# Can Risk Be Shared across Investor Cohorts? Evidence from a Popular Savings Product 

## Internet Appendix

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This internet appendix contains supplementary material for "Can Risk Be Shared across Investor Cohorts? Evidence from a Popular Savings Product."

- Section A contains the proofs of all the analytical results in the paper.
- Section B presents the institutional environment.
- Section C describes the data sources and variable construction.
- Section D presents the extension of the model to the case where contracts are held for several periods.
- Section E shows additional analysis of demand for euro contracts.
- Section F presents the analysis of reserves mean reversion.
- Section G presents the decomposition of cross-sectional variation in reserves.
- Section H analyzes tax-induced switching costs.

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## A Proofs

## A. 1 Proof of Proposition 1

We denote by $\mathcal{H}^{t}=\left(x^{t}, \xi^{t}\right)$ the history of shocks up to time $t$. Intermediary $j$ chooses the contract return policy $\left\{y_{j, t}\left(\mathcal{H}^{t}\right)\right\}_{t \geq 1}$ to maximize expected discounted profit

$$
\begin{equation*}
E_{0}\left[\sum_{t=1}^{+\infty} \frac{1}{(1+r)^{t}} \frac{\phi}{1-\phi} y_{j, t} V_{j, t-1}\left(\left\{E_{t-1}\left[u\left(y_{k, t}\right)\right]\right\}_{k=0, \ldots, J}\right)\right], \tag{A.1}
\end{equation*}
$$

where we omit argument $\mathcal{H}^{t}$ in $y_{j, t}$, and demand for contract $j$ in period $t$ is the function of the collection of expected utility of all contracts' returns

$$
\begin{equation*}
V_{j, t-1}\left(\left\{U_{k}\right\}_{k=0, \ldots, J}\right)=\frac{\exp \left\{\alpha U_{j}+\xi_{j, t-1}\right\}}{\sum_{k=0}^{J} \exp \left\{\alpha U_{k}+\xi_{k, t-1}\right\}} \tag{A.2}
\end{equation*}
$$

This maximization problem is subject to the intertemporal budget constraint

$$
\begin{equation*}
\sum_{t=1}^{+\infty} \frac{1}{(1+r)^{t}}\left[\left(x_{j, t}-\frac{1}{1-\phi} y_{j, t}\right) V_{j, t-1}\left(\left\{E_{t-1}\left[u\left(y_{k, t}\right)\right]\right\}_{k=0, \ldots, J}\right) \prod_{s=t+1}^{+\infty} \frac{1+x_{j, s}}{1+r}\right] \geq 0, \quad \forall \mathcal{H}^{t} \tag{A.3}
\end{equation*}
$$

(A.1) is obtained by plugging per-period profit (7) into intertemporal profit (8). (A.3) is obtained by plugging profit (7) into the sequential budget constraint (4), consolidating the budget constraint intertemporally, and using the transversality condition (9). (A.3) must hold for all histories $\mathcal{H}^{T}$. We denote by $\lambda_{j}\left(\mathcal{H}^{T}\right)$ the Lagrange multiplier of (A.3) divided by the probability of history $\mathcal{H}^{T}$. Therefore, the Lagrangian associated with intermediary $j$ 's problem is equal to (A.1) plus the time- 0 expectation of $\lambda_{j}\left(\mathcal{H}^{T}\right)$ times the LHS of (A.3). The first-order condition with respect to $y_{j, t}\left(\mathcal{H}^{t}\right)$ is

$$
\begin{gather*}
E_{t-1}\left[\frac{\phi}{1-\phi} y_{j, t}+\lambda_{j}\left(x_{j, t}-\frac{1}{1-\phi} y_{j, t}\right) \prod_{s=t+1}^{+\infty} \frac{1+x_{j, s}}{1+r}\right] V_{j, t-1}^{(j)}\left(\left\{E_{t-1}\left[u\left(y_{k, t}\right)\right]\right\}_{k=0, \ldots, J}\right) u^{\prime}\left(y_{j, t}\right) \\
+\left(\frac{\phi}{1-\phi}-\frac{1}{1-\phi} E_{t}\left[\lambda_{j} \prod_{s=t+1}^{+\infty} \frac{1+x_{j, s}}{1+r}\right]\right) V_{j, t-1}\left(\left\{E_{t-1}\left[u\left(y_{k, t}\right)\right]\right\}_{k=0, \ldots, J}\right)=0, \tag{A.4}
\end{gather*}
$$

where we continue to omit argument $\mathcal{H}^{t}$ in $y_{j, t}$ and $\operatorname{argument} \mathcal{H}^{T}$ in $\lambda_{j}, V_{j, t-1}^{(k)}=\partial V_{j, t-1} / \partial U_{k}$ denotes the partial derivative of demand, and $V_{j, t-1}^{(k \ell)}=\partial^{2} V_{j, t-1} /\left(\partial U_{k} \partial U_{\ell}\right)$ the cross-derivative. We write
asset returns and demand shocks, as follows:

$$
\begin{align*}
& x_{j, t}=r+\sigma \epsilon_{j, t}^{x}  \tag{A.5}\\
& \xi_{j, t}=\xi_{j}+\sigma \zeta_{j, t}=\xi_{j}+\sigma \sum_{s=1}^{t} \epsilon_{j, s}^{\xi} \tag{A.6}
\end{align*}
$$

for $j=0, \ldots, J$ and $t \geq 1$, where $\sigma>0, \epsilon_{j, t}^{x}$ and $\epsilon_{j, t}^{\xi}$ are realized at the end of period $t$, have bounded support, zero mean, and are equal to zero for $t>T$. We solve the model using a firstorder approximation for small shocks. We guess that, when $\sigma$ goes to zero,

$$
\begin{align*}
& y_{j, t}=y_{j, t}^{0}+\sigma y_{j, t}^{\prime}+O\left(\sigma^{2}\right),  \tag{A.7}\\
& \lambda_{j}=\lambda_{j}^{0}+\sigma \lambda_{j}^{\prime}+O\left(\sigma^{2}\right), \tag{A.8}
\end{align*}
$$

where $y_{j, t}^{0}$ and $\lambda_{j}^{0}$ are deterministic, $y_{j, t}^{\prime}$ and $\lambda_{j}^{\prime}$ are functions of $\mathcal{H}^{T}$, and $O\left(\sigma^{2}\right)$ denote functions of $\left(\mathcal{H}^{T}, \sigma\right)$ at the order of $\sigma^{2}$, that is, there exist $K>0$ and $\bar{\sigma}>0$ such that $O\left(\sigma^{2}\right) \leq K \sigma^{2}$ for all $\mathcal{H}^{T}$ and $\sigma<\bar{\sigma}$. We determine $y_{j, t}^{0}$ and $\lambda_{j}^{0}$ by letting $\sigma$ go to zero in the intertemporal budget constraint (A.3) and first-order condition (A.4). The latter yields

$$
\begin{equation*}
\left(\phi y_{j, t}^{0}+\lambda_{j}^{0}\left((1-\phi) r-y_{j, t}^{0}\right)\right) V_{j}^{(j)}\left(\left\{u\left(y_{k, t}^{0}\right)\right\}_{k=0, \ldots, J}\right) u^{\prime}\left(y_{j, t}^{0}\right)+\left(\phi-\lambda_{j}^{0}\right) V_{j}\left(\left\{u\left(y_{k, t}^{0}\right)\right\}_{k=0, \ldots, J}\right)=0 \tag{A.9}
\end{equation*}
$$

where $V_{j}(.) \equiv V_{j, 0}($.$) denotes the demand function when demand shocks \zeta_{k, t}$ are set to zero for all $k$. Since (A.9) does not depend on $t, y_{k, t}^{0}$ does not depend on $t$, and we denote it by $y_{k}^{0}$. Letting $\sigma$ go to zero in (A.3), we obtain

$$
\begin{equation*}
\sum_{t=1}^{+\infty} \frac{1}{(1+r)^{t}}\left(r-\frac{1}{1-\phi} y_{j}^{0}\right) V_{j}\left(\left\{u\left(y_{k}^{0}\right)\right\}_{k=0, \ldots, J}\right)=0 \tag{A.10}
\end{equation*}
$$

The solution to (A.10) is symmetric across intermediaries, and is given by

$$
\begin{equation*}
y_{j}^{0}=(1-\phi) r \tag{A.11}
\end{equation*}
$$

Substituting $y_{j}^{0}$ into (A.9), we obtain

$$
\begin{equation*}
\lambda_{j}^{0}=\phi+\phi(1-\phi) r \frac{V_{j}^{(j)}}{V_{j}} u^{\prime} \tag{A.12}
\end{equation*}
$$

where we omit argument $y_{j}^{0}$ in $u($.$) and its derivatives, and we omit argument \left\{u\left(y_{k}^{0}\right)\right\}_{k=0, \ldots, J}$ in $V_{j}($.$) and its derivatives. We determine y_{j, t}^{\prime}$ and $\lambda_{j}^{\prime}$ by calculating first-order approximations of the intertemporal budget constraint (A.3) and first-order condition (A.4). Let us first write down a first-order approximation of the demand function (A.2):

$$
\begin{align*}
& V_{j, t-1}\left(\left\{E_{t-1}\left[u\left(y_{k, t}\right)\right]\right\}_{k=0, \ldots, J}\right)=V_{j}\left(\left\{u\left(y_{k}^{0}\right)\right\}_{k=0, \ldots, J}\right) \\
&+\sigma \sum_{k=0}^{J} V_{j}^{(k)}\left(\left\{u\left(y_{k}^{0}\right)\right\}_{k=0, \ldots, J}\right) u^{\prime}\left(y_{k}^{0}\right)\left(E_{t-1}\left[y_{k, t}^{\prime}\right]+\frac{\zeta_{k, t-1}}{\alpha u^{\prime}\left(y_{k}^{0}\right)}\right)+O\left(\sigma^{2}\right) \tag{A.13}
\end{align*}
$$

The analogous approximation holds for $V_{j, t-1}^{(j)}$. A first-order approximation of the budget constraint (A.3) gives

$$
\begin{equation*}
\sum_{t=1}^{+\infty} \frac{1}{(1+r)^{t}}\left(\epsilon_{j, t}^{x}-\frac{1}{1-\phi} y_{j, t}^{\prime}\right) V_{j}=0 \tag{A.14}
\end{equation*}
$$

We denote $y_{j, t}^{s}=E_{s}\left[y_{j, t}^{\prime}\right]-E_{s-1}\left[y_{j, t}^{\prime}\right]$ as the time- $s$ innovation of $y_{j, t}^{\prime}$ for all $1 \leq s \leq t$. Calculating $E_{s}[$.$] of (A.14) minus E_{s-1}[$.$] of (A.14), we obtain$

$$
\begin{equation*}
\sum_{t=s}^{+\infty} \frac{y_{j, t}^{s}}{(1+r)^{t-s}}=(1-\phi) \epsilon_{j, s}^{x}, \quad s \leq t . \tag{A.15}
\end{equation*}
$$

A first-order approximation of the first-order condition (A.4) gives

$$
\begin{gather*}
\frac{\phi}{1-\phi} E_{t-1}\left[y_{j, t}^{\prime}\right] V_{j}^{(j)} u^{\prime}-\lambda_{j}^{0} \frac{1}{1-\phi} E_{t-1}\left[y_{j, t}^{\prime}\right] V_{j}^{(j)} u^{\prime}+\phi r \sum_{k=0}^{J} V_{j}^{(j k)}\left(u^{\prime}\right)^{2}\left(E_{t-1}\left[y_{k, t}^{\prime}\right]+\frac{\zeta_{k, t-1}}{\alpha u^{\prime}}\right)+\phi r V_{j}^{(j)} u^{\prime \prime} y_{j, t}^{\prime} \\
-\frac{1}{1-\phi} E_{t}\left[\lambda_{j}^{\prime}\right] V_{j}+\left(\frac{\phi}{1-\phi}-\frac{1}{1-\phi} \lambda_{j}^{0}\right) \sum_{k=0}^{J} V_{j}^{(k)} u^{\prime}\left(E_{t-1}\left[y_{k, t}^{\prime}\right]+\frac{\zeta_{k, t-1}}{\alpha u^{\prime}}\right)=0, \tag{A.16}
\end{gather*}
$$

where we have multiplied the first-order condition by $(1+r)^{t}$ and we have used $y_{j}^{0}=(1-\phi) r$. Using (A.12), we substitute $\lambda_{j}^{0}$ into (A.16). We then divide by $\phi r V_{j}^{(j)} u^{\prime \prime}$, to obtain

$$
\begin{align*}
& y_{j, t}^{\prime}+\frac{V_{j}^{(j)}}{V_{j}} \frac{\left(u^{\prime}\right)^{2}}{-u^{\prime \prime}} E_{t-1}\left[y_{j, t}^{\prime}\right]+\sum_{k=0}^{J}\left(\frac{V_{j}^{(k)}}{V_{j}}-\frac{V_{j}^{(j k)}}{V_{j}^{(j)}}\right) \frac{\left(u^{\prime}\right)^{2}}{-u^{\prime \prime}}\left(E_{t-1}\left[y_{k, t}^{\prime}\right]+\frac{\zeta_{k, t-1}}{\alpha u^{\prime}}\right) \\
&=\frac{V_{j}}{(1-\phi) \phi r V_{j}^{(j)} u^{\prime \prime}} E_{t}\left[\lambda_{j}^{\prime}\right] . \tag{A.17}
\end{align*}
$$

We denote $\lambda_{j}^{s}=E_{s}\left[\lambda_{j}^{\prime}\right]-E_{s-1}\left[\lambda_{j}^{\prime}\right]$ the time- $s$ innovation of $\lambda_{j}^{\prime}$ for all $1 \leq s \leq T$. Calculating (A.17) minus $E_{t-1}$ [.] of (A.17), we obtain

$$
\begin{equation*}
y_{j, t}^{t}=\frac{V_{j}}{(1-\phi) \phi r V_{j}^{(j)} u^{\prime \prime}} \lambda_{j}^{t} . \tag{A.18}
\end{equation*}
$$

Calculating $E_{s}[$.$] of (A.17) minus E_{s-1}[$.$] of (A.17) for s<t$, we obtain

$$
\begin{equation*}
\left(1+\frac{V_{j}^{(j)}}{V_{j}} \frac{\left(u^{\prime}\right)^{2}}{-u^{\prime \prime}}\right) y_{j, t}^{s}+\sum_{k=0}^{J}\left(\frac{V_{j}^{(k)}}{V_{j}}-\frac{V_{j}^{(j k)}}{V_{j}^{(j)}}\right) \frac{\left(u^{\prime}\right)^{2}}{-u^{\prime \prime}}\left(y_{k, t}^{s}+\frac{\epsilon_{k, s}^{\xi}}{\alpha u^{\prime}}\right)=\frac{V_{j}}{(1-\phi) \phi r V_{j}^{(j)} u^{\prime \prime}} \lambda_{j}^{s}, \quad s<t . \tag{A.19}
\end{equation*}
$$

The derivatives of the logit demand function are
$\frac{V_{j}^{(j)}}{V_{j}}=\alpha\left(1-s_{j}\right), \quad \frac{V_{j}^{(j j)}}{V_{j}^{(j)}}=\alpha\left(1-2 s_{j}\right), \quad \frac{V_{j}^{(k)}}{V_{j}}=-\alpha s_{k}, \quad \frac{V_{j}^{(j k)}}{V_{j}^{(j)}}=-\alpha s_{k}\left(1-\frac{s_{j}}{1-s_{j}}\right), \quad k \neq j$,
where

$$
\begin{equation*}
s_{j} \equiv V_{j}=\frac{\exp \left\{\xi_{j}\right\}}{\sum_{k=0}^{J} \exp \left\{\xi_{k}\right\}} \tag{A.20}
\end{equation*}
$$

is intermediary $j$ 's market share when all intermediaries offer the same contract return and all demand shocks are set to zero. We use these expressions, and (A.18) to substitute $\lambda_{j}^{s}$ on the right-hand side of (A.19). We obtain

$$
\begin{equation*}
y_{j, t}^{s}=\frac{\rho_{j}}{\alpha+\rho_{j}} y_{j, s}^{s}+\frac{\alpha \delta_{j}}{\alpha+\rho_{j}} \sum_{k=0}^{J} s_{k} y_{k, t}^{s}-\frac{\frac{1}{u^{\prime}} \delta_{j}}{\alpha+\rho_{j}}\left(\epsilon_{j, s}^{\xi}-\sum_{k=1}^{J} s_{k} \epsilon_{k, s}^{\xi}\right), \quad s<t, \tag{A.21}
\end{equation*}
$$

where

$$
\begin{align*}
\rho_{j} & =\frac{1}{1+\frac{s_{j}^{2}}{1-s_{j}}} \frac{1}{(1-\phi) r} \frac{-u^{\prime \prime}[(1-\phi) r](1-\phi) r}{u^{\prime}[(1-\phi) r]},  \tag{A.22}\\
\delta_{j} & =\frac{1}{1+\frac{s_{j}^{2}}{1-s_{j}}} \frac{s_{j}}{1-s_{j}} \tag{A.23}
\end{align*}
$$

where we have used the normalization $u^{\prime}[(1-\phi) r]=1$. To calculate the term $\sum_{k=0}^{J} s_{k} y_{k, t}^{s}$ in (A.21), we first note that $y_{0, t}^{s}=0$ for $s<t$. Then, multiplying (A.21) by $s_{j}$, and summing over $j=1, \ldots, J$,
we obtain

$$
\begin{equation*}
(1-A) \sum_{k=0}^{J} s_{k} y_{k, t}^{s}=\sum_{k=1}^{J} \frac{\rho_{k} s_{k}}{\alpha+\rho_{k}} y_{k, s}^{s}-\sum_{k=1}^{J} \frac{\frac{1}{u^{\prime}} \delta_{k} s_{k}}{\alpha+\rho_{k}}\left(\epsilon_{k, s}^{\xi}-\sum_{\ell=1}^{J} s_{\ell} \epsilon_{\ell, s}^{\xi}\right), \quad s<t . \tag{A.24}
\end{equation*}
$$

where $A=\sum_{k=1}^{J} \frac{\alpha \delta_{k} s_{k}}{\alpha+\rho_{k}}$. Substituting the expression of $\sum_{k=0}^{J} s_{k} y_{k, t}^{s}$ given by (A.24) into (A.21), and collecting the terms $\epsilon_{k, s}^{\xi}$, we obtain

$$
\begin{equation*}
y_{j, t}^{s}=\frac{\rho_{j}}{\alpha+\rho_{j}} y_{j, s}^{s}+\frac{1}{1-A} \frac{\alpha \delta_{j}}{\alpha+\rho_{j}} \sum_{k=1}^{J} \frac{\rho_{k} s_{k}}{\alpha+\rho_{k}} y_{k, s}^{s}-\frac{\frac{1}{u^{\prime}} \delta_{j}}{\alpha+\rho_{j}}\left(\epsilon_{j, s}^{\xi}-\bar{\epsilon}_{s}^{\xi}\right), \quad s<t, \tag{A.25}
\end{equation*}
$$

where

$$
\begin{equation*}
\bar{\epsilon}_{s}^{\xi}=\frac{1}{1-A} \sum_{k=1}^{J}\left(1-\frac{\alpha \delta_{k}}{\alpha+\rho_{k}}\right) s_{k} \epsilon_{k, s}^{\xi} . \tag{A.26}
\end{equation*}
$$

Substituting the expression of $y_{j, t}^{s}$ given by (A.25) into the budget constraint (A.15), we obtain

$$
\begin{equation*}
\frac{\alpha+\frac{1+r}{r} \rho_{j}}{\alpha+\rho_{j}} y_{j, s}^{s}+\frac{1}{1-A} \frac{\frac{1}{r} \alpha \delta_{j}}{\alpha+\rho_{j}} \sum_{k=1}^{J} \frac{\rho_{k} s_{k}}{\alpha+\rho_{k}} y_{k, s}^{s}=(1-\phi) \epsilon_{j, s}^{x}+\frac{\frac{1}{r u^{\prime}} \delta_{j}}{\alpha+\rho_{j}}\left(\epsilon_{j, s}^{\xi}-\bar{\epsilon}_{s}^{\xi}\right) . \tag{A.27}
\end{equation*}
$$

To determine $\sum_{k=1}^{J} \frac{\rho_{k} s_{k}}{\alpha+\rho_{k}} y_{k, s}^{s}$, we multiply both sides of (A.27) by $\frac{\rho_{j} s_{j}}{\alpha+\frac{1+r}{r} \rho_{j}}$, sum over $j=1, \ldots, J$, and rearrange terms, to obtain

$$
\begin{equation*}
\frac{1-B}{1-A} \sum_{k=1}^{J} \frac{\rho_{k} s_{k}}{\alpha+\rho_{k}} y_{k, s}^{s}=(1-\phi) \widehat{\epsilon}_{s}^{x}+\frac{1}{\alpha u^{\prime}} \widehat{\epsilon}_{s}^{\xi}, \tag{A.28}
\end{equation*}
$$

where $B=\sum_{k=1}^{J} \frac{\alpha \delta_{k} s_{k}}{\alpha+\frac{1+r+}{r} \rho_{k}}$, and

$$
\begin{align*}
\widehat{\epsilon}_{s}^{x} & =\sum_{k=1}^{J} \frac{\rho_{k} s_{k}}{\alpha+\frac{1+r}{r} \rho_{k}} \epsilon_{k, s}^{x},  \tag{A.29}\\
\widehat{\epsilon}_{s}^{\xi} & =\sum_{k=1}^{J} \frac{\rho_{k} s_{k}}{\alpha+\frac{1+r}{r} \rho_{k}} \frac{\frac{\alpha}{r} \delta_{k}}{\alpha+\rho_{k}}\left(\epsilon_{k, s}^{\xi}-\bar{\epsilon}_{s}^{\xi}\right) . \tag{A.30}
\end{align*}
$$

Substituting (A.28) back into (A.27), we obtain

$$
\begin{equation*}
y_{j, s}^{s}=\frac{\alpha+\rho_{j}}{\alpha+\frac{1+r}{r} \rho_{j}}(1-\phi) \epsilon_{j, s}^{x}-\frac{\frac{1}{r} \alpha \delta_{j}}{\alpha+\frac{1+r}{r} \rho_{j}} \frac{1}{1-B}(1-\phi) \widehat{\epsilon}_{s}^{x}+\frac{\frac{1}{r u^{\prime}} \delta_{j}}{\alpha+\frac{1+r}{r} \rho_{j}}\left(\epsilon_{j, s}^{\xi}-\bar{\epsilon}_{s}^{\xi}-\frac{1}{1-B} \widehat{\epsilon}_{s}^{\xi}\right) . \tag{A.31}
\end{equation*}
$$

Substituting the expression of $y_{j, s}^{s}$ given by (A.31) into (A.25) we obtain

$$
\begin{equation*}
y_{j, t}^{s}=\frac{\rho_{j}}{\alpha+\frac{1+r}{r} \rho_{j}}(1-\phi) \epsilon_{j, s}^{x}+\frac{\alpha \delta_{j}}{\alpha+\frac{1+r}{r} \rho_{j}} \frac{1}{1-B}(1-\phi) \widehat{\epsilon}_{s}^{x}-\frac{\frac{1}{u^{\delta}} \delta_{j}}{\alpha+\frac{1+r}{r} \rho_{j}}\left(\epsilon_{j, s}^{\xi}-\bar{\epsilon}_{s}^{\xi}-\frac{1}{1-B} \widehat{\epsilon}_{s}^{\xi}\right), \quad s<t . \tag{A.32}
\end{equation*}
$$

Finally, we use (A.11), (A.31), and (A.32) to calculate $y_{j, t}=y_{j, t}^{0}+\sigma \sum_{s=1}^{t} y_{j, t}^{s}+O\left(\sigma^{2}\right)$. We obtain:

$$
\begin{equation*}
y_{j, t}=(1-\phi)\left[r+\sum_{s=1}^{t-1} \frac{\rho_{j}}{\alpha+\frac{1+r}{r} \rho_{j}}\left(x_{j, s}-r\right)+\frac{\alpha+\rho_{j}}{\alpha+\frac{1+r}{r} \rho_{j}}\left(x_{j, t}-r\right)\right]+f_{j, t}\left(\bar{x}^{t}-r, \xi^{t}\right)+O\left(\sigma^{2}\right), \tag{A.33}
\end{equation*}
$$

where $f_{j, t}($.$) is a function of the history of average asset return shocks \bar{x}^{t}-r$ and demand shocks $\xi^{t}$ :

$$
\begin{align*}
& f_{j, t}\left(\bar{x}^{t}-r, \xi^{t}\right)=\frac{\alpha \delta_{j}}{\alpha+\frac{1+r}{r} \rho_{j}} \frac{\sum_{k=1}^{J} \frac{\rho_{k} s_{k}}{\alpha+\frac{1+r}{r} \rho_{k}}}{1-\sum_{k=1}^{J} \frac{\alpha \delta_{s_{k}}}{\alpha+\frac{1+r}{r} \rho_{k}}}(1-\phi)\left(\sum_{s=1}^{t-1}\left(\bar{x}_{s}-r\right)-\frac{1}{r}\left(\bar{x}_{t}-r\right)\right)+g_{j, t}(  \tag{t}\\
& \bar{x}_{t}=\sum_{k=1}^{J} \frac{\frac{\rho_{k} s_{k}}{\alpha+\frac{1+r}{r} \rho_{k}}}{\sum_{\ell=1}^{J} \frac{\rho_{\ell} s_{\ell}}{\alpha+\frac{1+r}{r} \rho_{\ell}}} x_{k, t}, \tag{A.35}
\end{align*}
$$

and $s_{j}, \rho_{j}$, and $\delta_{j}$ are given by (A.20), (A.22), and (A.23), respectively.

## A. 2 Sufficient Condition for Binding Regulatory Constraint

Suppose that instead of modeling the regulatory constraint (7) as an equality (with "="), we model it as an inequality (with " $\leq$ ") as in the actual regulation of euro contracts described in Appendix B.2. In this case, we show that a sufficient condition for this constraint to be binding, and therefore equivalent to the constraint modeled with an equality, is:

$$
\begin{equation*}
\frac{\phi \alpha}{1-\phi}<1 . \tag{A.36}
\end{equation*}
$$

Suppose (A.36) holds. Let $\kappa \in\left(1, \frac{1-\phi}{\phi \alpha}\right)$. To show that the regulatory constraint is binding, we need to show that intermediaries can increase their intertemporal profit by violating the constraint. Consider a marginal increase $d \pi_{j, t}>0$ in the fraction of account value that goes to intermediary $j$ in period $t$ and a reduction in contract return $d y_{j, t}=-\kappa d \pi_{j, t}$. Investor demand in period $t$ changes by $-\kappa d \pi_{j, t}\left[V_{j}^{(j)}+O(\sigma)\right]$. The budget constraint (A.3) is strictly relaxed because $\kappa>1$ and $x_{j, t}-\frac{1}{1-\phi} y_{j, t}=O(\sigma)$. The intermediary's intertemporal profit (A.1) changes by

$$
\frac{1}{(1+r)^{t}}\left[d \pi_{j, t} V_{j}-\frac{\phi}{1-\phi} \kappa d \pi_{j, t} V_{j}^{(j)}\right]=\frac{1}{(1+r)^{t}}\left[1-\frac{\phi}{1-\phi} \kappa \alpha\left(1-s_{j}\right)\right] d \pi_{j, t} V_{j},
$$

which is positive, because $\kappa<\frac{1-\phi}{\phi \alpha}$ and $1-s_{j}<1$. Therefore, the regulatory constraint is binding. (A.36) is arguably satisfied in our empirical setup since we estimate $\alpha \simeq 0$.

## A. 3 Proof of Proposition 2

Reserves evolve according to $R_{j, t}=\left(1+x_{j, t}\right) R_{j, t-1}+\left(x_{j, t}-\frac{1}{1-\phi} y_{j, t}\right) V_{j, t-1}$ with $R_{j, 0}=0$. Therefore, a first-order approximation of $R_{j, t}$ is

$$
\begin{equation*}
R_{j, t}=\sigma h_{j, t} V_{j}+O\left(\sigma^{2}\right), \tag{A.37}
\end{equation*}
$$

where

$$
\begin{align*}
h_{j, t} & =\sum_{s=1}^{t}(1+r)^{t-s}\left(\epsilon_{j, s}^{x}-\frac{y_{j, s}^{\prime}}{1-\phi}\right) \\
& =\sum_{s=1}^{t}(1+r)^{t-s}\left(\epsilon_{j, s}^{x}-\frac{y_{j, s}^{s}}{1-\phi}\right)-\sum_{s=1}^{t}(1+r)^{t-s} \sum_{\tau=1}^{s-1} \frac{y_{j, s}^{\tau}}{1-\phi} \\
& =\sum_{s=1}^{t}(1+r)^{t-s}\left(\epsilon_{j, s}^{x}-\frac{y_{j, s}^{s}}{1-\phi}\right)-\sum_{s=1}^{t} \sum_{\tau=s+1}^{t}(1+r)^{t-\tau} \frac{y_{j, \tau}^{s}}{1-\phi} \\
& =\sum_{s=1}^{t}(1+r)^{t-s}\left(\epsilon_{j, s}^{x}-\sum_{\tau=s}^{t}(1+r)^{s-\tau} \frac{y_{j, \tau}^{s}}{1-\phi}\right) \\
& =\sum_{s=1}^{t}(1+r)^{t-s} \sum_{\tau=t+1}^{+\infty}(1+r)^{s-\tau} \frac{y_{j, \tau}^{s}}{1-\phi} \\
h_{j, t} & =\frac{1}{r} \sum_{s=1}^{t} \frac{y_{j, t+1}^{s}}{1-\phi}, \tag{A.38}
\end{align*}
$$

where we move from the first line to the second line using $y_{j, s}^{\prime}=\sum_{\tau=1}^{s} y_{j, s}^{\tau}$, to the third line by switching indices $s$ and $\tau$, to the fourth line by putting the two sums over $s$ together, to the fifth line using the budget constraint (A.15), and to the sixth line using that $y_{j, \tau}^{s}$ does not depend on $\tau$ for all $\tau>s$, so $y_{j, \tau}^{s}$ can be replaced by $y_{j, t+1}^{s}$. We have

$$
\begin{aligned}
y_{j, t}-(1-\phi) r= & \sigma y_{j, t}^{\prime}+O\left(\sigma^{2}\right)=\sigma \sum_{s=1}^{t-1} y_{j, t}^{s}+\sigma y_{j, t}^{t}+O\left(\sigma^{2}\right) \\
= & \frac{(1-\phi) r}{1+r}\left((1+r) \frac{R_{j, t-1}}{V_{j}}+\sigma \epsilon_{j, t}^{x}\right)+\frac{1-\phi}{1+r} \frac{\alpha}{\alpha+\frac{1+r}{r} \rho_{j}} \sigma \epsilon_{j, t}^{x} \\
& -(1-\phi) \frac{\frac{1}{r} \alpha \delta_{j}}{\alpha+\frac{1+r}{r} \rho_{j}} \frac{1}{1-B} \sigma \widehat{\epsilon}_{t}^{x}+\frac{\frac{1}{r} \delta_{j}}{\alpha+\frac{1+r}{r} \rho_{j}} \sigma\left(\epsilon_{j, t}^{\xi}-\bar{\epsilon}_{t}^{\xi}-\frac{1}{1-B} \widehat{\epsilon}_{t}^{\xi}\right)+O\left(\sigma^{2}\right)
\end{aligned}
$$

where we move from the first line to the second line using (A.37) and (A.38) to substitute $\sum_{s=1}^{t-1} y_{j, t}^{s}$, and using (A.31) to substitute $y_{j, t}^{t}$. Thus:

$$
\begin{align*}
y_{j, t}=(1-\phi) r+\frac{1-\phi}{1+r} \frac{\alpha}{\alpha+\frac{1+r}{r} \rho_{j}}\left(x_{j, t}-r\right)+\frac{(1-\phi) r}{1+r} & \left(\frac{R_{j, t^{-}}}{V_{j, t-1}}-r\right) \\
& +\mu_{j}\left(\bar{x}_{t}-r\right)+\nu_{j} \Delta \xi_{j, t}+O\left(\sigma^{2}\right) \tag{A.39}
\end{align*}
$$

where

$$
\begin{equation*}
\mu_{j}=-(1-\phi) \frac{\frac{1}{r} \alpha \delta_{j}}{\alpha+\frac{1+r}{r} \rho_{j}} \frac{\sum_{k=1}^{J} \frac{\rho_{k} s_{k}}{\alpha+\frac{1+r}{r} \rho_{k}}}{1-\sum_{k=1}^{J} \frac{\delta_{k} s_{k}}{\alpha+\frac{1+r}{r} \rho_{k}}}, \tag{A.40}
\end{equation*}
$$

$s_{j}, \rho_{j}$, and $\delta_{j}$ are given by (A.20), (A.22), and (A.23), respectively, $\bar{x}_{t}$ is a weighted average of asset returns $x_{k, t}$ over $k=1, \ldots, J$ defined in (A.35), $\nu_{j}=\frac{1}{r} \delta_{j} /\left(\alpha+\frac{1+r}{r} \rho_{j}\right), \Delta \xi_{j, t}=\epsilon_{j, t}^{\xi}-\bar{\epsilon}_{t}^{\xi}-\frac{1}{1-B} \epsilon_{t}^{\xi}$, and $\bar{\epsilon}_{t}^{\xi}$ and $\widehat{\epsilon}_{t}^{\xi}$ are weighted averages of demand shocks $\epsilon_{k, t}^{\xi}$ over $k=1, \ldots, J$ defined in (A.26) and (A.30), respectively.

## A. 4 Proof of Proposition 3

(16) implies that the contract return can be written as

$$
\begin{equation*}
y_{j, t}=a+b x_{j, t}+c \bar{x}_{t}+O\left(\sigma^{2}\right), \tag{A.41}
\end{equation*}
$$

where

$$
\begin{align*}
a & =(1-\phi) r-(1-\phi) \frac{\alpha+\rho_{j}}{\alpha+\frac{1+r}{r} \rho_{j}} r+(1-\phi) r \mathcal{R}_{j, t-1}-\mu_{j} r  \tag{A.42}\\
b & =(1-\phi) \frac{\alpha+\rho_{j}}{\alpha+\frac{1+r}{r} \rho_{j}}  \tag{A.43}\\
c & =\mu_{j} \tag{A.44}
\end{align*}
$$

and $\mathcal{R}_{j, t-1}=R_{j, t-1} / V_{j, t-1}$. Therefore, the contract return can be replicated up to a constant with a portfolio with weight $b$ in the insurer's assets generating return $x_{j, t}$, weight $c$ in the average insurer portfolio generating return $\bar{x}_{t}$, and weight $1-b-c$ in the risk-free asset generating return $r_{f}$. The return difference between the contract and the replicating portfolio is the constant $a-(1-b-c) r_{f}$,
which is equal to

$$
\begin{equation*}
\left[1-(1-\phi) \frac{\alpha+\rho_{j}}{\alpha+\frac{1+r}{r} \rho_{j}}-\mu_{j}\right]\left(r-r_{f}\right)+(1-\phi) r \mathcal{R}_{j, t-1}-\phi r . \tag{A.45}
\end{equation*}
$$

## A. 5 Proof of Relation 1

We consider the case where the number of intermediaries, $J$, is large, so that market shares, $s_{j}$, are small. Formally, we assume there exists $\bar{s}>0$ such that $s_{j}<\bar{s} J^{-1}$ for all $J>1$. Let $O\left(J^{-1}\right)$ denote functions of the order of $J^{-1}$, that is, there exist $K>0$ and $\bar{J}$ such that $O\left(J^{-1}\right) \leq K J^{-1}$ for all $J>\bar{J}$. It follows from (A.22) that $\rho_{j}=-u^{\prime \prime} /\left(u^{\prime}\right)^{2}+O\left(J^{-2}\right)$, from (A.23) that $\delta_{j}=O\left(J^{-1}\right)$, and from (A.40) that $\mu_{j}=\mu+O\left(J^{-1}\right)$. Therefore, (A.39) can be rewritten as:

$$
\begin{equation*}
y_{j, t}=c s t e+\frac{1-\phi}{1+r} \frac{\alpha}{\alpha+\frac{1+r}{r} \rho} x_{j, t}+\frac{(1-\phi) r}{1+r} \mathcal{R}_{j, t^{-}}+\mu \bar{\epsilon}_{t}^{x}+\varepsilon_{j, t}+O\left(\sigma^{2}\right), \tag{A.46}
\end{equation*}
$$

where

$$
\begin{equation*}
\rho=\frac{-u^{\prime \prime}}{u^{\prime}}, \tag{A.47}
\end{equation*}
$$

and

$$
\begin{equation*}
\varepsilon_{j, t}=\nu_{j} \Delta \xi_{j, t}+O\left(J^{-1}\right) x_{j, t}+O\left(J^{-1}\right)\left(\bar{x}_{t}-r\right) . \tag{A.48}
\end{equation*}
$$

Since demand shocks entering into the expression of $\Delta \xi_{j, t}$ are uncorrelated with asset return $x_{j, t}$, the covariance between $x_{j, t}$ and $\varepsilon_{j, t}$ is $O\left(J^{-1}\right)$. Since $\mathcal{R}_{j, t^{-}}=\mathcal{R}_{j, t-1}+x_{j, t}\left(1+\mathcal{R}_{j, t-1}\right)$ and $\mathcal{R}_{j, t-1}$ is uncorrelated with $\varepsilon_{j, t}$, the covariance between $\mathcal{R}_{j, t^{-}}$and $\varepsilon_{j, t}$ is also $O\left(J^{-1}\right)$.

## A. 6 Proof of Relation 2

A first-order approximation of $\log$ demand of intermediary $j$ in period $t$ is

$$
\begin{equation*}
\log \left(V_{j, t-1}\right)=\log \left(V_{j}\right)+\sum_{k=1}^{J} \frac{V_{j}^{(k)}}{V_{j}}\left(E_{t-1}\left[y_{k, t}\right]-(1-\phi) r+\frac{\xi_{k, t-1}}{\alpha u^{\prime}}\right) u^{\prime}+O\left(\sigma^{2}\right) \tag{A.49}
\end{equation*}
$$

The expectation of contract return (A.39) is equal to

$$
\begin{equation*}
E_{t-1}\left[y_{k, t}\right]=(1-\phi) r+(1-\phi) r \mathcal{R}_{k, t-1}+O\left(\sigma^{2}\right) \tag{A.50}
\end{equation*}
$$

where we have used $\mathcal{R}_{k, t^{-}}=\left(1+x_{k, t}\right) \mathcal{R}_{k, t-1}+x_{k, t}$ and $E_{t-1}\left[x_{k, t}\right]=r$. Plugging (A.50) into (A.49), substituting the derivative of logit demand, and using the normalization $u^{\prime}((1-\phi) r)=1$, we obtain

$$
\begin{equation*}
\log \left(V_{j, t-1}\right)=\log \left(V_{j}\right)+\psi_{t-1}+\alpha(1-\phi) r \mathcal{R}_{j, t-1}+\xi_{j, t-1}+O\left(\sigma^{2}\right) \tag{A.51}
\end{equation*}
$$

where $\psi_{t-1}=-\sum_{k=1}^{J}\left(\alpha(1-\phi) r \mathcal{R}_{k, t-1}+\xi_{k, t-1}\right) s_{k}$. To calculate the covariance between $\mathcal{R}_{j, t-1}$ and $\xi_{j, t-1}$, we use (A.37) and (A.38) to write

$$
\begin{equation*}
\mathcal{R}_{j, t-1}=\sigma \frac{1}{r} \sum_{s=1}^{t-1} \frac{y_{j, t}^{s}}{1-\phi}+O\left(\sigma^{2}\right) . \tag{A.52}
\end{equation*}
$$

Substituting $y_{j, t}^{s}$ using (A.32), and focusing on terms $\epsilon_{j, s}^{\xi}$, we obtain

$$
\mathcal{R}_{j, t-1}=\ldots-\sigma \frac{1}{r} \sum_{s=1}^{t-1} \frac{1}{1-\phi} \frac{\frac{1}{u^{\prime}} \delta_{j}}{\alpha+\frac{1+r}{r} \rho_{j}} \epsilon_{j, s}^{\xi}+O\left(\sigma^{2}\right)=\ldots-\frac{1}{(1-\phi) r} \frac{\frac{1}{u^{\prime}} \delta_{j}}{\alpha+\frac{1+r}{r} \rho_{j}} \xi_{j, t-1}+O\left(\sigma^{2}\right) .
$$

Therefore

$$
\begin{equation*}
\operatorname{Cov}\left(\mathcal{R}_{j, t-1}, \xi_{j, t-1}\right) \simeq-\frac{1}{(1-\phi) r} \frac{\frac{1}{u^{\prime}} \delta_{j}}{\alpha+\frac{1+r}{r} \rho_{j}}<0 \tag{A.53}
\end{equation*}
$$

Finally, let us now show that $x_{j, t-1}$ is a valid instrument for $\mathcal{R}_{j, t-1}$. The relevance condition is satisfied, because it follows from the budget constraint (4) that $\operatorname{Cov}\left(x_{j, t-1}, \mathcal{R}_{j, t-1}\right)>0$. The exclusion restriction is satisfied, because $\operatorname{Cov}\left(\epsilon_{j, t-1}^{x}, \epsilon_{j, t-1}^{\xi}\right)=0$. Thus, the IV estimate of the flowreserves relation (A.51) using lagged asset return to instrument for reserves is unbiased.

## A. 7 Is Arbitrage Profitable?

In this appendix, we calculate arbitrage profits from buying euro contracts and shorting the replicating portfolio, for any value of $\alpha$. We then calibrate it in the relevant case $\alpha \simeq 0$. Following the proof of Proposition 3 in Appendix A.4, the contract return can be written as

$$
\begin{equation*}
y_{j, t}=a+b x_{j, t}+c \bar{x}_{t}+O\left(\sigma^{2}\right) \tag{A.54}
\end{equation*}
$$

where $a, b>0$, and $c<0$ are given by (A.42), (A.43), and (A.44), respectively. Consider the hedged, zero-cost portfolio that goes long one euro in contract $j$, short $(1-\tau) b$ euros in intermediary $j$ 's asset portfolio, long $(1-\tau)|c|$ euros in the weighted-average intermediary portfolio, and borrows $1-(1-\tau) b+(1-\tau)|c|$ at the risk-free rate. The return on the long position in the euro contract is
taxed at rate $\tau$. The return on the long position in the weighted-average intermediary portfolio is taxed if the position cannot be netted against the short position in intermediary $j$ 's asset, but the part of the long position that can be netted is not taxed. We make the conservative assumption (in the sense that it maximizes the profitability of the arbitrage strategy) that the long position in the weighted-average intermediary portfolio can be fully netted against the short position in intermediary $j$ 's asset, and thus is not taxed. The arbitrage profit is equal to

$$
\begin{align*}
& \pi_{j, t}^{a r b}=(1-\tau) y_{j, t}-(1-\tau) b x_{j, t}+(1-\tau)|c| \bar{x}_{t}-(1-(1-\tau) b+(1-\tau)|c|) r_{f} \\
= & {\left[1-(1-\tau)(1-\phi) \frac{\alpha+\rho_{j}}{\alpha+\frac{1+r}{r} \rho_{j}}+(1-\tau)\left|\mu_{j}\right|\right]\left(r-r_{f}\right)+(1-\tau)(1-\phi) r \mathcal{R}_{j, t-1}-\tau r-(1-\tau) \phi r . } \tag{A.55}
\end{align*}
$$

When $\alpha \simeq 0$, (A.40) implies $\mu_{j} \simeq 0$, and

$$
\begin{equation*}
\pi_{j, t}^{a r b} \simeq\left[1-\frac{(1-\tau)(1-\phi) r}{1+r}\right]\left(r-r_{f}\right)+(1-\tau)(1-\phi) r \mathcal{R}_{j, t-1}-\tau r-(1-\tau) \phi r \tag{A.56}
\end{equation*}
$$

We calibrate the expected asset return using the sample average asset return ( $4.9 \%$ in Table 1 ), and noting that it is likely realized asset returns have been above expected returns during the sample period. As discussed in Section 1.2, the reserve ratio rose by 25 basis points per year, while positive net flows should have diluted reserves at a rate of 25 basis points per year. Therefore, insurers have retained in reserves approximately 50 basis points of the realized asset returns in excess of expected returns. Thus, we set $r=4.4 \%$. Using $r_{f}=3 \%$, the risk premium is $1.4 \%$. We set $\phi=0.15$ based on the regulatory framework described in Section 1.1. To focus on a situation that makes the arbitrage most profitable, we assume the reserve ratio is 10 percentage points above target. This represents 1.5 standard deviations of the reserve ratio (Table 1). It also amounts to the difference between the highest point of the aggregate reserve ratio (reached in 2014, see Appendix Figure B.2) and its sample average. Thus, we set $\mathcal{R}_{j, t-1}=0.1$. Substituting these calibrated values in (A.56), arbitrage opportunities are eliminated for $\tau>0.26$.

## A. 8 Calibration of $\rho$ for the Welfare Analysis

(A.47) gives the expression for $\rho$ as a function of the indirect utility function $u(y)$ defined over the investment return $y$. To map $\rho$ into a standard risk aversion parameter based on a direct utility function defined over consumption, suppose that the investor derives utility $U(c)$ from consumption
at the end of the period $c$; that consumption is equal to the investor's final wealth; and that a share $S_{\text {contract }}$ of the investor's wealth is invested in euro contract such that $d \log (c) / d y=S_{\text {contract }}$. Therefore, (A.47) can be rewritten

$$
\rho=\frac{-u^{\prime \prime}(y)}{u^{\prime}(y)}=\frac{-U^{\prime \prime}(c)\left(\frac{\partial c}{\partial y}\right)^{2}}{U^{\prime}(c) \frac{\partial c}{\partial y}}=\frac{-U^{\prime \prime}(c) c_{i, t}}{U^{\prime}(c)} \times S_{\text {contract }} .
$$

Therefore, $\rho$ is equal to the coefficient of relative risk aversion times the share of the investor's total wealth invested in euro contract.

## B Institutional Environment

## B. 1 Aggregate Account Value

The combination of positive net flows and compounded contract returns generates an increasing trend in aggregate account value plotted as shown in Figure B.1. Aggregate account value grows from 500 billion euros in 2000 to 1,200 billion euros in 2015 (all amounts are in constant 2015 euros). Aggregate growth reflects the internal growth of existing life insurers rather than the entry of new insurers. The number of insurers in the sample is 65 at the beginning of the period and 61 at the end. Market concentration is relatively low, with a Herfindahl-Hirschman Index around 800 and total market shares of the top five insurers slightly below $50 \%$.

Figure B.1: Aggregate Account Value. The figure shows aggregate account value of euro contracts in billion 2015 (solid blue) euros and the number of insurers in the sample (dashed red).


## B. 2 Reserves

Reserves of euro contract funds represent the difference between the value of assets held by the fund and total account value. Reserves have three components, whose creation and use is determined by the regulation guiding how asset income can be split between investors and the insurer.

Regulatory framework. At least $85 \%$ of financial income plus $90 \%$ of technical income (or $100 \%$ if it is negative) must be distributed to investors. Financial income is equal to asset yield (dividends on non-fixed income securities plus yield on fixed income securities) plus realized gains and losses on non-fixed income securities plus net income from reinsurance. Technical income is equal to fees paid by investors minus operating costs. The amount distributed to investors is split into two parts: one part credited immediately to investors' accounts and another part credited to, or debited from, a reserve account called the profit-sharing reserve (provision pour participation aux bénéfices). This profit-sharing account is the first component of reserves. We have:

Contract return $+\Delta$ Profit-sharing reserve $\geq 85 \%$ Financial income $+90 \%$ Technical income. (B.1)

The profit-sharing reserve account can only be used for future distribution to investor accounts. Therefore, the profit-sharing reserve effectively belongs to (current and future) investors. The profitsharing reserve is pooled across all contracts. When an investor redeems her contract, she gives up her right to future distribution of the profit-sharing reserve. Conversely, when a new investor buys a contract, she shares in the outstanding profit-sharing reserve. Therefore, the profit-sharing reserve is passed on between successive cohorts of contract holders. ${ }^{1}$ The second component of reserves is called the capitalization reserve account (réserve de capitalisation). It is made of realized gains and losses on fixed income securities, which are not booked as financial income but are credited to, or debited from, this account. The capitalization reserve account can only be used to offset future losses on fixed income securities and cannot be credited to investors' accounts or to insurer income. The third component of reserves is made of the unrealized capital gains on the funds' assets, which are not booked as financial income. ${ }^{2}$ Both the capitalization reserve and unrealized

[^1]capital gains represent deferred financial income. Since at least $85 \%$ of the financial income must be distributed to investors, at least $85 \%$ of the capitalization reserve and unrealized capital gains effectively belong to (current and future) investors. Since all three components of reserves are eventually owed to investors and are pooled across investor cohorts, the composition of reserves is immaterial for intercohort risk sharing. For this reason, our empirical analysis focuses on total reserves.

Summary statistics. Reserves represent on average $10.9 \%$ of account value, of which $7.5 \%$ are unrealized capital gains, $2.1 \%$ are profit-sharing reserves, and $1.4 \%$ are capitalization reserves. Figure B. 2 plots the time-series of aggregate reserves and its three sub-components as a fraction of account value.

[^2]Figure B.2: Reserves. The figure shows total reserves as a fraction of account value (solid blue) and the breakdown into the three components of reserves: unrealized gains (long dashed red); profit-sharing reserves (dashed green); and capitalization reserves (short dashed orange).


## B. 3 Capital Requirements

As described in Section 1.1, the Solvency I regulation requires insurers to hold a minimum amount of capital such that the sum of capital and unrealized capital gains is at least equal to $4 \%$ of total account value. These capital requirements do not depend on the portfolio asset composition or the minimum return guarantees. Figure B. 3 shows the (value-weighted) ratio of equity to total account value from 2006-2015 (the capital position data are only available starting in 2006). The figure shows that even in 2008 and 2011-2012, this ratio is above the $4 \%$ capital requirement. Thus, insurers' equity plus unrealized capital gains are far above the $4 \%$ capital requirement throughout the sample period.

Figure B.3: Ratio of Insurer Equity Over Total Account Value. The figure shows the (value-weighted) average insurer equity as a fraction of account value. Although the capital requirement imposes that equity plus unrealized gains must be equal to at least $4 \%$ of total account value, insurers' equity alone remains above the capital requirement throughout our sample period and even during the 2008 global financial crisis and the 2011-2012 European debt crisis.


## C Variables Construction

## C. 1 Regulatory filings

This section describes how we construct variables at the insurer-year level using the annual regulatory filings (Dossiers Annuels) from 1999 to 2015.

Account value Provisions d'assurance vie à l'ouverture (beginning-of-year account value) and Provisions d'assurance vie à la clôture (end-of-year account value) in C1V1-C1V3 statements summed over contract categories $1,2,4,5$, and 7 , which is the set of contracts backed by the same pool of underlying assets and associated to the same pool of reserves. The main excluded contract categories are 8 and 9 , which are unit-linked contracts.

Profit-sharing reserves Provisions pour participations aux bénéfices et ristournes in BILPV statement.

Capitalization reserves Réserve de capitalisation in C5P1 statement.

Unrealized gains Book value (Valeur nette) minus market value (Valeur de réalisation) of assets underlying life insurance contracts measured as Placements représentatifs des provisions techniques minus Actifs représentatifs des unités de compte in N3BJ statement.

Inflows Sous-total primes nettes in C1V1-C1V3 statements summed over contract categories $1,2,4,5$, and 7 . It includes initial cash deposits at subscription and subsequent cash deposits in existing contracts. The inflow rate is calculated as inflow amount divided by beginning-of-year account value plus one half of net flows.

Outflows Sinistres et capitaux payés plus Rachats payés in C1V1-C1V3 statements summed over contract categories $1,2,4,5$, and 7 . It includes partial and full redemptions, either voluntary or at death of investor. The outflow rate is calculated as outflow amount divided by beginning-of-year account value plus one half of net flows.

Contract return We calculate the value-weighted average contract return as the amount credited to investor accounts divided by beginning-of-year account value plus one half of net flows
(i.e., we assume flows are uniformly distributed throughout the year and thus receive on average one half of the annual contract return). The amount credited to investor accounts is measured as Intérêts techniques incorporés aux provisions d'assurance vie plus Participations aux bénéfices incorporées aux provisions d'assurance vie plus Intérêts techniques inclus dans exercice prestations plus Participations aux bénéfices incorporées dans exercice prestations in C1V1-C1V3 statements summed over contract categories $1,2,4,5$, and 7 .

Asset return We sum the three components of asset returns, which are reported separately in insurers' financial statement. First, Produits des placements nets de charges in C1V1-C1V3 statements summed over contract categories $1,2,4,5$, and 7 , measures asset yield (dividends on non-fixed income securities plus yield on fixed income securities) and realized gains and losses on non-fixed income securities, net of operating costs. Second, the change in capitalization reserves account value reflects realized gains and losses on fixed income securities. Third, the change in unrealized gains captures unrealized gains. We calculate asset return as the sum of these three components divided by account value plus reserves.

## C. 2 Account Value by Cohort

We describe in this appendix how we estimate account value by cohort from insurer-level account value, inflows, and outflows, under parametric assumptions on the inflow rate and the outflow rate. Regarding inflows, we assume investors only make one-off investments. They make an initial deposit when they buy a contract and never deposit additional funds at subsequent dates. Regarding outflows, we assume investors only proceed to full redemptions and that the redemption rate does not depend on contract age for a given insurer in a given year. We omit the insurer index $j$ to simplify the formulas. We call cohort $\left(t_{0}, t_{1}\right)$ the set of investors who buy their contract in year $t_{0}$ and redeem it in year $t_{1}$, for $t_{0}<t_{1}$. We denote $V_{t}\left(t_{0}, t_{1}\right)$ the account value of cohort $\left(t_{0}, t_{1}\right)$ at the end of year $t$ and by $V_{t}^{+}\left(t_{0}, t_{1}\right)$ and $V_{t}^{-}\left(t_{0}, t_{1}\right)$ their inflows and outflows, respectively, during year $t$. Under the maintained assumption that inflows and outflows are uniformly distributed throughout the year and are entitled to one half of the annual contract return, account value of cohort $\left(t_{0}, t_{1}\right)$
evolves according to

$$
\begin{align*}
& V_{t_{0}-1}\left(t_{0}, t_{1}\right)=0  \tag{C.2}\\
& V_{t}\left(t_{0}, t_{1}\right)=\left(1+y_{t}\right) V_{t-1}\left(t_{0}, t_{1}\right)+\left(1+\frac{y_{t}}{2}\right)\left(V_{t}^{+}\left(t_{0}, t_{1}\right)-V_{t}^{-}\left(t_{0}, t_{1}\right)\right), \quad t=t_{0}, \ldots, t_{1}-1,  \tag{C.3}\\
& V_{t_{1}}\left(t_{0}, t_{1}\right)=0 \tag{C.4}
\end{align*}
$$

where $y_{t}$ is the net-of-fees contract return. The assumption of no inflow after initial subscription writes

$$
\begin{equation*}
V_{t}^{+}\left(t_{0}, t_{1}\right)=0, \quad t>t_{0} . \tag{C.5}
\end{equation*}
$$

The assumption of no partial redemption before exit writes

$$
\begin{equation*}
V_{t}^{-}\left(t_{0}, t_{1}\right)=0, \quad t<t_{1} . \tag{C.6}
\end{equation*}
$$

The assumption of outflow rate independent of contract age at the insurer-year level writes

$$
\begin{equation*}
\frac{V_{t}^{-}\left(t_{0}, t\right)}{V_{t-1}\left(t_{0}\right)}=\frac{V_{t}^{-}}{V_{t-1}}, \quad t>t_{0} . \tag{C.7}
\end{equation*}
$$

We now describe the procedure to calculate account value by cohort.

Net-of-fees returns The data only report gross-of-fees contract return. Since we observe beginning-of-year account value $V_{t-1}$, inflows $V_{t}^{+}$, outflows $V_{t}^{-}$, and end-of-year account value $V_{t}$, we back out the net-of-fees contract return $y_{t}$ from the law of motion of total account value

$$
\begin{equation*}
V_{t}=\left(1+y_{t}\right) V_{t-1}\left(1+\frac{y_{t}}{2}\right)\left(V_{t}^{+}-V_{t}^{-}\right) . \tag{C.8}
\end{equation*}
$$

Birth-cohort-level account value Define a birth-cohort $t_{0}$ as the set of cohorts $\left\{\left(t_{0}, t_{1}\right): t_{1}>\right.$ $\left.t_{0}\right\}$. Denoting by $T_{0}=1999$ and $T_{1}=2015$ the first year and last year when account value data are available, we redefine birth-cohort $T_{0}-1$ as the set of birth-cohorts $\left\{t_{0}: t_{0} \leq T_{0}-1\right\}$. We denote by $V_{t}\left(t_{0}\right), V_{t}^{+}\left(t_{0}\right)$, and $V_{t}^{-}\left(t_{0}\right)$ the end-of-year, inflows, and outflows, respectively, of birthcohort $t_{0} . V_{T_{0}-1}\left(T_{0}-1\right)$ is observed in the data as beginning-of-year account value in year $T_{0}$. (C.5) implies that, for all $t_{0} \geq T_{0}$, inflows of birth-cohort $t_{0}$ in year $t_{0}$ is $V_{t_{0}}^{+}\left(t_{0}\right)=V_{t_{0}}^{+}$, which is observed in the data as total outflow in year $t_{0}$. Then, we compute birth-cohort-level end-of-year account value and outflows in all years $t \in\left[T_{0}, T_{1}\right]$ by forward iteration. Once we have computed
birth-cohort-level end-of-year account value in year $t-1$, (C.6) and (C.7) imply that outflows of birth-cohort $t_{0}<t$ in year $t$ is $V_{t}^{-}\left(t_{0}\right)=\frac{V_{t-1}\left(t_{0}\right)}{V_{t-1}} V_{t}^{-}$, where the last term is total outflows in year $t$, which is observed in the data. End-of-year account value of birth-cohort $t_{0}<t$ in year $t$ is $V_{t}\left(t_{0}\right)=\left(1+y_{t}\right) V_{t-1}\left(t_{0}\right)-\left(1+\frac{y_{t}}{2}\right) V_{t}^{-}\left(t_{0}\right)$. End-of-year account value of birth-cohort $t$ in year $t$ is $V_{t}(t)=\left(1+\frac{y_{t}}{2}\right) V_{t}^{+}(t)$.

Cohort-level account value For $t_{1} \in\left[T_{0}, T_{1}\right]$, we redefine cohort $\left(T_{0}-1, t_{1}\right)$ as the set of cohorts $\left\{\left(t_{0}, t_{1}\right): t_{0} \leq T_{0}-1\right\}$. For $t_{0} \in\left[T_{0}, T_{1}\right]$, we redefine cohort $\left(t_{0}, T_{1}+1\right)$ as the set of cohorts $\left\{\left(t_{0}, t_{1}\right): t_{1} \geq T_{1}+1\right\}$. (C.6) implies that cohort-level outflows is $V_{t_{1}}^{-}\left(t_{0}, t_{1}\right)=V_{t_{1}}^{-}\left(t_{0}\right)$ for all $T_{0}-1 \leq t_{0}<t_{1} \leq T_{1}$. Then, we compute end-of-year account value for each cohort $\left(t_{0}, t_{1}\right)$ in all year $t \in\left[t_{0}, t_{1}-1\right]$ by backward iteration. If $t_{1} \leq T_{1}$, it follows from (C.3) and (C.4) that $V_{t_{1}-1}\left(t_{0}, t_{1}\right)=\left(1+\frac{y t_{1}}{2}\right) V_{t_{1}}^{-}\left(t_{0}\right) /\left(1+y_{t_{1}}\right)$. If $t_{1}=T_{1}+1, V_{T_{1}}\left(t_{0}, T_{1}+1\right)=V_{T_{1}}\left(t_{0}\right)$. Once we have computed the end-of-year account value of cohort $\left(t_{0}, t_{1}\right)$ in year $t$, we use (C.4) to calculate it in year $t-1: V_{t-1}\left(t_{0}, t_{1}\right)=V_{t}\left(t_{0}, t_{1}\right) /\left(1+y_{t}\right)$ for all $t \in\left[t_{0}+1, t_{1}-1\right]$. Finally, for $t_{0} \geq T_{0}$, it follows from (C.2) and (C.3) that inflows of cohort $\left(t_{0}, t_{1}\right)$ in year $t_{0}$ is $V_{t_{0}}^{+}\left(t_{0}, t_{1}\right)=V_{t_{0}}\left(t_{0}, t_{1}\right) /\left(1+\frac{y_{t_{0}}}{2}\right)$.

## D Generalized Model with Contracts Held For Several Periods

In this appendix, we extend the model to the case of contracts held for several periods. This requires to model inflow and outflow decisions separately. At the beginning of each period, a mass one of new investors choose which contract to buy among all intermediaries and the outside option. The inflow amount to intermediary $j$ at the beginning of period $t$ is given by the Logit demand function

$$
\begin{equation*}
\text { Inflow }_{j, t-1}=\frac{\exp \left\{\alpha E_{t-1}\left[u\left(y_{j, t}\right)\right]+\xi_{j, t-1}\right\}}{\sum_{k=0}^{J} \exp \left\{\alpha E_{t-1}\left[u\left(y_{k, t}\right)\right]+\xi_{k, t-1}\right\}}, \tag{D.1}
\end{equation*}
$$

where $\alpha$ captures the elasticity of inflow to expected indirect utility of return and $\xi_{j, t-1}$ is a demand shock realized at the end of period $t-1$. The vector of demand shocks follows a random walk as in the baseline version of the model: $E_{t-2}\left[\xi_{t-2}\right]=\xi_{t-1}$. We denote $s_{j}=e^{\xi_{j, 0}} / \sum_{k=0}^{J} e^{\xi_{k, 0}}$ as the market share when all expected contract returns are equalized and demand shocks are set to their unconditional expected value. At the end of each period, investors holding a contract decide whether to stay with their contract or leave for the outside option. The outflow rate is given by the Logit demand function

$$
\begin{equation*}
\text { OutflowRate }_{j, t-1}=1-\frac{\exp \left\{\beta E_{t-1}\left[u\left(y_{j, t}\right)\right]+\kappa\right\}}{\exp \left\{\beta E_{t-1}\left[u\left(y_{0, t}\right)\right]\right\}+\exp \left\{\beta E_{t-1}\left[u\left(y_{j, t}\right)\right]+\kappa\right\}} . \tag{D.2}
\end{equation*}
$$

When investors stay with their contract, they withdraw the contract return. Therefore, the account value at the beginning of period $t \geq 1$ is

$$
\begin{equation*}
V_{j, t-1}=\text { Inflow }_{j, t-1}+\left(1-\text { Out flowRate }_{j, t-1}\right) V_{j, t-2}, \tag{D.3}
\end{equation*}
$$

where $V_{j,-1}$ is given. We denote $\theta=e^{\kappa} /\left(1+e^{\kappa}\right)$ as one minus the outflow rate when expected returns are equalized between the intermediary and the outside option. The limit case $\theta=0$ corresponds to an outflow rate of one every period, i.e., to the baseline version of the model. In any given year an intermediary must pay the same contract returns to all its investors no matter when they purchased their contract. Each intermediary maximizes expected profit given by Equation (8) in the paper by choosing a contract return function $y_{j, t}$ subject to the budget constraint (4), the profit function (7), the transversality condition (9), and the demand system (D.1)-(D.2)-(D.3). Each intermediary takes other intermediaries' contract return policies as given. An equilibrium is defined as a fixed point of this problem. Finding a general solution to this problem is challenging when the space of contract return policies is all possible functions adapted to the filtration generated by the asset
return and demand shock processes. To simplify the problem, we focus on the case in which market shares $s_{j}$ are small and restrict the set of contract return policies to the functions that take the same form as the equilibrium contract return policies in the baseline version of the model (which we solve without restricting the contract return space), that is:

$$
\begin{equation*}
y_{j, t}=c_{j, t}+a_{j, t} x_{j, t}+\sum_{s=1}^{t-1} b_{j, s} x_{j, s} \tag{D.4}
\end{equation*}
$$

where $c_{j, t}, a_{j, t}$ and $b_{j, t}$ are constant terms to be determined in equilibrium. The following proposition extends Relation 1 by showing that the expression for equilibrium contract returns has the same form as in the baseline model (Equation (18)) where the coefficient before the current asset return now depends on the additional parameters $\beta$ and $\theta$ controlling the elasticity and level of outflows. The proof is provided in Section D.1.

Proposition D.1. The period-t contract return of intermediary $j$ is

$$
\begin{equation*}
y_{j, t} \simeq c s t e_{t}+A x_{j, t}+\frac{(1-\phi) r}{1+r} \mathcal{R}_{j, t^{-}} \tag{D.5}
\end{equation*}
$$

where cste ${ }_{t}$ is a period-specific constant and $A \in[0,1]$ is a constant given by Equation (D.26). Moreover:
(i) when contracts are held for one period, i.e., $\theta \rightarrow 0$, then $A$ goes to $\frac{1-\phi}{1+r} \frac{\alpha}{\alpha+\frac{1+r}{r} \rho}$ as in the baseline model;
(ii) when the elasticities of inflow and outflow go to zero at the same rate, i.e., $\frac{\partial \log \left(\text { Inflow }_{j, t}\right)}{\partial E_{t-1}\left[u\left(y_{j, t-1}\right)\right]}=$ $\alpha=\frac{\partial \log \left(1-\text { Outflow Rate }_{j, t}\right)}{\partial E_{t-1}\left[u\left(y_{j, t-1}\right)\right]}=(1-\theta) \beta \rightarrow 0$, then A goes to zero and $a_{j, t}=b_{j, s}$ for all $t$ and $s$;
(iii) when the elasticities of inflow and outflow go to infinity at the same rate, i.e., $\alpha=(1-\theta) \beta \rightarrow$ $\infty$, then $a_{j, t}$ goes to $1-\phi$ and $b_{j, s}$ goes to zero for all $t$ and $s$.

Point (i) of the proposition states that, when $\theta$ goes to zero, that is, when the outflow rate each period goes to one such that contracts are held for one period, we recover the equilibrium of the baseline model and $A=\frac{1-\phi}{1+r} \frac{\alpha}{\alpha+\frac{1+r}{r} \rho}$ as in (18). Point (ii) states that, when $\theta>0$ such that contracts are held for several periods, and the elasticity of inflow and the elasticity of outflow go to zero at the same rate, Proposition D. 1 extends the result established in the baseline model that risk is perfectly shared across investor cohorts: $A$ goes to zero and $a_{j, t}=b_{j, s}$ for all $t$ and $s$. Point (iii) states that, when $\theta>0$ and the elasticity of inflow and the elasticity of outflow go to infinity
at the same rate, we recover the result in the baseline model that intercohort risk sharing unravels: the pass-through from asset return to contract return $a_{j, t}$ goes to $1-\phi$ for all $t$. Using (D.5) to calculate a Taylor expansion of inflows and outflows, we show in Section D. 2 that the log inflow amount is given by

$$
\begin{equation*}
\log \left(\text { Inflow }_{j, t-1}\right) \simeq \text { cste }_{j}+\text { cste }_{t-1}+\alpha(1-\phi) r \mathcal{R}_{j, t-1}+\xi_{j, t-1} \tag{D.6}
\end{equation*}
$$

and the outflow rate is given by

$$
\begin{equation*}
\text { OutflowRate }_{j, t-1} \simeq \text { cste }-\beta \theta(1-\theta)(1-\phi) r \mathcal{R}_{j, t-1} \tag{D.7}
\end{equation*}
$$

## D. 1 Proof of Proposition D. 1

The proof follows the same steps as the proofs of Propositions 1 and 2.

Notations To shorten equations, we denote $I_{j, t} \equiv$ Inflow $_{j, t}$ and $S_{j, t} \equiv 1$ - OutflowRate ${ }_{j, t}$ where $S$ stands for $S$ tayers. We rewrite the contract return (D.8) as

$$
\begin{equation*}
y_{j, t}=y_{j, t}^{0}+a_{j, t} \sigma \epsilon_{j, t}^{x}+\sum_{s=1}^{t-1} b_{j, s} \sigma \epsilon_{j, s}^{x} \tag{D.8}
\end{equation*}
$$

where $y_{j, t}^{0} \equiv c_{j, t}+a_{j, t} r+\sum_{s=1}^{t-1} b_{j, s} r$.

Intermediary's problem Intermediary $j$ maximizes

$$
\begin{equation*}
E_{0}\left[\sum_{\tau=1}^{\infty} \frac{1}{(1+r)^{\tau}} \frac{\phi}{1-\phi} y_{j, \tau} V_{j, \tau-1}\right] \tag{D.9}
\end{equation*}
$$

subject to the intertemporal budget constraint

$$
\begin{equation*}
\sum_{\tau=1}^{\infty} \frac{1}{(1+r)^{\tau}}\left[\left(x_{j, \tau}-\frac{1}{1-\phi} y_{j, \tau}\right) V_{j, \tau-1} \prod_{u=\tau+1}^{\infty} \frac{1+x_{j, u}}{1+r}\right] \geq 0, \quad \forall \mathcal{H}^{t} \tag{D.10}
\end{equation*}
$$

where $y_{j, \tau}$ is given by (D.8), and where we omit argument $\mathcal{H}_{t}$ in contract returns. We denote by $\lambda_{j}\left(\mathcal{H}^{T}\right)$ the Lagrange multiplier of (D.10) divided by the probability of history $\mathcal{H}^{T}$. Therefore, the Lagrangian associated with intermediary $j$ 's problem is equal to (D.9) plus the time-0 expectation of $\lambda_{j}\left(\mathcal{H}^{T}\right)$ times the LHS of (D.10).

Solving for $y_{j, t}^{0}$ Using (D.3), the account value at the beginning of period $\tau \geq t$ can be written as

$$
V_{j, \tau-1}=V_{j, t-1} \prod_{t+1 \leq u \leq \tau} S_{j, u-1}+\sum_{t+1 \leq s \leq \tau} I_{j, s-1} \prod_{s+1 \leq u \leq \tau} S_{j, u-1} .
$$

Therefore

$$
\frac{\partial V_{j, \tau-1}}{\partial y_{j, t}^{0}}=\frac{\partial V_{j, t-1}}{\partial y_{j, t}^{0}} \prod_{t+1 \leq u \leq \tau} S_{j, u-1} \quad \tau \geq t
$$

and

$$
\frac{\partial V_{j, \tau-1}}{\partial y_{j, t}^{0}}=0, \quad \tau<t
$$

Denoting by $V_{j, t-1}^{(j)}$ the partial derivative of $V_{j, t-1}$ with respect to $E_{t-1}\left[u^{\prime}\left(y_{j, t}\right)\right]$, we have

$$
\frac{\partial V_{j, t-1}}{\partial y_{j, t}^{0}}=V_{j, t-1}^{(j)} E_{t-1}\left[u^{\prime}\left(y_{j, t}\right)\right] .
$$

Therefore, for $\tau \geq t$,

$$
\begin{equation*}
\frac{\partial V_{j, \tau-1}}{\partial y_{j, t}^{0}}=1_{\tau \geq t}\left(\prod_{t+1 \leq u \leq \tau} S_{j, u-1}\right) V_{j, t-1}^{(j)} E_{t-1}\left[u^{\prime}\left(y_{j, t}\right)\right] \tag{D.11}
\end{equation*}
$$

We are now ready to take the first order condition of the Lagrangian with respect to $y_{j, t}^{0}$ :

$$
\begin{align*}
E_{0}\left[\sum _ { \tau \geq t } \frac { 1 } { ( 1 + r ) ^ { \tau - t } } \left(\frac{\phi}{1-\phi} y_{j, \tau}+\lambda_{j}\left(x_{j, \tau}\right.\right.\right. & \left.\left.-\frac{1}{1-\phi} y_{j, \tau}\right) \prod_{u=\tau+1}^{\infty} \frac{1+x_{j, u}}{1+r}\right)\left(\prod_{t+1 \leq u \leq \tau} S_{j, u-1}\right) V_{j, t-1}^{(j)} E_{t-1}\left[u^{\prime}\left(y_{j, t}\right)\right] \\
& \left.+\left(\frac{\phi}{1-\phi}-\frac{\lambda_{j}}{1-\phi} \prod_{u=t+1}^{\infty} \frac{1+x_{j, u}}{1+r}\right) V_{j, t-1}\right]=0 \tag{D.12}
\end{align*}
$$

where we have used (D.11) to calculate the derivative of $V_{j, \tau-1}$, and we continue to omit argument $\mathcal{H}^{\tau}$ in $y_{j, \tau}$ and argument $\mathcal{H}^{T}$ in $\lambda_{j}$. Taking the limit $\sigma \rightarrow 0$ in (D.12), we obtain

$$
\begin{equation*}
\sum_{\tau \geq t} \frac{\theta^{\tau-t}}{(1+r)^{\tau-t}}\left(\frac{\phi}{1-\phi} y_{j, \tau}^{0}+\lambda_{j}^{0}\left(r-\frac{1}{1-\phi} y_{j, \tau}^{0}\right)\right) \frac{V_{j, t-1}^{(j)}}{V_{j, t-1}} u^{\prime}\left(y_{j, t}^{0}\right)+\left(\frac{\phi}{1-\phi}-\frac{\lambda_{j}^{0}}{1-\phi}\right)=0 . \tag{D.13}
\end{equation*}
$$

We denote a first-order expansion of $\lambda_{j}$ as

$$
\lambda_{j}=\lambda_{j}^{0}+\sigma \lambda_{j}^{\prime}+O\left(\sigma^{2}\right)
$$

where $\lambda_{j}^{0}$ is deterministic and $\lambda_{j}^{\prime}$ is a function of $\mathcal{H}^{T}$. Taking the limit $\sigma \rightarrow 0$ in the intertemporal budget constraint (D.10), we obtain

$$
\begin{equation*}
\sum_{\tau=1}^{\infty} \frac{1}{(1+r)^{\tau}}\left(r-\frac{1}{1-\phi} y_{j, \tau}^{0}\right) V_{j, \tau-1}=0 \tag{D.14}
\end{equation*}
$$

The solution to (D.13) and (D.14) is

$$
\begin{equation*}
y_{j, t}^{0}=(1-\phi) r \tag{D.15}
\end{equation*}
$$

and

$$
\begin{equation*}
\lambda_{j}^{0}=\phi+\frac{1+r}{1+r-\theta}(1-\phi) \phi r \alpha u^{\prime} . \tag{D.16}
\end{equation*}
$$

First order condition with respect to $a_{j, t}$ The first order condition of the Lagrangian with respect to $a_{j, t}$ is

$$
\begin{array}{r}
E_{0}\left[\sum_{\tau \geq t} \frac{1}{(1+r)^{\tau-t}}\left(\frac{\phi}{1-\phi} y_{j, \tau}+\lambda_{j}\left(x_{j, \tau}-\frac{1}{1-\phi} y_{j, \tau}\right) \prod_{u=\tau+1}^{\infty} \frac{1+x_{j, u}}{1+r}\right)\left(\prod_{t+1 \leq u \leq \tau} S_{j, u-1}\right) V_{j, t-1}^{(j)} E_{t-1}\left[u^{\prime}\left(y_{j, t}\right) \epsilon_{j, t}^{x}\right]\right. \\
\left.-\frac{\lambda_{j}}{1-\phi} \epsilon_{j, t}^{x} V_{j, t-1} \prod_{u=t+1}^{\infty} \frac{1+x_{j, u}}{1+r}\right]=0 . \quad \text { (D.17) } \tag{D.17}
\end{array}
$$

We have

$$
E_{t-1}\left[u^{\prime}\left(y_{j, t}\right) \epsilon_{j, t}^{x}\right]=u^{\prime \prime} a_{j, t} \sigma_{x, j}^{2} \sigma+O\left(\sigma^{2}\right),
$$

where we denote $\sigma_{x, j}^{2} \equiv \operatorname{Var}\left(\epsilon_{j, t}^{x}\right)$. A first order approximation of (D.17) is

$$
\frac{1+r}{1+r-\theta} \phi r \alpha s_{j} u^{\prime \prime} a_{j, t} \sigma_{x, j}^{2} \sigma-\frac{1}{1-\phi} E_{0}\left[\lambda_{j}^{\prime} \epsilon_{j, t}^{x}\right] s_{j} \sigma=O\left(\sigma^{2}\right),
$$

which implies

$$
\begin{equation*}
u^{\prime \prime} a_{j, t} \sigma_{x, j}^{2}=\frac{1+r-\theta}{1+r} \frac{1}{(1-\phi) \phi r \alpha} E_{0}\left[\lambda_{j}^{\prime} \epsilon_{j, t}^{x}\right] . \tag{D.18}
\end{equation*}
$$

First order condition with respect to $b_{j, s}$ The first order condition of the Lagrangian with respect to $b_{j, s}$ is

$$
\begin{gather*}
E_{0}\left[\sum_{t>s} \sum_{\tau \geq t} \frac{1}{(1+r)^{\tau-s}}\left(\frac{\phi}{1-\phi} y_{j, \tau}+\lambda_{j}\left(x_{j, \tau}-\frac{1}{1-\phi} y_{j, \tau}\right) \prod_{u=\tau+1}^{\infty} \frac{1+x_{j, u}}{1+r}\right)\left(\prod_{t+1 \leq u \leq \tau} S_{j, u-1}\right) V_{j, t-1}^{(j)} E_{t-1}\left[u^{\prime}\left(y_{j, t}\right)\right] \epsilon_{j, s}^{x}\right. \\
\left.+\sum_{\tau>s} \frac{1}{(1+r)^{\tau-s}}\left(\frac{\phi}{1-\phi} \epsilon_{j, s}^{x}-\frac{\lambda_{j}}{1-\phi} \epsilon_{j, s}^{x} \prod_{u=\tau+1}^{\infty} \frac{1+x_{j, u}}{1+r}\right) V_{j, \tau-1}\right]=0 . \quad \text { (D.19) } \tag{D.19}
\end{gather*}
$$

Towards calculating a first order approximation of (D.19), we show in Section D.1.1 that

$$
\begin{aligned}
& V_{j, t-1}^{(j)}=s_{j}(\beta \theta+\alpha)+\left\{s_{j} u^{\prime} \sum_{s=1}^{t-1}\left(\left(1-\theta^{t-1-s}\right) \beta \theta(\beta \theta+\alpha)+\beta^{2} \theta(1-2 \theta)+\alpha^{2}\right) b_{j, s} \epsilon_{j, s}^{x}\right. \\
& \left.-s_{j} u^{\prime} \sum_{s=1}^{t-1}\left(\alpha \beta \theta\left(1-\theta^{t-1-s}\right)+\alpha^{2}\right) \overline{b \epsilon}_{s}^{x}+\alpha s_{j} \beta \theta(1-\theta) u^{\prime} \sum_{s=1}^{t-1} \theta^{t-1-s} \delta_{s-1}+\alpha^{2} s_{j} u^{\prime} \delta_{t-1}\right\} \sigma+O\left(\sigma^{2}\right),
\end{aligned}
$$

and we have

$$
E_{t-1}\left[u^{\prime}\left(y_{j, t}\right)\right]=u^{\prime}+u^{\prime \prime} \sum_{s=1}^{t-1} b_{j, s} \epsilon_{j, s}^{x} \sigma+O\left(\sigma^{2}\right) .
$$

Therefore, a first order approximation of the term on the first line of (D.19) is

$$
\begin{aligned}
& \sum_{t>s} \sum_{\tau \geq t} \frac{1}{(1+r)^{\tau-s}}\left[\frac{\phi-\lambda^{0}}{1-\phi} b_{j, s} \sigma_{x, j}^{2} \theta^{\tau-t} s_{j}(\beta \theta+\alpha) u^{\prime}\right. \\
& \quad+\phi r \theta^{\tau-t} s_{j} u^{\prime}\left(\left(1-\theta^{t-1-s}\right) \beta \theta(\beta \theta+\alpha)+\beta^{2} \theta(1-2 \theta)+\alpha^{2}\right) b_{j, s} \sigma_{x, j}^{2} u^{\prime} \\
&\left.+\phi r \theta^{\tau-t} s_{j}(\beta \theta+\alpha) u^{\prime \prime} b_{j, s} \sigma_{x, j}^{2}\right]
\end{aligned}
$$

Calculating the sums and rearranging:

$$
\begin{align*}
\frac{1+r}{1+r-\theta}\left[\frac{1}{r} \frac{\phi-\lambda^{0}}{(1-\phi) \phi r}(\beta \theta+\alpha) u^{\prime}\right. & +\frac{1}{r}\left(\beta \theta(\beta \theta+\alpha)+\beta^{2} \theta(1-2 \theta)+\alpha^{2}\right)\left(u^{\prime}\right)^{2} \\
& \left.-\frac{\theta}{1+r-\theta} \beta(\beta \theta+\alpha)\left(u^{\prime}\right)^{2}+\frac{1}{r}(\beta \theta+\alpha) u^{\prime \prime}\right] \phi r s_{j} \sigma_{x, j}^{2} b_{j, s} \tag{D.20}
\end{align*}
$$

We show in Section D.1.1 that

$$
\begin{aligned}
& V_{j, \tau-1}=\frac{s_{j}}{1-\theta}+\left\{\frac{s_{j}}{1-\theta} u^{\prime} \sum_{s=1}^{\tau-1}\left(1-\theta^{\tau-s}\right)(\beta \theta+\alpha) b_{j, s} \epsilon_{j, s}^{x}\right. \\
&\left.-\frac{s_{j}}{1-\theta} u^{\prime} \sum_{s=1}^{\tau-1}\left(1-\theta^{\tau-s}\right) \alpha \overline{b \epsilon}_{s}^{x}+\alpha s_{j} u^{\prime} \sum_{s=1}^{\tau} \theta^{\tau-s} \delta_{s-1}\right\} \sigma+O\left(\sigma^{2}\right) .
\end{aligned}
$$

Therefore, a first order approximation of the term on the second line of (D.19) is

$$
\begin{array}{r}
-\frac{1}{1-\phi} \sum_{\tau>s} \frac{1}{(1+r)^{\tau-s}} E_{0}\left[\lambda_{j} \epsilon_{j, s}^{x}\right] \frac{s_{j}}{1-\theta}+\frac{\phi-\lambda^{0}}{1-\phi} \sum_{\tau>s} \frac{1}{(1+r)^{\tau-s}} \frac{s_{j}}{1-\theta} u^{\prime}\left(1-\theta^{\tau-s}\right)(\beta \theta+\alpha) b_{j, s} \sigma_{x, j}^{2} \\
\quad=-\frac{1}{1-\phi} \frac{1}{r} E_{0}\left[\lambda_{j} \epsilon_{j, s}^{x}\right] \frac{s_{j}}{1-\theta}+\frac{\phi-\lambda^{0}}{1-\phi} \frac{s_{j}}{1-\theta} u^{\prime}\left(\frac{1}{r}-\frac{\theta}{1+r-\theta}\right)(\beta \theta+\alpha) b_{j, s} \sigma_{x, j}^{2} . \quad \text { (D.21) } \tag{D.21}
\end{array}
$$

Using (D.16) to substitute $\frac{\phi-\lambda_{j}}{1-\phi}=-\frac{1+r}{1+r-\theta} \phi r \alpha u^{\prime}$ in (D.20) and (D.21), and using (D.18) to substitute $E_{0}\left[\lambda_{j} \epsilon_{j, s}^{x}\right]$ in (D.21), we obtain:

$$
\begin{equation*}
\left[\frac{\alpha+\beta \theta}{1+r-\theta}(2 \alpha(1+r)+\beta \theta r)-\alpha(\alpha+\beta \theta)-\beta^{2} \theta(1-\theta)+(\alpha+\beta \theta) \rho\right] b_{j, s}=\frac{\alpha \rho}{1-\theta} a_{j, s} \tag{D.22}
\end{equation*}
$$

where $\rho=-u^{\prime \prime} / u^{\prime}$. A first-order approximation of the intertemporal budget constraint (D.10) is

$$
\sum_{s=1}^{\infty} \frac{1}{(1+r)^{s}}\left(1-\phi-a_{j, s}-\frac{b_{j, s}}{r}\right) \sigma \epsilon_{j, s}^{x}=O\left(\sigma^{2}\right)
$$

which implies

$$
\begin{equation*}
a_{j, s}=1-\phi-\frac{b_{j, s}}{r} . \tag{D.23}
\end{equation*}
$$

(D.22) and (D.23) imply that $a_{j, s}$ and $b_{j, s}$ are constant independent of $j$ and $s: a_{j, s}=a$ and $b_{j, s}=b$. Using (D.8), we can write contract return as a function of current and past asset return:

$$
\begin{equation*}
y_{j, t}=(1-\phi) r+a \sigma \epsilon_{j, t}^{x}+\sum_{s=1}^{t-1} b \sigma \epsilon_{j, s}^{x}, \tag{D.24}
\end{equation*}
$$

where $a$ and $b$ are given by (D.22) and (D.23).

Contract return as function of reserves Reserves evolve according to $R_{j, t}=\left(1+x_{j, t}\right) R_{j, t-1}+$ $\left(x_{j, t}-\frac{1}{1-\phi} y_{j, t}\right) V_{j, t-1}$ with $R_{j, 0}=0$. Therefore, a first-order approximation of $R_{j, t}$ is

$$
R_{j, t}=\sigma h_{j, t} V_{j}+O\left(\sigma^{2}\right),
$$

where, following the same steps as in the proof of Proposition 2, we have

$$
h_{j, t}=\frac{1}{r} \sum_{s=1}^{t} \frac{b}{1-\phi} \epsilon_{j, s}^{x} .
$$

Substituting in (D.24), we obtain

$$
\begin{aligned}
y_{j, t} & =(1-\phi) r+a \sigma \epsilon_{j, t}^{x}+(1-\phi) r \frac{R_{j, t-1}}{V_{j, t-1}}+O\left(\sigma^{2}\right) \\
& \simeq(1-\phi) r+\left(a-(1-\phi) \frac{r}{1+r}\right)\left(x_{j, t}-r\right)+(1-\phi) \frac{r}{1+r}\left(\mathcal{R}_{j, t^{-}}-r\right),
\end{aligned}
$$

where $a$ is determined by (D.22) and (D.23). Therefore

$$
\begin{equation*}
y_{j, t} \simeq c s t e_{t}+A x_{j, t}+\frac{(1-\phi) r}{1+r} \mathcal{R}_{j, t^{-}} \tag{D.25}
\end{equation*}
$$

where

$$
\begin{equation*}
A=\frac{(1-\phi) r}{1+r} \frac{\left[\frac{\alpha+\beta \theta}{1+r-\theta}(2 \alpha(1+r)+\beta \theta r)-\alpha(\alpha+\beta \theta)-\beta^{2} \theta(1-\theta)+(\alpha+\beta \theta) \rho\right]-\frac{\alpha \rho}{1-\theta}}{\left[\frac{\alpha+\beta \theta}{1+r-\theta}(2 \alpha(1+r)+\beta \theta r)-\alpha(\alpha+\beta \theta)-\beta^{2} \theta(1-\theta)+(\alpha+\beta \theta) \rho\right] r+\frac{\alpha \rho}{1-\theta}} . \tag{D.26}
\end{equation*}
$$

## Limit cases

(i) $\theta \rightarrow 0$

Letting $\theta$ go to zero in (D.26), we obtain that $A$ goes to $\frac{1-\phi}{1+r} \frac{\alpha}{\alpha+\frac{1+r}{r} \rho}$.
(ii) $\alpha=(1-\theta) \beta \rightarrow 0$

Substituting $\alpha=(1-\theta) \beta$ in (D.26) and letting $\beta$ go to zero, we obtain that $A$ goes to zero.
Doing in the same in (D.22) and (D.23), we obtain $a=a_{j, t}=b=b_{j, s}$ for all $t$ and $s$.
(iii) $\alpha=(1-\theta) \beta \rightarrow \infty$

Substituting $\alpha=(1-\theta) \beta$ in (D.26) and letting $\beta$ go to infinity, we obtain $A$ goes to $\frac{1-\phi}{1+r}$. Doing in the same in (D.22) and (D.23), we obtain $a=a_{j, t}$ goes to $1-\phi$ and $b=b_{j, s}$ goes to zero.

## D.1.1 Supplementary Proofs

First order approximation of $V_{j, t-1}$ Account value is given by

$$
V_{j, t-1}=\frac{s_{j}}{1-\theta} \prod_{1 \leq u \leq t} S_{j, u-1}+\sum_{1 \leq s \leq t} I_{j, s-1} \prod_{s+1 \leq u \leq t} S_{j, u-1}
$$

where we have used that $V_{j,-1}=\frac{s_{j}}{1-\theta}$. We write first order approximations as

$$
\begin{aligned}
V_{j, t-1} & =V_{j, t-1}^{0}+V_{j, t-1}^{\prime} \sigma+O\left(\sigma^{2}\right) \\
S_{j, t-1} & =\theta+S_{j, t-1}^{\prime} \sigma+O\left(\sigma^{2}\right) \\
I_{j, t-1} & =s_{j}+I_{j, t-1}^{\prime} \sigma+O\left(\sigma^{2}\right)
\end{aligned}
$$

We have

$$
V_{j, t-1}^{0}=\frac{s_{j}}{1-\theta} \theta^{t}+\sum_{1 \leq s \leq t} s_{j} \theta^{t-s}=\frac{s_{j}}{1-\theta}
$$

and

$$
\begin{align*}
V_{j, t-1}^{\prime} & =\frac{s_{j}}{1-\theta} \sum_{1 \leq u \leq t} S_{j, u-1}^{\prime} \theta^{t-1}+\sum_{1 \leq s \leq t} s_{j} \sum_{s+1 \leq u \leq t} S_{j, u-1}^{\prime} \theta^{t-s-1}+\sum_{1 \leq s \leq t} I_{j, s-1}^{\prime} \theta^{t-s} \\
& =\sum_{1 \leq u \leq t} S_{j, u-1}^{\prime}\left[\frac{s_{j}}{1-\theta} \theta^{t-1}+\sum_{1 \leq s \leq u-1} s_{j} \theta^{t-s-1}\right]+\sum_{1 \leq s \leq t} I_{j, s-1}^{\prime} \theta^{t-s} \\
& =\frac{s_{j}}{1-\theta} \sum_{1 \leq u \leq t} \theta^{t-u} S_{j, u-1}^{\prime}+\sum_{1 \leq u \leq t} \theta^{t-u} I_{j, u-1}^{\prime} \tag{D.27}
\end{align*}
$$

where we move from the first line to the second line by collecting the terms $S_{j, u-1}^{\prime}$, and to the third line by calculating the inner sum. Using

$$
E_{t-1}\left[y_{j, t}\right]=(1-\phi) r+\sigma \sum_{s=1}^{t-1}\left(b_{j, s} \epsilon_{j, s}^{x}+d_{j, s} \bar{\epsilon}_{s}^{x}\right)+O\left(\sigma^{2}\right)
$$

and the derivative of the Logit demand function, we obtain

$$
\begin{equation*}
S_{j, t-1}^{\prime}=\beta \theta(1-\theta) u^{\prime}\left(\sum_{s=1}^{t-1}\left(b_{j, s} \epsilon_{j, s}^{x}+d_{j, s} \bar{\epsilon}_{s}^{x}\right)\right) \tag{D.28}
\end{equation*}
$$

Similarly, using

$$
\sum_{k=1}^{J} s_{k} E_{t-1}\left[y_{k, t}\right]=\left(1-s_{0}\right)(1-\phi) r+\sigma \sum_{s=1}^{t-1}\left(\overline{b \epsilon}_{s}^{x}+\overline{d \epsilon}_{s}^{x}\right)+O\left(\sigma^{2}\right)
$$

where $\overline{b \epsilon}_{s}^{x} \equiv \sum_{k=1}^{J} s_{k} b_{k, s} \epsilon_{k, s}^{x}$ and $\overline{d \epsilon}_{s}^{x} \equiv \sum_{k=1}^{J} s_{k} d_{k, s} \bar{\epsilon}_{s}^{x}$, we obtain

$$
\begin{equation*}
I_{j, t-1}^{\prime}=\alpha s_{j} u^{\prime}\left(\sum_{s=1}^{t-1}\left(\left(b_{j, s} \epsilon_{j, s}^{x}+d_{j, s} \bar{\epsilon}_{s}^{x}\right)-\left(\overline{b \epsilon}_{s}^{x}+\overline{d \epsilon}_{s}^{x}\right)\right)+\delta_{t-1}\right) \tag{D.29}
\end{equation*}
$$

where $\delta_{t-1} \equiv \frac{1}{\alpha u^{\prime}}\left(\xi_{j, t-1}-\sum_{k=1}^{J} s_{k} \xi_{k, t-1}\right)$. Using (D.31) and (D.32) to substitute $S_{j, t-1}^{\prime}$ and $I_{j, t-1}^{\prime}$ in (D.27), we obtain

$$
\begin{aligned}
V_{j, t-1}^{\prime} & =s_{j} \beta \theta u^{\prime} \sum_{s=1}^{t-1} \frac{1-\theta^{t-s}}{1-\theta}\left(b_{j, s} \epsilon_{j, s}^{x}+d_{j, s} \bar{\epsilon}_{s}^{x}\right)+\alpha s_{j} u^{\prime}\left(\sum_{s=1}^{t-1} \frac{1-\theta^{t-s}}{1-\theta}\left(\left(b_{j, s} \epsilon_{j, s}^{x}+d_{j, s} \epsilon_{s}^{x}\right)-\left(\overline{b \epsilon_{s}}+\overline{d \epsilon}_{s}^{x}\right)\right)+\sum_{s=1}^{t} \theta^{t-s} \delta_{s-1}\right) \\
& =\frac{s_{j}}{1-\theta} u^{\prime} \sum_{s=1}^{t-1}\left(1-\theta^{t-s}\right)(\beta \theta+\alpha)\left(b_{j, s} \epsilon_{j, s}^{x}+d_{j, s} \epsilon_{s}^{x}\right)-\frac{s_{j}}{1-\theta} u^{\prime} \sum_{s=1}^{t-1}\left(1-\theta^{t-s}\right)\left(\alpha \overline{b 匕}_{s}^{x}+\alpha \overline{d \epsilon}_{s}^{x}\right)+\alpha s_{j} u^{\prime} \sum_{s=1}^{t} \theta^{t-s} \delta_{s}(\mathrm{D} .30)
\end{aligned}
$$

First order approximation of $V_{j, t-1}^{(j)}$ The derivative of account value $V_{j, t-1}$ with respect to expected utility of return $E_{t-1}\left[u\left(y_{j, t}\right)\right]$ is given by

$$
V_{j, t-1}^{(j)}=V_{j, t-2} S_{j, t-1}^{(j)}+I_{j, t-1}^{(j)}
$$

We write first order approximations as

$$
\begin{aligned}
V_{j, t-1}^{(j)} & =V_{j, t-1}^{(j) 0}+V_{j, t-1}^{(j)^{\prime}} \sigma+O\left(\sigma^{2}\right) \\
S_{j, t-1}^{(j)} & =\beta \theta(1-\theta)+S_{j, t-1}^{(j)} \sigma+O\left(\sigma^{2}\right) \\
I_{j, t-1}^{(j)} & =\alpha s_{j}+I_{j, t-1}^{(j)^{\prime}} \sigma+O\left(\sigma^{2}\right)
\end{aligned}
$$

where

$$
V_{j, t-1}^{(j) 0}=\frac{s_{j}}{1-\theta} \beta \theta(1-\theta)+\alpha s_{j}=s_{j}(\beta \theta+\alpha)
$$

and

$$
V_{j, t-1}^{(j)^{\prime}}=V_{j, t-2}^{\prime} \beta \theta(1-\theta)+\frac{s_{j}}{1-\theta} S_{j, t-1}^{(j)^{\prime}}+I_{j, t-1}^{(j)^{\prime}}
$$

Using

$$
\begin{align*}
S_{j, t-1}^{(j)^{\prime}} & =\beta^{2} \theta(1-\theta)(1-2 \theta) u^{\prime}\left(\sum_{s=1}^{t-1}\left(b_{j, s} \epsilon_{j, s}^{x}+d_{j, s} \bar{\epsilon}_{s}^{x}\right)\right)  \tag{D.31}\\
I_{j, t-1}^{(j)^{\prime}} & =\alpha^{2} s_{j} u^{\prime}\left(\sum_{s=1}^{t-1}\left(\left(b_{j, s} \epsilon_{j, s}^{x}+d_{j, s} \bar{\epsilon}_{s}^{x}\right)-\left(\overline{b \epsilon}_{s}^{x}+\overline{d \epsilon} \epsilon_{s}^{x}\right)\right)+\delta_{t-1}\right) \tag{D.32}
\end{align*}
$$

we calculate

$$
\begin{aligned}
V_{j, t-1}^{(j)^{\prime}}= & {\left[\frac{s_{j}}{1-\theta} u^{\prime} \sum_{s=1}^{t-2}\left(1-\theta^{t-1-s}\right)(\beta \theta+\alpha) b_{j, s} \epsilon_{j, s}^{x}+\frac{s_{j}}{1-\theta} u^{\prime} \sum_{s=1}^{t-2}\left(1-\theta^{t-1-s}\right)(\beta \theta+\alpha) d_{j, s} \bar{\epsilon}_{s}^{x}\right.} \\
& \left.-\frac{s_{j}}{1-\theta} u^{\prime} \sum_{s=1}^{t-2}\left(1-\theta^{t-1-s}\right)\left(\alpha \overline{b 匕 \epsilon}_{s}^{x}+\alpha \overline{\epsilon \epsilon \epsilon}_{s}^{x}\right)+\alpha s_{j} u^{\prime} \sum_{s=1}^{t-1} \theta^{t-1-s} \delta_{s-1}\right] \beta \theta(1-\theta) \\
& +\frac{s_{j}}{1-\theta} \beta^{2} \theta(1-\theta)(1-2 \theta) u^{\prime}\left(\sum_{s=1}^{t-1} b_{j, s} \epsilon_{j, s}^{x}+\sum_{s=1}^{t-1} d_{j, s} \bar{\epsilon}_{s}^{x}\right) \\
& +\alpha^{2} s_{j} u^{\prime}\left(\sum_{s=1}^{t-1} b_{j, s} \epsilon_{j, s}^{x}+\sum_{s=1}^{t-1} d_{j, s} \bar{\epsilon}_{s}^{x}-\sum_{s=1}^{t-1} \bar{b}_{s}^{x}-\sum_{s=1}^{t-1} \overline{\epsilon \epsilon}_{s}^{x}+\delta_{t-1}\right)
\end{aligned}
$$

Rearranging terms:

$$
\begin{aligned}
V_{j, t-1}^{(j))^{\prime}}= & s_{j} u^{\prime} \sum_{s=1}^{t-1}\left(\left(1-\theta^{t-1-s}\right) \beta \theta(\beta \theta+\alpha)+\beta^{2} \theta(1-2 \theta)+\alpha^{2}\right)\left(b_{j, s} \epsilon_{j, s}^{x}+d_{j, s} \bar{\epsilon}_{s}^{x}\right) \\
& -s_{j} u^{\prime} \sum_{s=1}^{t-1}\left(\alpha \beta \theta\left(1-\theta^{t-1-s}\right)+\alpha^{2}\right)\left(\overline{b \epsilon}_{s}^{x}+\overline{d \epsilon}_{s}^{x}\right)+\alpha s_{j} \beta \theta(1-\theta) u^{\prime} \sum_{s=1}^{t-1} \theta^{t-1-s} \delta_{s-1}+\alpha^{2} s_{j} u^{\prime} \delta_{t-1}
\end{aligned}
$$

## D. 2 Flow-Reserves Relations

## D.2.1 Inflows

Using a first order expansion of the Logit demand function, the inflow amount is given by

$$
\text { Inflow }_{j, t-1} \simeq s_{j}+\alpha s_{j} u^{\prime}\left(E_{t-1}\left[y_{j, t}\right]-(1-\phi) r-c s t e_{t}\right) .
$$

Therefore, $\log$ inflow is given by

$$
\log \left(\text { Inflow }_{j, t-1}\right) \simeq \log \left(s_{j}\right)+\alpha u^{\prime}\left(E_{t-1}\left[y_{j, t}\right]-(1-\phi) r-\text { cste }_{t}\right) .
$$

The expected contract return can be calculated by taking time $t-1$ expectation in (D.25):

$$
\begin{equation*}
E_{t-1}\left[y_{j, t}\right] \simeq(1-\phi) r+(1-\phi) r \frac{R_{j, t-1}}{V_{j}} \tag{D.33}
\end{equation*}
$$

Therefore:

$$
\begin{equation*}
\log \left(\text { Inflow }_{j, t-1}\right) \simeq \log \left(s_{j}\right)+\alpha(1-\phi) r \frac{R_{j, t-1}}{V_{j}}+c s t e_{t} \tag{D.34}
\end{equation*}
$$

where we have used the normalization $u^{\prime}=1$.

## D.2.2 Outflow

Using a first order expansion of the Logit demand function, the outflow rate is

$$
\text { OutflowRate }_{j, t-1} \simeq 1-\theta-\beta \theta(1-\theta) u^{\prime}\left(E_{t-1}\left[y_{j, t}\right]-(1-\phi) r\right)
$$

Using (D.33) to substitute $E_{t-1}\left[y_{j, t}\right]$, we obtain

$$
\begin{equation*}
\text { OutflowRate }_{j, t-1} \simeq 1-\theta-\beta \theta(1-\theta)(1-\phi) r \frac{R_{j, t-1}}{V_{j}} \tag{D.35}
\end{equation*}
$$

## E Additional Analysis of Demand for Euro Contracts

## E. 1 First Stage of IV

Column 1 of Table E. 1 shows the first stage of the flow-reserves IV regressions presented in Panel B of Table 5 . The first stage is highly significant: the $t$-stat is equal to 5.7 with standard errors twoway clustered by insurer and by year. In Columns 2 and 3, we regress entry fees and management fees on the lagged reserves ratio. The coefficients are small and statistically insignificant, which is in line with the IV's exclusion restriction that past asset return does not impact investor flow through fees.

Table E.1: First Stage. Panel regressions at the insurer-year level for 76 insurers over 2001-2015. In Column 1, the dependent variable is the beginning-of-year reserve ratio. In Columns 2 (3), the dependent variable is the average entry (management) fee constructed as the average fee of contracts sold by the insurer in the current year; the number of observations is smaller because the fee data do not cover all insurers. Lagged asset return is the return on the insurer's asset portfolio in the previous year. Lagged 'asset class' share is the share of the insurer's asset portfolio in each asset class at the beginning of the previous year. All regressions include insurer and year fixed effects and are weighted by the insurer share in aggregate account value in the current year. Standard errors two-way clustered by insurer and year are reported in parenthesis. ${ }^{*} p<.1 ;{ }^{* *} p<.05 ;{ }^{* * *} p<.01$.

|  | Reserve ratio <br> $(1)$ | Entry fee <br> $(2)$ | Management fee <br> $(3)$ |
| :--- | :---: | :---: | :---: |
| Lagged asset return | $.56^{* * *}$ | -.006 | -.0002 |
|  | $(.096)$ | $(.005)$ | $(.00069)$ |
| Lagged bond share | $.12^{* *}$ | -.0054 | -.00077 |
|  | $(.051)$ | $(.0095)$ | $(.00073)$ |
| Lagged stock share | $.25^{* * *}$ | $-.025^{*}$ | $-.0014^{* *}$ |
| Lagged real estate share | $(.069)$ | $(.014)$ | $(.00056)$ |
|  | -.016 | -.028 | .0011 |
| Lagged loan share | $(.33)$ | $(.025)$ | $(.0018)$ |
|  | .1 | -.041 | $.017^{* *}$ |
| Year FE | $. .15)$ | $(.043)$ | $(.0056)$ |
| Insurer FE | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $R^{2}$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| Observations | .88 | .93 | .96 |

## E. 2 Additional Covariates in Flow Regressions

Table E. 2 reports the same flow regressions as in Table 5 with additional control variables. We consider the capital ratio of the insurer measured at the end of the previous year, a dummy variable equal to one if the insurer is a bank-insurer conglomerate, and the contract return in the previous year. We find that bank-insurance conglomerates enjoy a level of inflows $e^{1.5}=4.5$ times larger than stand-alone insurers (Column 10) as well as a lower outflow rate (Column 7). This suggests that the distribution channel is a key determinant of investor demand, with bank-insurance conglomerates benefiting from the advantage of tunneling depositors towards the contracts of their subsidiaries. The effect of the insurer's capital ratio on flow rates is insignificant except in one specification in which a higher capital ratio is associated with a lower outflow rate. In the specifications in level, a higher capital ratio is associated with lower flow amounts, reflecting the fact that smaller insurers have higher capital ratios. The effect of the lagged contract return is insignificant. We note, however, that the regression coefficient on lagged contract return is a biased estimate of the sensitivity of demand to lagged contract return because contract return is a choice variable of the insurer. Proposition 2 shows that insurers increase the contract return when demand is higher (holding fixed asset return and reserves). Therefore, the relation between flows and contract return is contaminated with reverse causality. Reverse causality is not a problem for the flow-performance relation studied in the mutual fund literature, because mutual funds have no flexibility to adjust the return paid to investors: for mutual funds, the contract return is pinned down by the realized asset return. By contrast, euro contract returns are disconnected from the current asset return (as shown by Figure 1 and Table 3) and chosen by the insurer.
Table E.2: Investor Flows with Additional Covariates. Panel regressions at the insurer-year level for 76 insurers over 2000-2015. Inflows is total premia normalized by total account value. Redemptions is voluntary redemptions normalized by total account value. Termination is involuntary redemptions at contract termination (investor death) normalized by total account value. Net flows is Inflows minus Redemptions minus Termination. Lagged reserves is the beginning-of-year level of reserves normalized by total account value. Capital ratio is the lagged capital normalized by total account value. Bank-insurer is a dummy variable equal to one if the insurer is a bank-insurer conglomerate. Lagged contract return in contract return in previous year. Panel A shows OLS regressions. Panel B shows IV regressions in which the insurer's beginning-of-year reserve ratio is instrumented using the insurer's asset return in the previous year (the first year of data for each insurer is therefore dropped from the second stage). All regressions are weighted by the insurer share in aggregate account value in the current year. Standard errors two-way clustered by insurer and year are reported in parenthesis. ${ }^{*} p<.1 ;{ }^{* *} p<.05 ;{ }^{* * *} p<.01$.

|  | Panel A: OLS Regressions |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Net flows |  |  | Inflows |  |  | Outflows |  |  | Log(Inflows) |  |  |
|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| Lagged reserves | -.11* | . 042 | . 043 | -. 07 | . 036 | . 038 | . 036 | -. 0066 | -. 005 | 3.9 | -. 81 | -. 72 |
|  | (.05) | (.039) | (.041) | (.061) | (.038) | (.039) | (.039) | (.018) | (.019) | (2.5) | (1) | (.98) |
| Capital ratio | . 015 | -. 15 | -. 15 | -. 15 | -. 087 | -. 088 | -. $16^{* * *}$ | . 06 | . 059 | $-18 * * *$ | -8.8*** | $-8.8{ }^{* * *}$ |
|  | (.095) | (.15) | (.15) | (.099) | (.12) | (.12) | (.056) | (.055) | (.055) | (4.1) | (2.8) | (2.9) |
| Bank-insurer | . $013{ }^{*}$ | (.15) |  | . 0061 | (.12) | (.12) | -.0068* | (.055) | (.055) | $1.5{ }^{* * *}$ | (2.8) | (2.9) |
| Lagged contract return | (.007) |  |  | (.0086) |  |  | (.0037) |  |  | (.3) |  |  |
| Lagged contract return |  |  | $\text { -. } .531$ |  |  | $\begin{aligned} & -.13 \\ & (.41) \end{aligned}$ |  |  | -.091 |  |  | $\begin{gathered} -5.2 \\ (8.9) \end{gathered}$ |
| Year FE Insurer FE | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\stackrel{R}{\text { Observations }}$ | 978 | 63 9 9 | 63 9 | 24 978 | 73 9 | 73 978 | $\stackrel{12}{9} 78$ | 66 9 | 66 9 9 | 9 978 | 92 978 | -92 |


|  | Panel B: IV Regressions |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Net flows |  |  | Inflows |  |  | Outflows |  |  | Log(Inflows) |  |  |
|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| Lagged reserves | . 022 | . 12 | . 12 | -. 026 | . 069 | . 07 | -. 048 | -. 051 | -. 051 | 2.4 | . 32 | . 34 |
|  | (.069) | (.088) | (.088) | (.079) | (.075) | (.075) | (.029) | (.037) | (.037) | (2.2) | (1.2) | (1.2) |
| Capital ratio | -. 011 | -. 19 | -. 19 | -. 16 | -. 12 | -. 12 | -.14** | . 066 | . 066 | -20*** | -10*** | -10*** |
|  | (.094) | (.16) | (.16) | (.11) | (.13) | (.13) | (.059) | (.048) | (.047) | (4.3) | (3.3) | (3.3) |
| Bank-insurer | (.0097 |  |  | . 00059 |  |  | -..0038 |  |  | $1.7 * * *$ |  |  |
| Lagged contract return | (.0074) |  | $\begin{aligned} & -.28 \\ & (.62) \end{aligned}$ | (.0075) |  | $\begin{gathered} -.32 \\ (.47) \end{gathered}$ | (.0036) |  | $\begin{gathered} -.04 \\ (.15) \end{gathered}$ | (.3) |  | $\begin{aligned} & -5.2 \\ & (8.2) \end{aligned}$ |
| Year FE Insurer FE | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $\stackrel{R^{2}}{\text { Observations }}$ | 42 910 | 65 910 | 65 910 | 36 910 | 76 910 | 76 910 | 15 910 | 67 910 | 67 910 | .53 910 | 92 910 | 92 910 |

## E. 3 Two-Year Lagged Instrument

Our baseline IV specification instruments reserves using the one-year lagged asset return. It already controls for the (one-year lagged) asset portfolio weights to mitigate the concern that insurers may adjust asset risk exposure to expected demand. To additionally control for this potential threat to the validity of the instrument, in this appendix, we instrument reserves using the two-year lagged asset return controlling for the two-year lagged asset portfolio weights. The first stage becomes less significant ( $t$-stat 3.2) because lagging the instrument mechanically reduces its power but also because each additional lag requires to drop one year of data. We also experimented with a threeyear lag but the first stage becomes insignificant at conventional statistical levels ( $t$-stat 1.5). The results of the second stage using the two-year lagged instrument are shown in Table E.3. As in the OLS and as in the baseline IV specification, these specifications imply that the estimated sensitivity of flows to reserves is economically small and statistically insignificant.

Table E.3: Investor Flows with Two-Year Lagged Instrument. Panel regressions at the insurer-year level for 76 insurers over 2000-2015. Lagged reserves are instrumented using the insurer's asset return in year $t-2$ controlling for shares of the insurer's asset portfolio in five broad asset classes at the beginning of year $t-2$. The first two years of data for each insurer are therefore dropped. Inflows is total premia normalized by total account value. Redemptions is voluntary redemptions normalized by total account value. Termination is involuntary redemptions at contract termination (investor death) normalized by total account value. Net flows is Inflows minus Redemptions minus Termination. Lagged reserves is the beginning-of-year level of reserves normalized by total account value. All regressions are weighted by the insurer share in aggregate account value in the current year. Standard errors two-way clustered by insurer and year are reported in parenthesis. ${ }^{*} p<.1 ;{ }^{* *} p<.05 ;{ }^{* * *} p<.01$.

|  | Net flows <br>  <br>  <br> $(1)$ | Inflows <br> $(2)$ | Redemptions <br> $(3)$ | Termination <br> Log(Inflows) |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Lagged reserves | -.11 | -.005 | .14 | -.033 | -.69 |
|  | $(.2)$ | $(.13)$ | $(.093)$ | $(.036)$ | $(2.8)$ |
| Year FE | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| Insurer FE | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $R^{2}$ | .66 | .78 | .72 | .8 | .93 |
| Observations | 772 | 772 | 772 | 772 | 772 |

## E. 4 Regulated Interest Rate

In this appendix, we show that flows to euro contracts decrease when the interest rate on a regulated savings product which competes with euro contracts increases. The regulated savings product is called Livret A. It has an investment limit of 23,000 euros per household member including children and is demandable on short notice. Interests are tax free. In 2015, there were 60 million Livret A accounts for a population of 66 million, representing an outstanding investment of 250 billion euros. Livret A accounts are distributed by banks but the interest rate is fixed by the Ministry of Finance. The rate is revised twice a year based on the short-term interest rate and the inflation rate. Changes in the regulated rate are discussed in the press, making them salient events. Since euro contract returns are smooth and the regulated rate tracks the short-term rate, there is time-series variation in the difference between euro contract returns and the regulated rate. Figure E. 2 suggests that aggregate inflow into euro contracts decreases when the regulated rate increases relative to euro contract returns. ${ }^{3}$ We confirm the graphical analysis with regressions in Table E.4. We use two different approaches to control for time trends. In Panel A, we work with variables in first difference. In Panel B, we work with variables in level and include a quadratic time trend. Both approaches yield consistent results. Inflows to euro contract increase when the return differential between euro contract and regulated savings increases (i.e., when the regulated rate decreases), whereas the opposite holds for outflows.

[^3]Figure E.2: Flows and Regulated Interest Rate. Aggregate inflow to euro contracts normalized by aggregate account value (solid blue), weighted-average euro contract return (dashed red), and yearly average interest rate of regulated savings product Livret A (dashed green).


Table E.4: Flows and Regulated Interest Rate. Time-series regressions over 2000-2015. Regressions in Panel A are in first difference. Regressions in Panel B are in level and include a quadratic time trend. Inflows is aggregate premia normalized by aggregate account value. Redemptions is aggregate voluntary redemptions normalized by aggregate account value. Termination is aggregate involuntary redemptions at contract termination (investor death) normalized by aggregate account value. Net flows is Inflows minus Redemptions minus Termination. $\log ($ Inflows $)$ is the log inflow amount. Contract return minus Regulated rate is contract return averaged across insurers minus regulated interest rate averaged over the year. Newey-West standard errors with two lags are reported in parenthesis. ${ }^{*} p<.1 ;{ }^{* *} p<.05 ;{ }^{* * *} p<.01$.

|  | Panel A: First difference |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Net flows | Inflows | Redemptions | Termination | $\log$ (Inflows) |
|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ |
| Return spread | $-1.6^{* * *}$ | $-1^{* * *}$ | $.65^{* *}$ | -.011 | $-8.1^{*}$ |
|  | $(.47)$ | $(.29)$ | $(.23)$ | $(.019)$ | $(3.9)$ |
| Observations | 16 | 16 | 16 | 16 | 16 |
|  |  | Panel B: Level with quadratic time trend |  |  |  |
|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ |
| Return spread | $-1.9^{* * *}$ | $-1.2^{* * *}$ | $.69^{* * *}$ | -.031 | $-11^{* * *}$ |
|  | $(.52)$ | $(.32)$ | $(.22)$ | $(.039)$ | $(3.5)$ |
| Observations | 17 | 17 | 17 | 17 | 17 |

Table E.5: Flow-Reserves Relation During Financial Crises. The specifications are the same as in Table 5 except that lagged reserves is interacted with a crisis dummy equal to one for the years 2008, 2011, and 2012. These years correspond to the Global Financial Crisis and the European sovereign debt crisis. In the IV regressions, both lagged reserves and its interactions with the crisis dummy are instrumented with lagged asset return and its interactions with the crisis dummy. ${ }^{*} p<.1 ;{ }^{* *} p<.05 ;{ }^{* * *} p<.01$.

|  | Panel A: OLS Regressions |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Net flows <br> (1) | Inflows <br> (2) | Redemptions <br> (3) | Termination <br> (4) | $\log$ (Inflows) <br> (5) |
| Lagged reserves | . 019 | . 024 | -. 0056 | . 013 | -1 |
|  | (.038) | (.038) | (.017) | (.0098) | (.97) |
| Lagged reserves $\times$ Crisis | . $16^{* *}$ | . 07 | -.078** | -.012*** | -2.3 |
|  | (.069) | (.046) | (.036) | (.003) | (1.5) |
| Year FE | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| Insurer FE | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $R^{2}$ <br> Observations | . 64 | . 73 | . 75 | . 79 | . 91 |
|  | 978 | 978 | 978 | 978 | 978 |
|  | Panel B: IV Regressions |  |  |  |  |
|  | Net flows | Inflows | Redemptions | Termination | $\log$ (Inflows) |
|  | (1) | (2) | (3) | (4) | (5) |
| Lagged reserves | .086* | . 063 | -. 011 | -. 0084 | . 46 |
|  | (.047) | (.065) | (.027) | (.014) | (1.1) |
| Lagged reserves $\times$ Crisis | . 33 | . 074 | -. 22 | -. 028 | -. 31 |
|  | (.23) | (.078) | (.13) | (.055) | (4.4) |
| Year FE | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| Insurer FE | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| $R^{2}$ | . 65 | . 76 | . 76 | . 8 | . 92 |
| Observations | 910 | 910 | 910 | 910 | 910 |

## F Mean Reversion of Reserves

This appendix presents the calculation of the mean reversion rate of the reserve ratio that we use in Section 4.4. Using (7) to substitute insurer profit in the sequential budget constraint (4), the evolution of reserves is given by

$$
\begin{equation*}
\left(1+y_{j, t}+\mathcal{F}_{j, t}\right) \mathcal{R}_{j, t}=x_{j, t}+\left(1+x_{j, t}\right) \mathcal{R}_{j, t-1}-\frac{1}{1-\phi} y_{j, t} \tag{F.1}
\end{equation*}
$$

where $\mathcal{R}_{j, t}=R_{j, t} / V_{j, t}$ is the reserve ratio and $\mathcal{F}_{j, t}=\left(V_{j, t}-\left(1+y_{j, t}\right) V_{j, t-1}\right) / V_{j, t-1}$ is net flow. Taking the conditional expectation $E_{t-1}[$.$] of (F.1) and linearizing (F.1) around the steady state, we obtain$

$$
\begin{equation*}
\mathcal{R}_{j, t}-\overline{\mathcal{R}}=\frac{1+\bar{x}}{1+\bar{y}+\overline{\mathcal{F}}}\left(\mathcal{R}_{j, t-1}-\overline{\mathcal{R}}\right)-\frac{\frac{1}{1-\phi}+\overline{\mathcal{R}}}{1+\bar{y}+\overline{\mathcal{F}}}\left(y_{j, t}-\bar{y}\right)-\frac{\overline{\mathcal{R}}}{1+\bar{y}+\overline{\mathcal{F}}}\left(\mathcal{F}_{j, t}-\overline{\mathcal{F}}\right), \tag{F.2}
\end{equation*}
$$

where upper bars denote steady state values. The empirical estimate of the contract return policy in Table 3 implies

$$
\begin{equation*}
E_{t-1}\left[y_{j, t}\right]=\bar{y}+\frac{\partial y}{\partial \mathcal{R}} \times\left(\mathcal{R}_{j, t-1}-\overline{\mathcal{R}}\right), \quad \text { where } \frac{\partial y}{\partial \mathcal{R}} \simeq 0.03 \tag{F.3}
\end{equation*}
$$

The empirical estimate of the flow-reserves relation in Table 5 implies

$$
\begin{equation*}
E_{t-1}\left[\mathcal{F}_{j, t}\right]=\overline{\mathcal{F}}+\frac{\partial \mathcal{F}}{\partial \mathcal{R}} \times\left(\mathcal{R}_{j, t-1}-\overline{\mathcal{R}}\right), \quad \text { where } \frac{\partial \mathcal{F}}{\partial \mathcal{R}} \simeq 0 \tag{F.4}
\end{equation*}
$$

Using (F.3) and (F.4) to substitute $E_{t-1}\left[y_{j, t}\right]$ and $E_{t-1}\left[\mathcal{F}_{j, t}\right]$, respectively, in (F.2), we obtain

$$
\begin{align*}
E_{t-1}\left[\mathcal{R}_{j, t}-\overline{\mathcal{R}}\right]=\left(\frac{1+\bar{x}}{1+\bar{y}+\overline{\mathcal{F}}}-\frac{\frac{1}{1-\phi}+\overline{\mathcal{R}}}{1+\bar{y}+\overline{\mathcal{F}}} \frac{\partial y}{\partial \mathcal{R}}-\frac{\overline{\mathcal{R}}}{1+\bar{y}+\overline{\mathcal{F}}} \frac{\partial \mathcal{F}}{\partial \mathcal{R}}\right) & \left(\mathcal{R}_{j, t-1}-\overline{\mathcal{R}}\right) \\
& \equiv(1-\delta)\left(\mathcal{R}_{j, t-1}-\overline{\mathcal{R}}\right) \tag{F.5}
\end{align*}
$$

(F.1) implies that the steady-state contract return $\bar{y}$ must satisfy $(1+\bar{y}+\overline{\mathcal{F}}) \overline{\mathcal{R}}=\bar{x}+(1+\bar{x}) \overline{\mathcal{R}}-$ $\bar{y} /(1-\phi)$. A first order approximation of (F.5) for small values of $\frac{\partial y}{\partial \mathcal{R}}, \phi, \overline{\mathcal{F}}, \bar{x}$, and $\overline{\mathcal{R}}$, implies that the reserve ratio mean reverts at rate

$$
\begin{equation*}
\delta \simeq \frac{\partial y}{\partial \mathcal{R}}+\overline{\mathcal{F}}+\frac{\partial \mathcal{F}}{\partial \mathcal{R}} \overline{\mathcal{R}} \tag{F.6}
\end{equation*}
$$

The first term of (F.6) arises because the fraction $\frac{\partial y}{\partial \mathcal{R}}$ of reserves are distributed to investors. The second and third terms reflect reserve dilution by flows. The second term arises because unconditional flows dilute reserves at a rate equal to the unconditional net flow rate $\overline{\mathcal{F}}$. The third term arises because conditional flows dilute reserves at a rate equal to the sensitivity of flows to reserves $\frac{\partial \mathcal{F}}{\partial \mathcal{R}}$ times the unconditional reserve ratio $\overline{\mathcal{R}}$. Using $\frac{\partial y}{\partial \mathcal{R}}=0.03$ (Table 3), $\overline{\mathcal{F}}=0.024$ (Table 1), and $\frac{\partial \mathcal{F}}{\partial \mathcal{R}}=0$ (Table 5), the reserve ratio mean reverts at a rate of $\delta \simeq 5.4 \%$ per year.

## G Asset Return Decomposition

Section 4.5 shows that most of the cross-sectional variation in reserves comes from heterogeneity in asset return. In this appendix, we refine the decomposition of the cross-sectional variation in reserves in order to identify whether it comes from heterogeneity in systematic risk exposure or heterogeneity in idiosyncratic risk exposure. To do this, we use data from the regulatory filings on the composition of insurers' asset portfolio, reported in five broad asset classes ( $k=$ Bonds; Stocks; Real estate; Loans; Other assets), and we estimate

$$
x_{j, t}=\sum_{k} \gamma_{k, t} w_{j, k, t}+\epsilon_{j, t}
$$

where $w_{j, k, t}$ is insurer $j$ 's portfolio share in asset class $k$ at the beginning of year $t$ reported in the regulatory filings. The regression coefficient $\gamma_{k, t}$ reflects the average return of asset class $k$ in year $t$. We replace $x_{j, t}$ in (20) by the estimated systematic risk component $\sum_{k} \widehat{\gamma}_{k, t} w_{j, k}$ and the idiosyncratic risk component $\widehat{\epsilon}_{j, t}$. Table G. 2 decomposes the cross-sectional reserves variation using this refined decomposition of the asset return. The table shows that that heterogeneity in systematic risk exposure only explains a small part of the cross-sectional variation in reserves, so that the heterogeneity in asset return is mainly due to idiosyncratic risk. One caveat with this decomposition is that the breakdown across asset classes in the regulatory filings is quite coarse, so we may over-attribute return heterogeneity to the idiosyncratic risk component. For example, we do not have information on bond duration; therefore, the cross-sectional variation in asset return due to heterogeneous interest rate risk exposure is attributed to idiosyncratic risk. The European Sovereign Debt Crisis of 2011-2012 shed light on idiosyncratic risk heterogeneity, when the large exposure of certain insurers to PIGS countries was revealed. The popular press focused on Groupama as a salient example, after its downgrade by rating agencies highlighted Groupama's losses due to its heavy exposure to Greece. ${ }^{4}$ This anecdotal evidence illustrates two features of the market. First, different insurers hold different assets, even within an asset class, so that Groupama (and other insurers) were more exposed to the European Sovereign Debt Crisis. Second, before Groupama's downgrade, investors and the general public were mostly unaware of the insurer's asset composition. Yet, even when the news came out, investors did not withdraw en masse in response to this news, and Groupama has remained one of the largest insurers in France. These facts are consistent with

[^4]our broader result that despite differences in asset returns (hence reserves) across insurers, investors are inelastic to reserves.

Table G.2: The Role of Systematic vs. Idiosyncratic Asset Return In Explaining the Cross-Sectional Variation in Reserves. Each cell in Columns 1 to 6 corresponds to a different regression at the insurer-year level. The statistics reported in each cell is the $R^{2}$ of the regression of the $h$-year change in the reserve ratio on the variable indicated in the column heading, run after partialing out the initial reserve ratio, year fixed effects and the initial ratio interacted with year fixed effects, divided by the $R^{2}$ of the same regression but including the six explanatory variables. The statistics in Column 7 is calculated as one minus the sum of the other columns.

|  | Share of cross-sectional variance of $h$-year change in reserves explained by: |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Syst. asset return |  | Contract return | Insurer profit, costs, fees | Reins. | Account value growth | Covariance terms |
|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| $h=1$ year | 0.11 | 0.56 | $<0.01$ | 0.07 | $<0.01$ | 0.02 | 0.23 |
| $h=2$ years | 0.08 | 0.49 | 0.01 | 0.07 | 0.01 | 0.03 | 0.31 |
| $h=3$ years | 0.08 | 0.52 | 0.01 | 0.08 | 0.02 | 0.04 | 0.25 |
| $h=4$ years | 0.12 | 0.54 | 0.01 | 0.12 | 0.01 | 0.04 | 0.16 |
| $h=5$ years | 0.09 | 0.52 | $<0.01$ | 0.14 | 0.01 | 0.04 | 0.19 |
| $h=6$ years | 0.08 | 0.53 | $<0.01$ | 0.17 | $<0.01$ | 0.04 | 0.17 |
| $h=7$ years | 0.05 | 0.51 | $<0.01$ | 0.21 | $<0.01$ | 0.05 | 0.17 |

## H Taxes

Tax treatment of euro contracts Contract returns are automatically reinvested in the contract and are not taxable until cash is withdrawn. When an individual withdraws cash, contract returns associated with the withdrawal are taxable as capital income. The French tax system for capital income has a two-tier structure. The first tier is social security contributions, which is a flat tax on capital income whose rate has progressively increased from $10 \%$ in 1999 to $15.5 \%$ in 2015. The second tier is the income tax. Households can either include capital income in their taxable income, in which case it is taxed at the marginal income tax rate (between $0 \%$ and $45 \%$ depending on total taxable income and household size). Or they can choose to pay a flat withholding tax, whose rate depends on the savings vehicle. The withholding tax rate has been in the range $16 \%-19 \%$ for directly held stocks and mutual funds over 2004-2015. For euro contracts and unit-linked contracts, the withholding tax rate depends on the holding period of the contract: $35 \%$ if less than four years; $15 \%$ between four and eight years; $7.5 \%$ with a tax allowance of 4,600 euros if more than eight years. The withholding tax is the most favorable option for the majority of households (at least in value-weighted terms).

Tax cost of switching insurer The tax system creates a tax cost of switching insurer for two reasons. First, contract returns are taxed upon withdrawals. Therefore, switching contracts moves the tax bill forward in time, which increases the present value of the tax bill. Second, the tax rate is a (non-continuously) decreasing function of contract age upon withdrawal. Therefore, switching contract increases the applicable tax rate by resetting the tax clock. The total tax loss of switching contract depends on how long the investor has held the initial contract and how long she will hold the new contract. To quantify the tax cost of switching insurer, consider an investor who has been holding a contract for $m$ years and has a contract value of one euro in year $t$, i.e., she invested $(1+y)^{-m}$ euro in year $t-m$. We calculate the year $t$-present value of the tax bill in the following two scenarios: (1) she holds the contract for another $n$ years; (2) she switches to another insurer and holds the new contract for $n$ years. Returns are taxed upon withdrawals and the tax rate depends on the age of the contract at the time of withdrawals. During the sample period, the tax rate for a $k$ year old contract is $\tau(k)=35 \%$ if $k$ is less than four years; $\tau(k)=15 \%$ if $k$ is between four and eight years; and $\tau(k)=7.5 \%$ if $k$ is more than eight years. In scenario (1), the tax bill is

$$
\begin{equation*}
\tau(m+n)\left[(1+y)^{n}-(1+y)^{-m}\right] \quad \text { in year } t+n \tag{H.1}
\end{equation*}
$$

In scenario (2), the tax bill is

$$
\begin{cases}\tau(m)\left[1-(1+y)^{-m}\right] & \text { in year } t  \tag{H.2}\\ \tau(n)\left[(1+y)^{n}-1\right] & \text { in year } t+n\end{cases}
$$

The tax cost of switching insurer is the year $t$-present value of (H.2) minus that of (H.1). This tax cost is plotted in Figure H. 2 as a function of $n$, for $m \in\{0,4,8\}$. We compare the tax cost of switching insurer to the gain from switching from an insurer with a low reserve ratio $\mathcal{R}_{L}$ to an insurer with a higher reserve ratio $\mathcal{R}_{H}>\mathcal{R}_{L}$. The gain is calculated as present value of additional contract returns obtained by switching to the high reserve contract. We discount the expected return difference between the two contracts at the risk-free rate, because the market beta of the long high-reserves/short low-reserves portfolio is zero (Table 7). Using that reserves are distributed to investors at a rate of $\frac{\partial y}{\partial \mathcal{R}} \simeq 3 \%$ per year (Columns $1-2$ of Table 3) and that the reserve ratio decays at rate $\delta \simeq 5.4 \%$ per year (Appendix F ), the present value for an investment of $n$ years is

$$
\begin{equation*}
P V(n)=\frac{\partial y}{\partial \mathcal{R}} \times\left(\mathcal{R}_{H}-\mathcal{R}_{L}\right) \times \frac{1-\left(1-r_{f}-\delta\right)^{n}}{r_{f}+\delta} . \tag{H.3}
\end{equation*}
$$

The present value is evaluated at the sample standard deviation of the reserve ratio $\mathcal{R}_{H}-\mathcal{R}_{L}=0.068$ using $r_{f}=3 \%$. It is plotted in Figure H. 2 as a function of the investment horizon $n$.

Figure H.2: Tax Cost of Switching Contract. The figure plots the tax losses of switching from an insurer with low reserves to an insurer with high reserves as a function of the remaining holding periods. The solid blue line is the present value of expected additional returns. The dashed red (orange) line is the present value of the tax loss for an investor who has held her previous contract for eight (four) years. The dashed green line is the present value of the tax loss for an investor who does not already hold a contract.


Two main results can be taken away from Figure H.2. First, new investors (yellow line) face no tax distortions and thus should always select contracts with higher reserves. Second, for investors already holding a contract for four yours (orange line) or eight years (red line), the tax loss outweighs the gains from predictability (dashed blue line) if investors plan to liquidate their investment within eight years whereas the gain outweighs the loss if investors plan to invest for another eight years or more. Given that the average holding period is twelve years, the majority of investors already holding a contract should find switching contract to be profitable. Thus, tax distortions do not seem qualitatively large enough to explain inelastic flows even for investors already holding a contract.


[^0]:    *HEC Paris and CEPR, France
    ${ }^{\dagger}$ The Ohio State University, USA

[^1]:    ${ }^{1}$ Another regulation imposes that insurers must distribute the funds credited to the profit-sharing reserve to investors within eight years. This implies that insurers can hoard up to eight years worth of contract returns in the profit-sharing reserve. In practice, this constraint is never binding. The profit-sharing reserve represents less than one year of contract returns on average, and two years and a half at the 99th percentile.
    ${ }^{2}$ While unrealized capital gains are not booked as financial income, there exist two deviations from historical cost accounting principles that force insurers to recognize large unrealized losses. First, when an asset has "lasting and significant" unrealized capital losses, its book value is partially adjusted downwards through the creation of a provision

[^2]:    on the asset side of the balance sheet (provision pour dépréciation durable) to reflect the paper loss. This adjustment is booked as a realized loss. It thus increases unrealized gains (makes them less negative). If the return credited to investors' accounts and to the insurer's profit are held constant, this realized loss reduces the profit-sharing reserve, and total reserves are not affected. The goal of this provision is to induce insurers to reduce the return credited to investors' accounts and thus to reduce the profit-sharing reserve by less than the realized loss, increasing total reserves. The second deviation from historical cost accounting is that, when the market value of the portfolio of non-fixed income securities is less than the book value, the overall paper loss is recognized through a provision on the liability side of the balance sheet (provision pour risque d'exigibilité). This is booked as a loss. Therefore, if the return credited to investors' accounts and to the insurer's profit are held constant, this reduces the profit-sharing reserve and thus total reserves. The goal of this provision is to induce insurers to reduce the return credited to investors' accounts and thus to offset the reduction in the amount of reserves.

[^3]:    ${ }^{3}$ The regulated rate plotted on Figure E. 2 is free of fees and taxes, whereas the contract return is before fees and taxes, so the level of the return differential between euro contracts and Livret A is smaller than suggested by the figure.

[^4]:    ${ }^{4}$ See, e.g., https://www.insuranceinsider.com/article/28761kosd8azapmonzv9c/groupama-on-review-as-sovereign-debt-fears-continue

