J'(s) > J^{**}(s)$ for s in the left neighbourhood of 1. The second point of the proposition then follows.

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Supplementary Data

Supplementary data are available at *Review of Economic Studies* online.

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